Assignment-2.

Exercise 6.3.

2) Test the consistency of the following system. If they have solutions, solve by Gauss elimination method.

(5) $x_1 + x_2 - x_3 = 1$
$2x_1 + 3x_2 - 3x_3 = 3$
$x_1 - 3x_2 + 3x_3 = 2$

Sol:

Given system

$$x_1 + x_2 - x_3 = 1 \quad -(i)$$
$$2x_1 + 3x_2 - 3x_3 = 3 \quad -(ii)$$
$$x_1 - 3x_2 + 3x_3 = 2 \quad -(iii)$$

Multiplying (i) by 2 and subtracting from (ii)

$$\begin{array}{r} 2x_1 + 3x_2 - 3x_3 = 3 \\ 2x_1 + 2x_2 - 2x_3 = 2 \\ - \quad - \quad + \quad - \\ \hline x_2 - x_3 = 1 \end{array} \quad -(iv)$$

Subtracting (i) from (iii)

$$\begin{array}{r} x_1 - 3x_2 + 3x_3 = 2 \\ x_1 + x_2 - x_3 = 1 \\ - \quad - \quad + \quad - \\ \hline -4x_2 + 4x_3 = 1 \end{array} \quad -(v)$$

Multiplying (iv) by 4 by adding to (v)

$$-4x_2 + 4x_3 = 1$$
$$4x_2 - 4x_3 = 4$$
$$0 \cdot x_3 = 5$$

No value of x_3 satisfy the equation, hence the system is inconsistent & has no solution.

(a) Ans $x - y - 2z = -1$, $2x + y + z = 2$, $3x + 2y + 9z = 4$.

Sol^n.

The system are:

$x - y - 2z = -1$ — (i)

$2x + y + z = 2$ — (ii)

$3x + 2y + 9z = 4$ — (iii)

Multiplying eq^n (i) by 2 and substracting from eq^n (ii)

$$2x + y + z = 2$$
$$2x - 2y - 4z = -2$$
$$- \quad + \quad + \quad +$$
$$3y + 5z = 4 \quad \text{— (iv)}$$

Multiplying eq^n (i) by 3 and substracting from eq^n (iii)

$$3x + 2y + 9z = 4$$
$$3x - 3y - 6z = -3$$
$$- \quad + \quad + \quad +$$
$$5y + 15z = 7 \quad \text{— (v)}$$

Multiplying eq 2 (VII) by $5/3$ and subtracting from (VII)

$$5y + 15z = 7$$
$$5y + \frac{25}{3}z = \frac{20}{3}$$
$$- \quad - \quad -$$
$$\frac{20}{3}z = \frac{1}{3}$$
$$z = \frac{1}{20}$$

System after Gauss elimination.

$$x - y - 2z = -1 \quad - (VI)$$
$$3y + 5z = 4 \quad - (VII)$$
$$z = \frac{1}{20} \quad - (VIII)$$

from (VII)

$$3y + 5z = 4$$
$$3y + 5 \times \frac{1}{20} = 4$$
$$3y = 4 - \frac{1}{4} = \frac{15}{4}$$
$$y = \frac{5}{4}$$

from (vi) $x - y - 2z = -1$

$x - \frac{5}{4} - 2 \times \frac{1}{20} = -1$

$x = \frac{5}{4} + \frac{1}{10} - 1$

$= \frac{5 \times 10 + 1 \times 4 - 40}{40}$

$= \frac{54 - 40}{40} = \frac{7}{20}$

∴ Consistent solutions are $x = \frac{7}{20}$, $y = \frac{5}{4}$ and $z = \frac{1}{20}$

Exercise - 6.4

9) System of linear equation by Guass seidel method

4) $4x_1 + x_2 + x_3 = 7$, $2x_1 - 5x_2 + 2x_3 = 1$, $x_1 - x_2 + 3x_3 = 6$

Sol:-

Rewriting the system as,

$$x_1 = \frac{7 - x_2 - x_3}{4}$$

$$x_2 = \frac{-1 + 2x_1 + 2x_3}{5}$$

$$x_3 = \frac{6 - x_1 + x_2}{3}$$

Initial guess.

