Assignment - 1

Use the simplex method to find the optimal solution

a) Max. $Z = 9x_1 + x_2$ subject to $2x_1 + x_2 \le 8$, $4x_1 + 3x_2 \le 18$ and $x_1, x_2 \ge 0$

Soln

⇒ The standard eqn of above LPP are :-

$$2x_1 + x_2 + p = 8$$
$$4x_1 + 3x_2 + q = 18$$
$$-9x_1 - x_2 + z = 0$$

Initial simplex table is,

B.V	x_1	x_2	p	q	z	R.H.S
p	$\boxed{2}$	1	1	0	0	8
q	1	3	0	1	0	18
	$-9\uparrow$	-1	0	0	1	0

Initial solution :- $x_1 = 0$, $x_2 = 0$, $p = 8$, $q = 18$ and $z = 0$

In last row, most '-ve' entry $= -9$

$\therefore$ pivot column $= x_1$

Now,

$8/2 = 4$ and $18/1 = 18$

Smallest non negative ratio $= 8/2$

$\therefore$ pivot element $= 2$

Apply, $R_1 \to \frac{1}{2}R_1$

B.V.	x_1	x_2	p	q	z	R.H.S
x_1	$\boxed{1}$	$1/2$	$1/2$	0	0	4
q	1	3	0	1	0	18
	-9	-1	0	0	1	0

Apply, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 + 9R_1$

B.V.	x_1	x_2	p	q	z	R.H.S
x_1	1	$1/2$	$1/2$	0	0	4
q	0	$5/2$	$-1/2$	1	0	14
	0	$7/2$	$9/2$	0	1	36

No negative entry in last row,
$\therefore$ Maximum achieved.
Max $z = 36$
at $x_1 = 4$ and $x_2 = 0$

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b) Max, $p = 50x_1 + 80x_2$ subject to $x_1 + 2x_2 \leq 32$, $3x_1 + 4x_2 \leq 84$ and $x_1, x_2 \geq 0$.

Sol^n:

The standard equation of above LPP are:-

$$x_1 + 2x_2 + ps = 32$$
$$3x_1 + 4x_2 + qt = 84$$
$$-50x_1 - 80x_2 + p = 0$$

Initial simplex table

B.V	x_1	x_2	s	t	p	R.H.S
s	1	$\boxed{2}$	1	0	0	32
t	3	4	0	1	0	84
	-50	-80	0	0	1	0

Initial soln : $x_1 = 0$, $x_2 = 0$, $s = 32$, $t = 84$ and $p = 0$

Most -ve entry in last row $= -80$

$\therefore$ pivot column $= x_2$

Now, $\frac{32}{2} = 16$ and $\frac{84}{4} = 21$

The smallest non negative ratio is $32/2$.

$\therefore$ pivot element $= 2$

Apply, $R_1 \to \frac{1}{2} R_1$

B.V	x_1	x_2	s	t	p	R.H.S
x_2	1/2	$\boxed{1}$	1/2	0	0	16
t	3	4	0	1	0	84
	-50	-80	0	0	1	0

Apply, $R_2 \to R_2 - 4R_1$ and $R_3 \to R_3 + 80R_1$

B.V	x_1	x_2	s	t	p	R.H.S
x_2	1/2	1	1/2	0	0	16
t	$\boxed{1}$	0	-2	1	0	20
	-10	0	40	0	1	1280

Negative entry exist in last row,
So pivot column $= x_1$

Now,

$\frac{16}{1/2} = 32$ and $\frac{20}{1} = 20$

$\therefore$ The smallest non negative ratio element $= \frac{20}{1}$

$\therefore$ pivot element $= 1$

Apply, $R_1 \to R_1 - \frac{1}{2} R_2$ and $R_3 \to R_3 + 10 R_2$

B.V	x_1	x_2	s	t	p	R.H.S
x_2	0	1	3/2	-1/2	0	6
x_1	1	0	-2	1	0	20
	0	0	20	10	1	1480

No negative entry in last row,
$\therefore$ Maximum achieved
Max $p = 1480$
at $x_1 = 20$ and $x_2 = 6$

c) Max, $z = 3x_1 + 4x_2$ subject to $2x_1 + x_2 \leq 8$,
$x_1 + 3x_2 \leq 15$ and $x_1, x_2 \geq 0$.

