TFA - Assignment 2

① $s(t) = \left(\frac{2}{\pi}\right)^{1/4} e^{-10t^2 + 5jt^2 + 100jt}$

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} \left| \left(\frac{2}{\pi}\right)^{1/4} e^{-10t^2 + j(5t^2 + 100t)} \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{2}{\pi}\right)^{1/2} e^{-20t^2} \cdot dt \qquad \left[\because |e^{j\theta}| = 1\right]$$

$$= \left(\frac{2}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} e^{-20t^2} dt$$

We Know that $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$

$$\therefore E = \left(\frac{2}{\pi}\right)^{1/2} \cdot \sqrt{\frac{\pi}{20}} = \frac{1}{\sqrt{10}} \quad \text{— ①}$$

Now $\langle \omega \rangle = \dfrac{\int_{-\infty}^{\infty} \omega |s(\omega)|^2 d\omega}{E}$

But the given signal $s(t)$ is quite complex and calculating $s(\omega)$ will be tough.

$\therefore$ from Leon Cohen T.B, (Page 9)

$$\langle \omega \rangle = \frac{1}{E} \int s^*(t) \cdot \frac{1}{j} \frac{d}{dt} s(t) dt.$$

$$\therefore \frac{ds(t)}{dt} = \left(\frac{2}{\pi}\right)^{1/4} \cdot e^{-10t^2+5jt^2+100jt} \cdot (-20t+10jt+100j)$$

$$\frac{1}{j}\frac{ds(t)}{dt} = \frac{1}{j}(-20t+j(10t+100))\, s(t)$$

$$= \frac{1}{j}(j^2 20t + j(10t+100))\, s(t)$$

$$= (20jt + (10t+100))\, s(t) \quad \text{— (2)}$$

$$\therefore \langle \omega \rangle = \frac{1}{E}\int s^*(t) \cdot (20jt + 10t + 100)\, s(t) \cdot dt$$

$$= \frac{1}{E}\int (20jt + 10t + 100)\, |s(t)|^2\, dt$$

$\because |s(t)|^2 = s(t)\, s^*(t)$. & from above calculations

$$|s(t)|^2 = \left(\frac{2}{\pi}\right)^{1/2} \cdot e^{-20t^2}$$

$$\therefore \langle \omega \rangle = \frac{1}{E}\int_{-\infty}^{\infty} (20jt + 10t + 100) \left(\frac{2}{\pi}\right)^{1/2} e^{-20t^2} \cdot dt$$

$$\langle \omega \rangle = \frac{1}{E}\int_{-\infty}^{\infty} \left(\frac{2}{\pi}\right)^{1/2} \cdot \left(20jte^{-20t^2} + 10te^{-20t^2} + 100e^{-20t^2}\right) dt$$

$$= \sqrt{10}\left(\frac{2}{\pi}\right)^{1/2} \cdot \int_{-\infty}^{\infty} \cdot 100e^{-20t^2}\, dt \qquad \left[\because te^{-20t^2} = \text{odd function}\right]$$

$$= \sqrt{10}\left(\frac{2}{\pi}\right)^{1/2} \cdot 100 \cdot \sqrt{\frac{\pi}{20}} \qquad \left[E = 1/\sqrt{10} \text{ from (1)}\right]$$

$$\langle \omega \rangle = 100$$

Now $\langle \omega^2 \rangle = \frac{1}{E}\int \omega^2 |s(\omega)|^2 d\omega = \frac{1}{E}\int \left|\frac{d}{dt} s(t)\right|^2 dt.$

↳ from Leon Cohen, TB Page 9.

from (2), $\frac{d\, s(t)}{dt} = (-20t + j(10t+100))\, s(t)$.