$x_1 = 0$, $x_2 = 0$ & and $x_3 = 0$

Iteration I

$$x_1 = \frac{7 - 0 - 0}{4} = 1.7500$$

$$x_2 = \frac{-1 + 2 \times 1.7500 + 2 \times 0}{5}$$

$$= 0.5000$$

$$x_3 = \frac{6 - 1.75 + 0.5}{3}$$

$$= 1.5800$$

Iteration II

$$x_1 = \frac{7 - 0.5 - 1.58}{4} = 1.23$$

$$x_2 = \frac{-1 + 2 \times 1.23 + 2 \times 1.58}{5}$$

$$= 0.92$$

$$x_3 = \frac{6 - 1.23 + 0.92}{3}$$

$$= 1.9000$$

Iteration III

$$x_1 = \frac{7 - 0.92 - 1.9}{4}$$

$$= 1.0500$$

$$x_2 = \frac{-1 + 2 \times 1.05 + 2 \times 1.9}{5}$$

$$= 0.9800$$

$$x_3 = \frac{6 - 1.05 + 0.98}{3}$$

$$= 1.9800$$

Iteration IV

$$x_1 = \frac{7 - 0.98 - 1.98}{4}$$

$$= 1.0200$$

$$x_2 = \frac{-1 + 2x_1 + 2x_3}{5}$$

$$= \frac{-1 + 2 \times 1.018 + 2 \times 1.3298}{5}$$

$$= 0.80 \quad 1.00$$

$$x_3 = \frac{6 - 1.18 \quad 1.01 + 1.00}{3}$$

$$= 2.0000$$

Iteration V

$$x_1 = \frac{7 - 1 - 2}{4} = 1.0000$$

$$x_2 = \frac{-1 + 2 \times 1 + 2 \times 2}{5} = 1.0000$$

$$x_3 = \frac{6 - 1 + 1}{3} = 2.0000$$

Iteration VI

$$x_1 = \frac{7 - 1 - 2}{4} = 1.0000$$

$$x_2 = \frac{-1 + 2 \times 1 + 2 \times 2}{5} = 1.0000$$

$$x_3 = \frac{6 - 1 + 1}{3} = 2.0000$$

∴ Iteration (V) & (VI) are in 4 decimal place &
Soln: $x_1 = 1$
$x_2 = 1$
$x_3 = 2$

16) $4x + y + z = 8$

$2x + 5y + 2z = 3$

$x + 2y + 4z = 11$

Iteration 1

$$x = \frac{8-0-0}{4} = 2.0000$$

$$y = \frac{3-2\times2-2\times0}{5} = -0.2000$$

$$z = \frac{11-2-2\times(-0.2)}{4} = 2.3200$$

Iteration 2

$$x = \frac{8+0.2-2.32}{4} = 1.4700$$

$$y = \frac{3-2\times1.47-2\times2.32}{5} = -0.9200$$

$$z = \frac{11-1.47+2\times0.92}{4} = 2.8400$$

Iteration 3

$$x = \frac{8+0.92-2.84}{4} = 1.5200$$

$$y = \frac{3-2\times1.52-2\times2.84}{5} = -1.1400$$

$$z = \frac{11-1.52+2\times1.14}{4} = 2.9400$$

Iteration 4

$$x = \frac{8+1.14-2.94}{4} = 1.5500$$

$$y = \frac{3 - 2x1.55 - 2x2.94}{5} = -1.200$$

$$z = \frac{11 - 1.55 + 1.2x2}{4} = 2.960$$

Iteration 5

$$x = \frac{8 + x.2 - 2.96}{4} = 1.5600$$

$$y = \frac{3 - 2x1.56 - 2x2.96}{5}$$

$$= -1.2100$$

$$z = \frac{11 - 1.56 + 2x1.21}{4}$$

$$= 2.9700$$

Iteration 6

$$x = \frac{8 + 1.21 - 2.97}{4} = 1.5600$$

$$y = \frac{3 - 2x1.56 - 2x2.97}{5}$$

$$= -1.2100$$

$$z = \frac{11 - 1.56 + 2x1.21}{4}$$

$$= 2.9700$$

Iteration (5) & (6) are same to 4 place of decimal so,

solution $x = 1.56$

$y = -1.21$

$z = 2.97$