Soln:

The standard eqn of above LPP are:

$2x_1 + x_2 + p = 8$

$x_1 + 3x_2 + q = 15$

$-3x_1 - 4x_2 + z = 0$

Initial simplex table.

B.V	x_1	x_2	p	q	z	R.H.S
p	2	1	1	0	0	8
q	1	$\boxed{3}$	0	1	0	15
	-3	-4 ↑	0	0	1	0

Initial soln :- $x_1 = 0$, $x_2 = 0$, $p = 8$, $q = 15$ and $z = 0$

Most '-ve' entry in last row $= -4$

$\therefore$ pivot column $= x_2$

Now, $8/1 = 8$ and $15/3 = 5$

The smallest non negative ratio $= 15/3$

$\therefore$ pivot element $= 3$

$\therefore R_2 \to \frac{1}{3} R_2$

B.V	x_1	x_2	p	q	z	R.H.S
p	2	1	1	0	0	8
x_2	1/3	$\boxed{1}$	0	1/3	0	5
	-3	-4	0	0	1	0

Apply, $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 4R_2$.

B.V	x_1	x_2	p	q	z	R.H.S
p	$\boxed{5/3}$	0	1	-1/3	0	3
x_2	1/3	1	0	1/3	0	5
	-5/3 ↑	0	0	4/3	1	20

Negative entry exist in last row,
$\therefore$ pivot column $= x_1$

Now, $\frac{3}{5/3} = \frac{9}{5} = 1.8$ and $\frac{5}{1/3} = \frac{15}{1} = 15$

The smallest non negative ratio $= \frac{3}{5/3}$
$\therefore$ pivot element $= 5/3$, Apply $R_1 \to 3/5 R_1$

B.V	x_1	x_2	p	q	z	R.H.S
x_1	1	0	$3/5$	$-1/5$	0	$9/5$
x_2	$1/3$	1	0	$1/3$	0	5
	$-5/3$	0	0	$4/3$	1	20

Apply, $R_2 \to R_2 - \frac{1}{3} R_1$ and $R_3 \to R_3 + \frac{5}{3} R_1$

B.V	x_1	x_2	p	q	z	R.H.S
x_1	1	0	$3/5$	$-1/5$	0	$9/5$
x_2	0	1	$-1/5$	$6/15$	0	$22/5$
	0	0	1	1	1	23

No negative entry in last row,
$\therefore$ Maximum achieved

Max $z = 23$

at $x_1 = 9/5$ and $x_2 = 22/5$

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d) Max $f = 15x_1 + 10x_2$ subjected to $2x_1 + x_2 \leq 10$,
$x_1 + 3x_2 \leq 10$, $x_1, x_2 \geq 0$

The standard eq^n of above LPP are:-

$$2x_1 + x_2 + p = 10$$
$$x_1 + 3x_2 + q = 10$$
$$-15x - 10x_2 + f = 0$$

Initial simplex table is.

B.V	x_1	x_2	p	q	f	R.H.S
p	$\boxed{2}$	1	1	0	0	10
q	1	3	0	1	0	10
	$-15\uparrow$	-10	0	0	1	0

Initial sol^n, $x_1 = 0$, $x_2 = 0$, $p = 10$, $q = 10$ & $f = 0$

Most '-ve' element in last row, -15

$\therefore$ pivot column x_1

Now, $10/2 = 5$ and $10/1 = 10$

Smallest non negative ratio $= 10/2$

$\therefore$ pivot element $= 2$

Apply $R_1 \Rightarrow \frac{1}{2} R_1$

B.V	x_1	x_2	p	q	f	R.H.S
x_1	$\boxed{1}$	$1/2$	$1/2$	0	0	5
q	1	3	0	1	0	10
	-15	-10	0	0	1	0