$$\therefore \langle \omega^2 \rangle = \frac{1}{E}\int_{-\infty}^{\infty} \left|-20t + j(10t+100)\right|^2 |s(t)|^2\, dt$$

$$\cancel{= \frac{1}{E}\int_{-\infty}^{\infty} \left|(400t^2 - (10t+100)^2) + 40tj(10t+100)\right| |s(t)|^2\, dt}$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} (400t^2 + (10t+100)^2)\, |s(t)|^2\, dt$$

$$\left[\because |a+ib| = \sqrt{a^2+b^2}\right]$$

$$= \left(\frac{1}{\sqrt{10}}\right)^{-1} \int_{-\infty}^{\infty} 400t^2 \left(\frac{2}{\pi}\right)^{1/2} e^{-20t^2} + (10t+100)^2 \sqrt{\frac{2}{\pi}}\, e^{-20t^2}\, dt$$

$$= \sqrt{10}\cdot\sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} 400t^2 \cdot e^{-20t^2} + (100t^2 + 10^4 + 2000t)\, e^{-20t^2}\, dt$$

$$= \sqrt{\frac{20}{\pi}} \int_{-\infty}^{\infty} 400t^2 e^{-20t^2}\, dt + \int_{-\infty}^{\infty} 100t^2 e^{-20t^2}\, dt + \int_{-\infty}^{\infty} 10^4 e^{-20t^2}\, dt$$

$$= \sqrt{\frac{20}{\pi}} \int_{-\infty}^{\infty} 500\, t^2 e^{-20t^2}\, dt + \int_{-\infty}^{\infty} 10^4 e^{-20t^2}\, dt$$

$$= \sqrt{\frac{20}{\pi}} \left[\frac{1}{2}(500)\sqrt{\frac{\pi}{8000}} + 10^4\cdot\sqrt{\frac{\pi}{20}}\right]$$

$$= \sqrt{\frac{20}{\pi}} \left[\frac{25\cancel{0}}{2\cancel{0}}\cdot\sqrt{\frac{\pi}{20}} + 10^4\cdot\sqrt{\frac{\pi}{20}}\right] = 10^4 + 12.5$$

$$= \underline{\underline{10012.5}} = \langle \omega^2 \rangle$$

$$\therefore \sigma_\omega^2 = \langle \omega^2 \rangle - \langle \omega \rangle^2$$

$$= 10012.5 - 10000 = \underline{\underline{12.5}}$$

(2) $s(t) = \left(\frac{3}{\pi}\right)^{1/4}\left[e^{jm\sin\omega_m t - \frac{3t^2}{2}}\right] e^{j\beta t^2/2 + j\omega_0 t}$

$$E = \int_{-\infty}^{\infty} |s(t)|^2\, dt$$

$$= \int_{-\infty}^{\infty} \left|\left(\frac{3}{\pi}\right)^{1/4} \cdot e^{jm\sin\omega_m t - \frac{3t^2}{2}} \cdot e^{j[\beta t^2/2 + \omega_0 t]}\right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{3}{\pi}\right)^{1/2} \cdot e^{-\frac{3t^2}{1}}\, dt = \left(\frac{3}{\pi}\right)^{1/2} \cdot \sqrt{\frac{\pi \cdot 2}{3}}$$

$|s(t)|^2 = \left(\frac{3}{\pi}\right)^{1/2} e^{-\frac{3t^2}{2}}$

$$= \frac{\sqrt{2}}{\sqrt{2}} = 1 \quad \text{—— (1)}$$

$$\langle\omega\rangle = \frac{1}{E}\int s^*(t) \cdot \frac{1}{j}\frac{d}{dt}s(t)\, dt$$

$$\frac{d}{dt}s(t) = \left(\frac{3}{\pi}\right)^{1/4} e^{jm\sin\omega_m t - \frac{3t^2}{2} + j\beta t^2/2 + j\omega_0 t} \cdot$$
$$[jm\cos\omega_m t - 3t + j\beta t + j\omega_0]$$

$$= s(t) \cdot j\left[m\cos\omega_m t - \frac{3t}{j} + \beta t + \omega_0\right] \quad \text{—— (2)}$$

$$\frac{1}{j}\frac{d}{dt}s(t) = s(t)\left[m\cos\omega_m t + j\frac{3t}{1} + \beta t + \omega_0\right].$$

$$\therefore \langle\omega\rangle = \frac{1}{E}\int_{-\infty}^{\infty} s^*(t) \cdot \frac{1}{j}\frac{d}{dt}s(t)\, dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty}\left[m\cos\omega_m t + j3t + \beta t + \omega_0\right] \cdot |s(t)|^2\, dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty}\left[m\cos\omega_m t + j3t + \beta t + \omega_0\right]\left(\frac{3}{\pi}\right)^{1/2} e^{-3t^2/2}\, dt$$

