ARC Quiz 2

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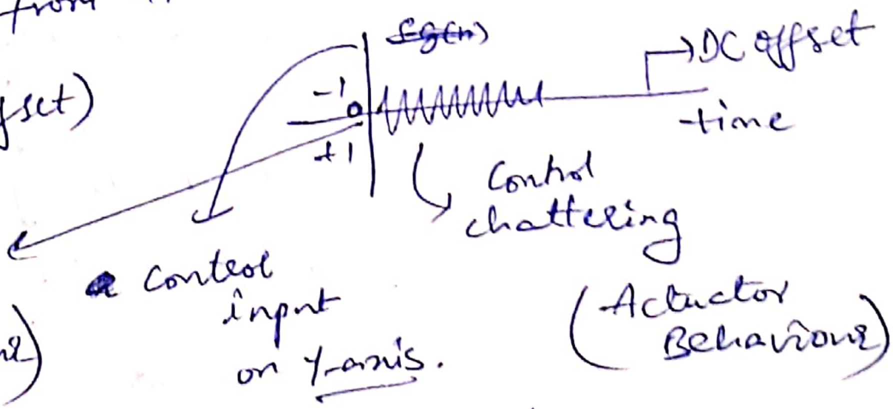
To answer (a), (d), (g), (j) parts

(3) Q. (a) Chattering:

→ It generally occurs in high frequency control. The type of ~~input~~ control input that we choose for this kind of control is generally of the form $u = -kx - \theta^* sgn(x)$ (or) $u = -k\theta^* sgn(x)$. It basically involves the signum f^n which is, discontinuous in nature.

$$sgn(x) = \frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$$

→ If frequency is high, there will be a lot of switching from +1 to -1 over the dc component (or the offset)

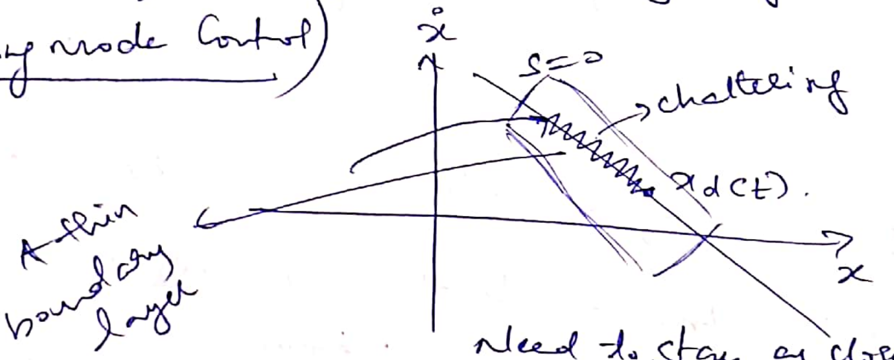


→ This switching is not suitable in many applications & actuator might wear off from heat. This is called control chattering.

To reduce chattering, in a crude sense ~~a broader idea~~ we can use high gain controller but that controller has its own disadvantages like Actuator saturation ($u = -kx$) & UUB stability.

→ Chattering is undesirable ~~due~~ basically due to high control activity. It can also excite the high frequency dynamics which might have been neglected in the modelling.

→ Chattering can be reduced by smoothing out the control discontinuity in a thin Boundary layer neighbouring the switching surface in (Sliding mode Control)



Need to stay as close to $s=0$ as possible.
leads to trade off b/w Control Bandwidth & Tracking precision.
(parametric uncertainity) (performance).

→ we can ~~use~~ use smooth interpolations & ~~approx~~ reduce or remove the discontinuities in the control ip, which will inturn reduce the high switching properties of a high freq. Controller.

~~long as there are no ... for~~

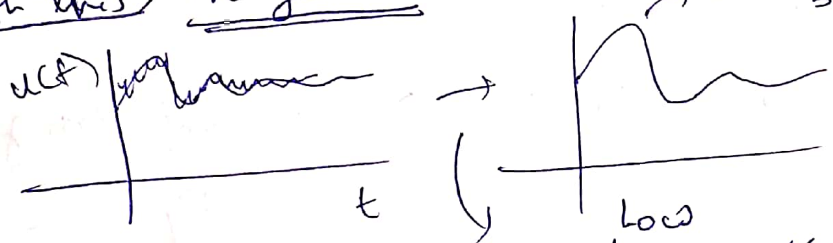
→ chattering can also be eliminated as long as high-freq. unmodelled dynamics are not excited.

→ In smooth. interpolating, we can replace ~~sgn(s)~~ signum by say a function $\frac{s}{\phi}$

s → sliding mode parameter

ϕ → Boundary layer thickness

~~with this~~ In general:



→ made smooth by smooth interpolation

Using a ~~high~~ Low pass filter

Low ~~pass~~ pass filter → basically removes noise which is generally at higher freq than other freq components of the signal.