Apply, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 + 15 R_1$

B.V	x_1	x_2	p	q	f	R.H.S
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	5
q	0	$\boxed{\frac{5}{2}}$	$-\frac{1}{2}$	1	0	5
	0	$-\frac{5}{2}$ ↑	$\frac{15}{2}$	0	1	75

Negative entry exist in last row
$\therefore$ pivot column $= x_2$

~~Apply~~ Now, $\frac{5}{\frac{1}{2}} = 10$ and $\frac{5}{\frac{5}{2}} = 2$

$\therefore$ Smallest non negative ratio $= \frac{5}{\frac{5}{2}}$

$\therefore$ pivot element $= \frac{5}{2}$.

Apply, $R_2 \to \frac{2}{5} R_2$.

B.V	x_1	x_2	p	q	f	R.H.S
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	5
x_2	0	$\boxed{1}$	$-\frac{1}{5}$	$\frac{2}{5}$	0	2
	0	$-\frac{5}{2}$	$\frac{15}{2}$	0	1	75

Apply, $R_1 \to R_1 - \frac{1}{2} R_2$ and $R_3 \to R_3 + \frac{5}{2} R_2$

B.V	x_1	x_2	p	q	f	R.H.S
x_1	1	0	$3/5$	$-1/5$	0	4
x_2	0	1	$-1/5$	$2/5$	0	2
	0	0	7	1	1	80

No negative entry in last row,
∴ Maximum achieved.

Max $f = 80$
at $x_1 = 4$ & $x_2 = 2$

(e) Max. $F = x + 7y$ subject to $-x + 2y \leq 8$,
$x - y \leq 4$ & $x, y \geq 0$

The standard eq^n of above LPP are:-

$$-x + 2y + p = 8$$
$$x - y + q = 4$$
$$-x - 7y + f = 0$$

Initial simplex table.

B.V	x_1	y	p	q	f	R.H.S
p	-1	$\boxed{2}$	1	0	0	8
q	1	-1	0	1	0	4
	-1	$-7^{\uparrow}$	0	0	1	0

Initial sol^n :- $x = 0$, $y = 0$ then $p = 8$, $q = 4$ &
$f = 0$

Most '-ve' entry in last row = -7

∴ pivot column = y

Now,

$\frac{8}{2} = 4$ and $\frac{4}{-1} = -4$

∴ Non negative ratio is $\frac{8}{2}$

∴ pivot element = 2

Apply $R_1 \to \frac{1}{2} R_1$

B.V	x	y	p	q	f	R.H.S
y	$-\frac{1}{2}$	$\boxed{1}$	$\frac{1}{2}$	0	0	4
q	1	-1	0	1	0	4
	-1	-7	0	0	1	0

Apply $R_2 \to R_2 + R_1$ and $R_3 \to R_3 + 7R_1$

B.V	x	y	p	q	f	R.H.S
y	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	0	4
q	$\boxed{\frac{1}{2}}$	0	$\frac{1}{2}$	1	0	8
	$-\frac{9}{2}$ ↑	0	$\frac{7}{2}$	0	1	28

Negative entry exist in last row,

∴ pivot column = ~~y~~ x

Now, $\frac{4}{-\frac{1}{2}} = -8$ and $\frac{8}{\frac{1}{2}} = 16$

∴ Non negative ratio = $\frac{8}{\frac{1}{2}}$

$\therefore$ pivot element $= \frac{1}{2}$.

Apply $R_2 \to 2R_2$

B.V	x	y	p	q	f	R.H.S
y	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	0	4
x	$\boxed{1}$	0	1	2	0	16
	$-\frac{9}{2}$	0	$\frac{7}{2}$	0	1	28

Apply $R_1 \to R_1 + \frac{1}{2}R_2$ and $R_3 \to R_3 + \frac{9}{2}R_2$

B.V	x	y	p	q	f	R.H.S
y	0	1	1	1	0	12
x	1	0	1	2	0	16
	0	0	8	9	1	100

No negative entry in last row.