$$= \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{3}{\pi}\right)^{1/2}\left[\int_{-\infty}^{\infty} m\cos\omega_m t\, e^{-\frac{3t^2}{2}} + \omega_0\int_{-\infty}^{\infty} e^{-\frac{3t^2}{1}}\, dt\right]$$

$$\langle \omega \rangle = \omega_0 + \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{3}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} m\cos\omega_m t \, e^{-\frac{3t^2}{12}} dt.$$

$$= \omega_0 + \frac{1}{\sqrt{2}}\left(\frac{3}{\pi}\right)^{1/2} \cdot \sqrt{2}\sqrt{\frac{\pi}{3}} \cdot m\omega_m e^{-\omega_m^2/12}$$

$$\therefore \langle \omega \rangle = \omega_0 + m\omega_m e^{-\frac{\omega_m^2}{12}}$$

$$\langle \omega^2 \rangle = \frac{1}{E}\int \omega^2 |s(\omega)|^2 d\omega \qquad \frac{d}{dt}s(t) = s(t)\cdot j\left(m\cos\omega_m t - \frac{3t}{j} + \beta t + \omega_0\right)$$

[from ②]

$$= \frac{1}{E}\int \left|\frac{d}{dt}s(t)\right|^2 dt$$

$$= \frac{1}{E}\int |s(t)|^2 \cdot \left(m\cos\omega_m t + j3t + \beta t + \omega_0\right)^2 \cdot dt$$

$$= \frac{1}{E}\left(\frac{3}{\pi}\right)^{1/2} \int e^{-\frac{3t^2}{12}} \left(\cancel{m^2\cos^2\omega_m t} + 9t^2 + (m\cos\omega_m t + \beta t + \omega_0)^2 + 9t^2\right) dt$$

$$= \left(\frac{3}{\pi}\right)^{1/2} \frac{1}{E}\int e^{-\frac{3t^2}{12}}\left[m^2\cos^2\omega_m t + \beta^2 t^2 + \omega_0^2 + \cancel{2m\beta t\cos\omega_m t} + \cancel{2\beta t\omega_0} + 2\omega_0 m\cos\omega_m t + 9t^2\right] dt$$

$$= \left(\frac{3}{\pi}\right)^{1/2} \frac{1}{E}\int e^{-3t^2/4}\left[m^2\cos^2\omega_m t + \omega_0^2 + 2\omega_0 m\cos\omega_m t\right] + (9+\beta^2)t^2 \, dt$$

$$= \left(\frac{3}{\pi}\right)^{1/2} \frac{1}{E}\left[\omega_0^2 \cdot \sqrt{\frac{\pi 2}{3}}\right] + \frac{2m\omega_0}{E}\int \cos\omega_m t \, e^{-3t^2/4} dt + \frac{9+\beta^2}{\sqrt{9 \cdot 2}} + \left(\frac{3}{\pi}\right)^{1/2} \frac{m^2}{E}\int e^{-\frac{3t^2}{12}} \cdot \cos^2\omega_m t \, dt$$

$$\underline{\langle \omega^2 \rangle} = \frac{9+\beta^2}{6} + \frac{m^2\omega_m^2\left(1+e^{-\frac{\omega_m^2}{3}}\right)}{2} + 2m\omega_m\omega_0 e^{-\frac{\omega_m^2}{12}} + \omega_0^2$$

$$\therefore \sigma_\omega^2 = \langle\omega^2\rangle - \langle\omega\rangle^2$$

$$= \frac{a+\beta^2}{6} + \cancel{\omega_0^2} + 2m\omega_m\omega_0 e^{-\frac{\omega_m^2}{12}} + \frac{m^2\omega_m^2}{2}\left(1+e^{-\frac{\omega_m^2}{3}}\right) - \cancel{\omega_0^2} - m^2\omega_m^2 e^{-\frac{\omega_m^2}{6}} - 2m\omega_m e^{-\frac{\omega_m^2}{12}}$$

$$= \frac{a+\beta^2}{6} + \frac{m^2\omega_m^2}{2} + \frac{m^2\omega_m^2}{2} e^{-\frac{\omega_m^2}{3}} + 2m\omega_m\omega_0 e^{-\frac{\omega_m^2}{12}} - 2m\omega_m \omega_0 e^{-\frac{\omega_m^2}{12}} - m^2\omega_m^2 e^{-\frac{\omega_m^2}{6}}.$$

upon solving. $\Rightarrow$

$$\sigma_\omega^2 = \frac{a+\beta^2}{6} + \frac{m^2\omega_m^2}{2}\left(1-e^{-\frac{\omega_m^2}{6}}\right)^2$$