Allows only freqs upto a certain cut off frequency.

→ Basically filtered tracking error (eg. Sliding mode)

$$R(s) = s\,e(s) + \lambda e(s)$$

$$\Rightarrow \frac{e(s)}{R(s)} = \frac{1}{s+\lambda}$$

① Given,

EL system $M\ddot{q} + H = \tau$ ↳ control input.

q ↳ position

$e = q - q_d$ ↳ desired

To develop a sliding mode controller for this system,

[(a) $r = q$, (d) $r = e$.]

$r \rightarrow$ filter tracking error (or) sliding manifold.

Lets consider H as,

$H = C\dot{q} + F(\dot{q}) + G(q)$ without loss of generality.

C ↳ $C(q, \dot{q})$

$r = q$ — $q = e + q_d$ — q_d given & known

laplace transform on b/s,

writing this in terms of error variable,

$r = e + q_d$

$\rightarrow R(s) = e(s) + \frac{q_d}{s}$

$\dot{r} = \dot{e} \Rightarrow \dot{r} = \dot{q}$ — ① $M\dot{r} = M\dot{q}$

$M\ddot{q} + C\dot{q} + F + G = \tau$ — ② $M\ddot{r} = M\ddot{q}$

$M\ddot{r} = \tau - C\dot{q} - F - G = \tau - C\dot{r} - F - G$.

Consider $V = \frac{1}{2} r^T M r$

$\dot{V} = r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r$.

$\dot{V} = r^T [\cancel{q} M\dot{q}] + \frac{1}{2} r^T \dot{M} r$

③ (a) $V = q$. assuming q is the only ~~[illegible]~~ state variable,

$q = 0 \Rightarrow V = 0$

$q > 0 \Rightarrow V > 0$

$\therefore$ It is positive definite

$\dot{V} = \dot{q}$ ↳ Not guaranteed to be NSD.

→ It is radially unbounded & continuously differentiable

$\therefore$ Not ~~[illegible]~~ suitable for lyapunov fn candidate.

if $\dot{q}$ is guaranteed to be $\dot{q} \leq 0 \ \forall t$ (or everywhere), then ~~[illegible]~~ $V = q$ can be chosen as a lyapunov fn..

(~~b~~ d) $V = e^2$

$V = 0$ if $e = 0$.

~~[illegible]~~ $V > 0$ if $e > 0$ & $e < 0$.

$\therefore V$ is PD.

$\dot{V} = 2e\dot{e}$

$\dot{V} = e\dot{e}$ is not guaranteed to be NSD.

Also we dont know if it is continuosly differentiable

It ~~[illegible]~~ radially unbounded may or may not be

$\therefore$ Unless $e\dot{e} \leq 0$ everywhere, ~~[illegible]~~ $V = e^2$ is not a suitable lyapunov function.

(g) $(\dot{e}+e)^2 = V$

$V = \dot{e}^2 + 2\dot{e}e + e^2$.

$V(e, \dot{e})$

always ≥ 0.

if $e = -\dot{e}$ ie, & $\dot{e} = -e$

$V = 0$.

$\dot{V} = 2(\dot{e}+e)(\ddot{e}+\dot{e})$

↳ -ve semi definite.

It is possible for $e = 0$ & $\dot{e} = 0$ & some other combination also.

∴ V is not Positive definite

It is positive semi definite.

It is radially unbounded

$V(x) \to \infty$ as $\|x\| \to \infty$.

(applied same for ques as well)

It may or may not be continuosly differentiable

it depends on e(t) & its differentiability.

∴ It is not a suitable lyapunov in any case

unless V can be guaranteed to be PD.

~~made~~ continuosly differentiable

assumed

(j) $\log(e+\dot{e}) = V$

$\dot{V} = \frac{1}{e+\dot{e}} (\dot{e}+\ddot{e})$

↳ zero for other

Combinations of $\dot{e}, \ddot{e}, e$. Sometimes undefined. $\therefore \dot{V}$ not NSD

$$V = \log(e + \dot{e})$$

↳ Not positive Definite

$V < 0$ also in some cases

ie when $(e + \dot{e}) < 1$.

$\therefore$ Not a suitable lyapunov candidate.

It is not radially unbounded also.

$C_?(V)$ it is continuosly differentiable.

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① Continuation

$$\dot{V} = r^T M \dot{q} + \frac{1}{2} r^T \dot{M} r.$$

we need design a τ for which $\dot{V} < 0$ is achievable.
or $\dot{V} \leq 0$

ie which ensures lyapunov stability

$$\dot{V} = r^T \left[M \dot{q} + \frac{1}{2} \dot{M} r \right]$$

↳