$\therefore$ Max^{m} achieved

Max $f = 100$

at $x = 16$ and $y = 12$

f) Max, $U = 25x + 45y$ subject to

$x + 3y \le 21$, $2x + 3y \le 24$

and $x, y \ge 0$

Standard equation of Lpp are:-

$$x + 3y + p = 21$$
$$2x + 3y + q = 24$$
$$-25x - 45y + U = 0$$

Initial simplex table,

B.V	x	y	p	q	U	R.H.S
p	1	$\boxed{3}$	1	0	0	21
q	2	3	0	1	0	24
	-25	-45	0	0	1	0

Initial soln : $x = 0$, $y = 0$, $p = 21$, $q = 24$ & $U = 0$

Most negative entry in last row $= -45$

$\therefore$ pivot column $= y$

Now,

$$21/3 = 7 \quad \text{and} \quad 24/3 = 8$$

Smallest non negative ratio $= 21/3$

$\therefore$ pivot element $= 3$

$R_1 \to \frac{1}{3} R_1$

B.V	x	y	p	q	U	R.H.S
y	$1/3$	$\boxed{1}$	$1/3$	0	0	7
q	2	3	0	1	0	24
	-25	-45	0	0	1	0

$R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 + 45R_1$

B.V	x	y	p	q	v	R.H.S
y	$\frac{1}{3}$	1	$\frac{1}{3}$	0	0	7
q	$\boxed{1}$	0	−1	1	0	3
	−10↑	0	15	0	1	315

Negative entry exist in last row,

$\therefore$ pivot column = x

Now,

$\frac{7}{1/3} = 21$ and $\frac{3}{1} = 3$

Smallest non negative ratio = $\frac{3}{1}$ =

$\therefore$ pivot element = 1

Apply $R_1 \to R_1 - \frac{1}{3}R_2$ and $R_3 \to R_3 + 10R_2$

B.V	x	y	p	q	v	R.H.S
y	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	6
x	1	0	−1	1	0	3
	0	0	5	10	1	345

No negative entry in last row

$\therefore$ Max v = 345

at x = 3 and y = 6

g) Max. $z = 3x_1 + 5x_2$ subject to $3x_1 + 2x_2 \le 18$
$x_1 \le 4$, $x_2 \le 6$ and $x_1, x_2 \ge 0$

⇒ The standard eq^n of above LPP are:-

$$3x_1 + 2x_2 + p = 18$$
$$x_1 + q = 4$$
$$x_2 + r = 6$$
$$-3x_1 - 5x_2 + z = 0$$

Initial simplex table,

B.V	x_1	x_2	p	q	r	z	RHS
p	3	2	1	0	0	0	18
q	1	0	0	1	0	0	4
r	0	$\boxed{1}$	0	0	1	0	6
	-3	-5↑	0	0	0	1	0

Initial sol^n :- $x_1 = 0$, $x_2 = 0$ then $p = 18$, $q = 4$,
$r = 6$ and $z = 0$

Most negative entry in last row $= -5$
∴ pivot column x_2

Now, $\frac{18}{2} = 9$, $\frac{4}{0} = \infty$, $\frac{6}{1} = 6$

Smallest non negative ratio $= \frac{6}{1}$

∴ pivot element = 1

Apply, $R_1 \to R_1 - 2R_3$, $R_4 \to R_4 + 5R_3$

B.V	x_1	x_2	p	q	r	z	R.H.S
p	$\boxed{3}$	0	1	0	-2	0	10
q	1	0	0	1	0	0	4
x_2	0	1	0	0	1	0	6
	-3^{A}	0	0	0	5	1	30

Negative entry exist in last row,

$\therefore$ pivot column $= x_1$

Now, $\frac{10}{3} = 3.33$, $\frac{4}{1} = 4$, $\frac{6}{0} = \infty$

Smallest non negative ratio $= \frac{10}{3}$.