(3) $s(t) = \sqrt{t}\cdot e^{j\phi(t)}$. $\qquad t_1 \le t \le t_2$

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |\sqrt{t}\, e^{j\phi(t)}|^2 dt$$

$$= \int_{-\infty}^{\infty} t\,dt = \int_{t_1}^{t_2} t\,dt = \left(\frac{t^2}{2}\right)_{t_1}^{t_2} = \frac{t_2^2-t_1^2}{2}$$

$$\therefore \langle t \rangle = \frac{1}{E}\int_{-\infty}^{\infty} t \cdot |s(t)|^2 \cdot dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} t \cdot |\sqrt{t}\, e^{j\phi(t)}|^2 dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} t \cdot t \cdot dt$$

$$= \frac{2}{t_2^2 - t_1^2} \cdot \int_{t_1}^{t_2} t^2 dt$$

$$\langle t \rangle = \frac{2}{t_2^2 - t_1^2} \cdot \left[\frac{t_2^3 - t_1^3}{3}\right]$$

$$\langle t \rangle = \frac{2}{3} \frac{(t_2^3 - t_1^3)}{(t_2^2 - t_1^2)}$$

$$\langle t^2 \rangle = \frac{1}{E}\int_{-\infty}^{\infty} t^2 |s(t)|^2 dt$$

$$= \frac{2}{t_2^2 - t_1^2} \cdot \int_{-\infty}^{\infty} t^2 \cdot t \, dt$$

$$= \frac{2}{t_2^2 - t_1^2} \cdot \int_{t_1}^{t_2} t^3 dt$$

$$= \frac{2}{t_2^2 - t_1^2} \cdot \left[\frac{t^4}{4}\right]_{t_1}^{t_2}$$

$$\langle t^2 \rangle = \frac{t_2^4 - t_1^4}{(t_2^2 - t_1^2)} \cdot \frac{1}{2}$$

$$\langle t^2 \rangle = \frac{1}{2}\, t_1^2 + t_2^2$$

$$\sigma_t^2 = \langle t^2 \rangle - \langle t \rangle^2$$

$$= \frac{1}{2}(t_1^2 + t_2^2) - \frac{4}{9}\left(\frac{t_2^3 - t_1^3}{t_2^2 - t_1^2}\right)^2$$

(1)

from ① ⇒

$$\frac{t_2^3 - t_1^3}{t_2^2 - t_1^2} = \frac{\cancel{(t_2 - t_1)}(t_1^2 + t_1 t_2 + t_2^2)}{\cancel{(t_2 - t_1)}(t_2 + t_1)}$$

$$= \frac{(t^2 + t_1 t_2 + t_1^2)}{(t_2 + t_1)}$$

$$\therefore \sigma_t^2 = \frac{1}{2}(t_1^2 + t_2^2) - \frac{4}{9}\frac{(t_2^2 + t_1 t_2 + t_1^2)^2}{(t_1^2 + t_2^2 + t_1 t_2 \cdot 2)}$$

$$= \frac{1}{2}(t_1^2 + t_2^2) - \frac{4}{9}\frac{t_2^4 + t_1^2 t_2^2 + t_1^4 + 2[t_1 t_2^3 + t_1^3 t_2 + t_1^2 t_2^2]}{t_1^2 + t_2^2 + t_1 t_2 \cdot 2}$$

$$= \frac{(9t_1^4 + 9t_1^2 t_2^2 + 18 t_1^3 t_2 + 9 t_1^2 t_2^2 + 9 t_2^4 + 18 t_1 t_2^3) - (8t_2^4 + 8 t_1^2 t_2^2 + 8 t_1^4 + 16 t_1 t_2^3 + 16 t_1^3 t_2 + 16 t_1^2 t_2^2)}{18(t_1 + t_2)^2}$$

