Assignment on wavelets

1. Give expression for the Mexihan-hat wavelet and its Fourier transform (specification upto a scale factor acceptable).

2. (a) Show that $f(t) = sinc(t)$ is not absolutely integrable but it is square integrable.

   (b) Evaluate the $\sum_{n \in Z} sinc(t - 0.2n)$

3. (a) Consider N unit vectors in R^2 specified in parametric form as $e_k = \left(cos(\frac{2\pi k}{N}), sin(\frac{2\pi k}{N}\right)$, where $k = 0, 1, 2, \ldots N-1$. Examine the conditions under which they form an orthonormal basis, a Riesz basis and a frame. Compute the frame bounds.

   (b) Show that in a space of finite dimensions N, a frame of $P \geq N$ normalized vectors has frame bounds A and B, which satisfy $A \leq \frac{P}{N} \leq B$.

4. Let $g(t)$ be a window supported over $\left[-\frac{\pi}{\xi_0}, \frac{\pi}{\xi_0}\right]$ prove if $\frac{2\pi}{\xi_0} \sum_{n \in Z} |g(t - nu_0)|^2 = A > 0 \ \forall t \in R$, then $\{g_{n,k}(t) = g(t - nu_0)e^{jk\xi_0 t}, (n, k) \in Z\}$ is a tight frame of $L^2(R)$

5. Prove Plancherel-Parseval identity: $\int_R f(t)g^*(t)dt = \frac{1}{2\pi} \int \hat{f}(\omega)g^*(\omega)d\omega$ from that compute $\int_R \frac{1}{t^2+a^2)(t^2+b^2)} dt$?

6. Show that a scaling function $\phi(t)$ obtained by aggregating wavelets at scales larger than unity satisfies $||\phi(t)|| = 1$

7. Consider the set of integer shifted functions $\{\theta(t - n), n \in Z\}$. Derive the condition on θ if the integer shifted functions have to be mutually orthogonal.

8. (a) Compute the support of n-th order spline '$\beta^{(n)}$-spline' and its Riesz bounds.

   (b) Compute $\sum_{k \in Z} \beta^{(n)}(t - k)$.

9. Let $\phi(t) = \sum_{n \in Z} h(n)\phi(2t - n)$, prove the below

   (a) If $\int \phi(t)dt \neq 0$ then $\sum_{n \in Z} h(n) = \sqrt{2}$

   (b) If $\{\phi(t-k), k \in Z\}$ is an orthonormal set, then $\sum_{n \in Z} h(n)h(n-2k) = \delta(k)$ and $\sum_{n \in Z} h(2n) = \sum_{n \in Z} h(2n+1) = \frac{1}{\sqrt{2}}$

   (c) If $\phi(t)$ has compact support on $0 \leq t \leq N-1$ and if $\{\phi(t-k), k \in z\}$ linear independent, then $h(n)$ has compact support over $0 \leq n \leq N-1$.

10. (a) Explain Daubechies method of constructing compactly supported wavelets?

    (b) Compute the Daubechies scaling and wavelet filters if the wavelet function possess two vanishing moments.