$\therefore$ pivot element $= 3$

Apply, $R_1 \to \frac{1}{3} R_1$

B.V	x_1	x_2	p	q	r	z	R.H.S
x_1	$\boxed{1}$	0	1/3	0	-2/3	0	10/3
q	1	0	0	1	0	0	4
x_2	0	1	0	0	1	0	6
	-3	0	0	0	5	1	30

Apply $R_2 \to R_2 - R_1$ and $R_4 \to R_4 + 3R_1$

B.V	x_1	x_2	p	q	r	z	R.H.S
x_1	1	0	$1/3$	0	$-2/3$	0	$10/3$
q	0	0	$-1/3$	1	$2/3$	0	$2/3$
x_2	0	1	0	0	1	0	6
	0	0	1	0	3	1	40

No negative entry in last row,

$\therefore$ Max $z = 40$

at $x_1 = 10/3$ & $x_2 = 6$

h) Max. $z = 2x_1 - x_2 + x_3$ subject to $-3x_1 + x_2 + x_3 \le 60$, $x_1 - x_2 + 2x_3 \le 10$ and $x_1 + x_2 - x_3 \le 20$

Soln.

The standard eqn of above LPP are:-

$$-3x_1 + x_2 + x_3 + p = 60$$
$$x_1 - x_2 + 2x_3 + q = 10$$
$$x_1 + x_2 - x_3 + r = 20$$
$$-2x_1 + x_2 - x_3 + z = 0$$

Initial simplex table.

B.V	x_1	x_2	x_3	p	q	r	z	R.H.S
p	-3	1	1	1	0	0	0	60
q	$\boxed{1}$	-1	2	0	1	0	0	10
r	1	1	-1	0	0	1	0	20
	$-2\uparrow$	1	-1	0	0	0	1	0

Most negative element in last row = -2

∴ pivot column = x_1

Now, $\frac{60}{-3} = -30$, $\frac{10}{1} = 10$ and $\frac{20}{1} = 20$

Smallest non negative ratio is $\frac{10}{1}$

∴ pivot element = 1.

Apply, $R_1 \Rightarrow R_1 + 3R_2$, $R_3 \Rightarrow R_3 - R_2$ and $R_4 \Rightarrow R_4 + 2R_2$

B.V	x_1	x_2	x_3	p	q	r	z	R.H.S
p	0	-2	7	1	3	0	0	90
x_1	1	-1	2	0	1	0	0	10
r	0	$\boxed{2}$	-3	0	-1	1	0	10
	0	-1 ↑	3	0	2	0	1	20

Negative entry exist in last row,

∴ pivot column = x_2

Now

$\frac{90}{-2} = -45$, $\frac{10}{-1} = -10$, $\frac{10}{2} = 5$

The non negative ~~rat~~ ratio = $\frac{10}{2}$

∴ pivot element = 2

$R_3 \to \frac{1}{2} R_3$

B.V	x_1	x_2	x_3	p	q	r	z	R.H.S
p	0	-2	7	1	3	0	0	90
x_1	1	-1	2	0	1	0	0	10
x_2	0	$\boxed{1}$	$-3/2$	0	$-1/2$	$\frac{1}{2}$	0	5
	0	-1	3	0	2	0	1	20

Now, Apply $R_1 \to R_1 + 2R_3$, $R_2 \to R_2 + R_3$ and
$R_4 \to R_4 + R_3$

B.V	x_1	x_2	x_3	p	q	r	z	R.H.S
p	0	0	4	1	2	1	0	110
x_1	1	0	$1/2$	0	$1/2$	$1/2$	0	15
x_2	0	1	$-3/2$	0	$-1/2$	$\frac{1}{2}$	0	5
	0	0	$3/2$	0	$3/2$	$\frac{1}{2}$	1	25

No negative entry in last row,
$\therefore$ Maximum achieved
Max $z = 25$
at $x_1 = 15$, $x_2 = 5$ and $x_3 = 0$