$$= \frac{t_1^4 + t_2^4 + t_1^2 t_2^2 + 2[t_1 t_2^3 + t^3 t_2 + t_1^2 t_2^2] - 9 t_1^2 t_2^2}{18(t_1 + t_2)^2}$$

$$= \cancel{\frac{(t_1^2 + t_2^2 + t_1 t_2)^2}{18(t_1 + t_2)^2}} = \frac{(t_1^2 + t_2^2 + t_1 t_2)^2 - (3 t_1 t_2)^2}{18(t_1 + t_2)^2}$$

$$\therefore \cancel{\sigma_t = \frac{1}{3\sqrt{2}}\left(\frac{t_1^2 + t_2^2 + t_1 t_2}{t_1 + t_2}\right)} = \frac{(t_1^2 + t_2^2 + t_1 t_2 - 3 t_1 t_2)(t_1^2 + t_2^2 + 4 t_1 t_2)}{18(t_1 + t_2)^2}$$

$$\therefore \sigma_t = \frac{|t_1 - t_2|\sqrt{t_1^2 + t_2^2 + 4 t_1 t_2}}{3\sqrt{2}(t_1 + t_2)}$$

(4) (a) $s(t) = te^{j\phi(t) + j10t + j\sin\omega_0 t}$ $\quad 1 \le t \le 4$

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} t^2 dt = \left[\frac{t^3}{3}\right]_1^4 = \frac{1}{3}\left[\frac{63}{3}\right] \cdot 3$$

$$= \frac{63 \times 3}{9} = 21$$

$$t_0 = \langle t \rangle = \frac{1}{E}\int_1^4 t\,|s(t)|^2 dt$$

$$= \frac{1}{7}\int_1^4 t \cdot t^2 dt$$

$$= \frac{1}{7}\int_1^4 \cdot t^3 dt = \frac{1}{21}\left[\frac{t^4}{4}\right]_1^4$$

$$= \frac{1}{\cancel{7}}\left[\frac{\cancel{255}}{4}\right] \frac{255}{7}$$

$$= \frac{\cancel{255}\,85}{28 \times \cancel{3}} = \frac{85}{28}$$

$$\langle t^2 \rangle = \frac{1}{7 \times 3}\int_1^4 t^2 \cdot t^2 dt$$

$$= \frac{1}{21}\left[\frac{t^5}{5}\right]_1^4 = \frac{1}{21}\left[\frac{1023}{5}\right]$$

$$= \frac{1023}{35 \times 3}$$

$$\therefore \sigma_t^2 = \frac{1023}{35 \times 3} - \left(\frac{85}{28}\right)^2 = \underline{\underline{0.527}}$$

(b) $s'(t) = s(t-2) = y(t)$

$$\therefore t_{0y} = \langle t \rangle = \frac{1}{E}\int_{-\infty}^{\infty} t\,|y(t)|^2\,dt$$

$$E_y = \int_{-\infty}^{\infty} |y(t)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |s(t-2)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |s(t)|^2\,dt = E_s$$

$$= \underline{\underline{21}}$$

$$t_{0y} = \langle t \rangle_y = \frac{1}{21}\int_{-\infty}^{\infty} t\,|s(t-2)|^2\,dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} t\,|s(t-2)|^2\,dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} (K+2)\,|s(K)|^2\,dK$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} K\,|s(K)|^2\,dK + \frac{2}{E}\int_{-\infty}^{\infty} |s(K)|^2\,dK$$

$$= t_{0s} + \frac{2}{E}\times E$$

$$= t_{0s} + 2$$

$$= \frac{85}{28} + 2 = 5.0357$$

$$\langle t^2 \rangle_y = \frac{1}{E}\int_{-\infty}^{\infty} t^2\,|s(t-2)|^2\,dt$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} (K+2)^2\,|s(K)|^2\,dK$$

$$= \frac{1}{E}\int_{-\infty}^{\infty} K^2\,|s(K)|^2 + \frac{4}{E}\int_{-\infty}^{\infty} K\,|s(K)|^2\,dK + \frac{4}{E}\int_{-\infty}^{\infty} |s(K)|^2\,dK$$

$$\sigma^2_{ty} = \langle t^2\rangle_y - \langle t\rangle_y^2$$

$$\sigma^2_{ty} = \left(\langle t^2\rangle_s + 4\langle t\rangle_s + 4\right) - \left(\langle t\rangle_s^2 + 4\langle t\rangle_s + 4\right)$$

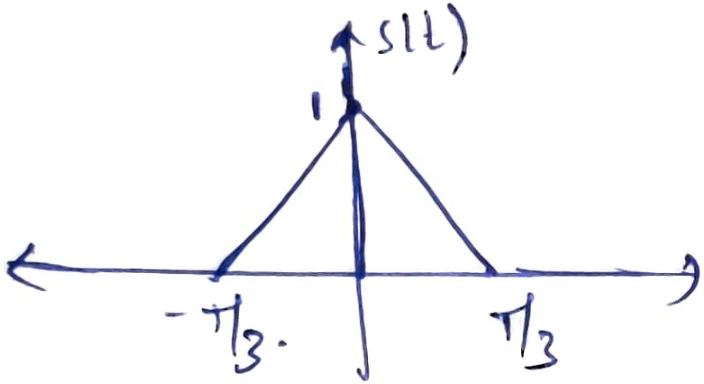
$$= \langle t^2\rangle_s - \langle t\rangle_s^2$$

$$= \sigma^2_{ts}$$

$$= \underline{\underline{0.527}}$$

(5) $s(t) = 1-|t|$, $-\frac{T}{3} \le t \le \frac{T}{3}$

$$s(t) = \begin{cases} 1-t & 0 \le t \le T/3 \\ 1+t & -\frac{T}{3} \le t < 0 \end{cases}$$



from figure above, $t_0 = \boxed{\langle t \rangle = 0}$

$$\sigma_t^2 = \langle t^2 \rangle - \langle t \rangle^2$$

$$= \langle t^2 \rangle$$

$$\frac{1}{E}\int_{-\infty}^{\infty} t^2 |s(t)|^2 dt = \langle t^2 \rangle$$

$$\therefore \langle t^2 \rangle = \frac{1}{E}\int_{-T/3}^{0} t^2 . |1+t|^2 dt + \int_{0}^{T/3} t^2 (1-t)^2 dt$$

$$= \left[\int_{-T/3}^{0} t^2(t^2+1+2t)\, dt + \int_{0}^{T/3} t^2(t^2-2t+1)\, dt\right]\frac{1}{E}$$

$$= \left[\int_{-T/3}^{0} t^4+t^2+2t^3\, dt + \int_{0}^{T/3} t^4-2t^3+t^2\, dt\right]\frac{1}{E}$$

$$= \frac{1}{E}\left[\frac{t^5}{5}+\frac{t^3}{3}+\frac{t^4}{2}\right]_{-T/3}^{0} + \frac{1}{E}\left[\frac{t^5}{5}-\frac{t^4}{2}+\frac{t^3}{3}\right]_{0}^{T/3}$$

$$= \left[0+\frac{T^5}{243\times 5}+\frac{T^3}{81}-\frac{T^4}{162}+\frac{T^5}{243\times 5}-\frac{T^4}{162}+\frac{T^3}{81}\right]\frac{1}{E}$$

$$= \frac{2}{E_t}\left[\frac{T^5}{243\times 5}+\frac{T^3}{81}-\frac{T^4}{162}\right] \quad \text{—— (1)}$$

$$E_t = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

$$= \int_{-T/3}^{0} (t^2+2t+1)dt + \int_{T/3}^{0} (t^2-2t+1)dt$$

$$= \left[\frac{t^3}{3}+\frac{2t^2}{2}+t\right]_{-T/3}^{0} + \left[\frac{t^3}{3}+t-\frac{2t^2}{2}\right]_{0}^{T/3}$$

$$= \left[\frac{T^3}{81}-\frac{T^2}{9}+\frac{T}{3}\right]+\left[\frac{T^3}{81}-\frac{T^2}{9}+\frac{T}{3}\right]$$

$$\boxed{E_t = \frac{2T}{3}\left[\frac{T^2}{27}-\frac{T}{3}+1\right]} \quad \text{—— (2)}$$