(i) Max. $z = x_1 + x_2 + 5x_3$ subject to
$x_1 + 2x_2 \le 10$, $3x_2 + x_3 \le 24$, $x_1, x_2, x_3 \ge 0$

Sol.:
The standard eq^n of above LPP are:-

$$x_1 + 2x_2 + p = 10$$
$$3x_2 + x_3 + q = 24$$
$$-x_1 - x_2 - 5x_3 + z = 0$$

The initial simplex table,

B.V	x_1	x_2	x_3	p	q	z	R.H.S
p	1	2	0	1	0	0	10
q	0	3	$\boxed{1}$	0	1	0	24
	-1	-1	-5	0	0	1	0

Initial soln, $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ then $p = 10$, $q = 24$ and $z = 0$

The smallest negative value in last row $= -5$

$\therefore$ pivot column $= x_3$

Now,

$\frac{10}{0} = \infty$ and $\frac{24}{1} = 24$

Smallest non negative ratio $= \frac{24}{1}$

$\therefore$ pivot element 1

Apply, $R_3 \to R_3 + 5R_2$

B.V	x_1	x_2	x_3	p	q	z	R.H.S
p	$\boxed{1}$	2	0	1	0	0	10
x_3	0	3	1	0	1	0	24
	-1	14	0	0	5	1	120

Negative entry exists in last row,

∴ pivot column = x_1

Now $\frac{10}{1} = 10$ and $\frac{24}{0} = \infty$

∴ Smallest non negative ratio $= \frac{10}{1} = 10$

∴ pivot element = 1

Apply: $R_3 \to R_3 + R_1$

B.V	x_1	x_2	x_3	p	q	z	R.H.S
x_1	1	2	0	1	6	0	10
x_3	0	3	1	0	1	0	24
	0	16	0	1	5	1	130

No negative entry in last row,

∴ Maximum achieved,

Max $Z = 130$

at $x_1 = 10, x_2 = 0$ & $x_3 = 24$.

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2. Find the optimal solution of the following LPP.

(a) Min. $C = x + 4y$ subject to $x + 2y \geq 8$, $3x + 2y \geq 12$ and $x, y \geq 0$.

Sol^n.

Given, $C = x + 4y$
st :
$$x + 2y \geq 8$$
$$3x + 2y \geq 12$$
Augmented matrix,

$$A = \left(\begin{array}{cc|c} 1 & 2 & 8 \\ 3 & 2 & 12 \\ \hline 1 & 4 & 0 \end{array}\right)$$

Transpose :-

$$\begin{array}{c} \\ A^T = \\ \\ p \end{array} \begin{array}{c} \begin{array}{ccc} u & v & \end{array} \\ \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & 2 & 4 \\ \hline 8 & 12 & 0 \end{array}\right) \end{array}$$

Dual maximize problem,
$$\text{Max } p = 8u + 12v$$
$$\text{st : } u + 3v \leq 1$$
$$2u + 2v \leq 4$$
$$u, v \geq 0$$
Standard eq^n are :-

$$u + 3v + x = 1$$
$$2u + 2v + y = 4$$
$$-8u - 12v + p = 0$$

Initial simplex table :-

B.V	u	v	x	y	p	R.H.S
	1	$\boxed{3}$	1	0	0	1
	2	2	0	1	0	4
	-8	$-12^{\uparrow}$	0	0	1	0

Initial sol^n, $u = 0, v = 0, x = 1, y = 4$ and $p = 0$

Smallest negative number in last row $= -12$

$\therefore$ pivot column $= v$

Now, $1/3 = 0.33$, $4/2 = 2$

Smallest non negative ratio is $1/3$

$\therefore$ pivot element $= 3$

Apply $R_1 \to \frac{1}{3} R_1$

B.V	u	v	x	y	p	R.H.S
	$1/3$	$\boxed{1}$	$1/3$	0	0	$1/3$
	2	2	0	1	0	4
	-8	-12	0	0	1	0