Keeping ② in ① →

$$\langle t^2\rangle = \frac{2.3}{2T}\cdot\left[\frac{T^5}{243\times5}+\frac{T^3}{81}-\frac{T^4}{162}\right]\cdot\frac{1}{\left(\frac{T^2}{27}-\frac{T}{3}+1\right)}$$

$$= \frac{3}{T}\cdot T^3\left[\frac{T^2}{243\times5}+\frac{1}{81}-\frac{T}{162}\right]\cdot\frac{1}{\left(\frac{T^2}{27}-\frac{T}{3}+1\right)}$$

$$= \frac{3T^2}{81}\left[\frac{T^2}{15}+1-\frac{T}{2}\right]\frac{1}{\left[\frac{T^2}{27}-\frac{T}{3}+1\right]}$$

$$\boxed{\langle t^2\rangle = \frac{T^2}{27}\left(\frac{T^2/15 - T/2 + 1}{T^2/27 - T/3 + 1}\right)}$$

Now, $\langle \omega\rangle = \int_{-\infty}^{\infty} s^*(t)\cdot\frac{1}{j}\frac{ds(t)}{dt}\,dt\,.$

$$\therefore \frac{ds(t)}{dt} = \begin{cases} -1 & 0\le t\le T/3 \\ 1 & -T/3\le t<0\end{cases}$$

$$\therefore \frac{1}{j}\frac{ds(t)}{dt} = \begin{cases} +\frac{j^2}{j} & 0\le t\le T/3 \\ -j^2/j & -T/3\le t<0\end{cases} = \begin{cases} j & 0\le t\le T/3 \\ -j & -T/3\le t<0\end{cases}$$

$$\therefore \langle\omega\rangle = \frac{1}{E_\omega}\int_{-T/3}^{0} -(1+t)(j)\,dt + (-1)\int_{T/3}^{0}(1-t)(j)\,dt$$

$$= \left[(-j)\left(\frac{t^2}{2}+t\right)_{-T/3}^{0} + j\cdot\left[t-\frac{t^2}{2}\right]_0^{T/3}\right]\frac{1}{E_\omega}$$

$$\langle w\rangle = \left[(-j)\left[0-\frac{T^2}{18}+\frac{T}{3}\right]+(j)\left[\frac{T}{3}-\frac{T^2}{18}\right]\right]\frac{1}{E_w}$$

$$= \left[\left[\frac{T^2}{18}-\frac{T}{3}\right]+\left[\frac{T}{3}-\frac{T^2}{18}\right]\right]\frac{j}{E_w}.$$

$$\boxed{\langle w\rangle = 0}$$

$$\langle w^2\rangle = \frac{1}{E_w}\int_{-\infty}^{\infty}\left|\frac{ds(t)}{dt}\right|^2 dt$$

From parsavel's theorem, $\int_{-\infty}^{\infty}|s(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|S(f)|^2 df$

$$\therefore E_T = \frac{1}{2\pi}\int_{-\infty}|S(f)|^2 df$$

$$\therefore \boxed{E_w = (2\pi)\cdot\left[\frac{2T}{3}\right]\left[\frac{T^2}{27}-\frac{T}{3}+1\right]}$$

$$\therefore \langle w^2\rangle = \frac{3}{4\pi T}\cdot\frac{2\pi}{(T^2/27-T/3+1)}\int_{-\infty}^{\infty}\left|\frac{ds(t)}{dt}\right|^2 dt$$

$$= \frac{3\times 2\pi}{4\pi T\left(\frac{T^2}{27}-\frac{T}{3}+1\right)}\int_{-T/3}^{0}dt+\int_{0}^{T/3}dt$$

$$= \frac{\cancel{3}}{2\cancel{4}\pi\cancel{T}\left(\frac{T^2}{27}-\frac{T}{3}+1\right)}\cdot\frac{2\cancel{T}}{\cancel{3}} = \boxed{\frac{\cancel{2\pi}\quad 1}{2\pi\left(\frac{T^2}{27}-\frac{T}{3}+1\right)} = \langle w^2\rangle}$$

$$\langle w^2\rangle = \frac{2T}{3E_t}$$