Apply, $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 12R_1$

B.V	u	v	x	y	p	R.H.S
	$\boxed{1/3}$	1	$1/3$	0	0	$1/3$
	$4/3$	0	$-2/3$	1	0	$10/3$
	-4	0	4	0	1	4

Negative entry exists in last row,

$\therefore$ pivot column $= u$

Now, $\frac{\frac{1}{3}}{\frac{1}{3}} = 1$, $\frac{\frac{10}{3}}{\frac{4}{3}} = 2.5$

Smallest non negative ratio is $\frac{\frac{1}{3}}{\frac{1}{3}}$

$\therefore$ pivot element $= \frac{1}{3}$.

Apply, $R_1 \to 3R_1$

B.V	u	v	x	y	p	R.H.S
	$\boxed{1}$	3	1	0	0	1
	$\frac{4}{3}$	0	$-\frac{2}{3}$	1	0	$\frac{10}{3}$
	-4	0	4	0	1	4

$R_2 \to R_2 - \frac{4}{3} R_1$ and $R_3 \to R_3 + 4R_1$

B.V	u	v	x	y	p	R.H.S
	1	3	1	0	0	1
	0	-4	-2	1	0	2
	0	12	8	0	1	8

No negative entry in last row,

Minimize $C : 8$

at $x = 8$ and $y = 0$

(b) Min. $C = 7x_1 + y_2$ subject to $3x - 2y \geq -6$, $x + 3y \geq 15$ and $x, y \geq 0$

Sol 2.

Augmented matrix of above LPP.

$$A = \left(\begin{array}{cc|c} 3 & -2 & -6 \\ 1 & 3 & 15 \\ \hline 7 & 1 & 0 \end{array}\right)$$

Traspose A^T,

$$\begin{array}{cc} & \begin{array}{ccc} u & v & \end{array} \\ & A^T = \left(\begin{array}{cc|c} 3 & 1 & 7 \\ -2 & 3 & 1 \\ \hline -6 & 15 & 0 \end{array}\right) \end{array}$$

Dual Maximize problem,

$$\text{Max } p = -6u + 15v$$

$$\text{st}: \quad 3u + v \leq 7$$

$$-2u + 3v \leq 1$$

standard eqn of above LPP are :-

$$3u + v + x = 7$$

$$-2u + 3v + y = 1$$

$$6u - 15v + p = 0$$

Initial simplex table,

B.V	u	v	x	y	p	R.H.S
	3	1	1	0	0	7
	-2	$\boxed{3}$	0	1	0	1
	6	-15	0	0	1	0

Negative entry exists in last row,

$\therefore$ pivot column $= v$

Now, $7/1 = 7$, $1/3 = 0.33$

Smallest non negative ratio $= 1/3$ 7

$\therefore$ pivot element $= 3$

Apply, $R_2 \to \frac{1}{3} R_2$

B.V	x	y	x	y	p	R.H.S
	3	1	1	0	0	7
	$-\frac{2}{3}$	$\boxed{1}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$
	6	-15	0	0	1	0

Apply; $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 15 R_2$

B V	u	v	x	y	p	R.H.S
	$\boxed{\frac{7}{3}}$	0	1	$-\frac{1}{3}$	0	$\frac{20}{3}$
	$-\frac{2}{3}$	1	0	$\frac{1}{3}$	0	$\frac{1}{3}$
	$-4\uparrow$	0	0	5	1	5

Negative entry exists in last row,

pivot column $= u$

Now, $\dfrac{\frac{20}{3}}{\frac{7}{3}} = \dfrac{20}{7} = 2.86$ and $\dfrac{\frac{1}{3}}{-\frac{2}{3}} = -\frac{1}{2}$

Non negative ratio $= \dfrac{\frac{20}{3}}{\frac{7}{3}}$

$\therefore$ pivot element $= \frac{7}{3}$

$R_1 \to \frac{3}{7} R_1$

B.V	u	v	x	y	p	R.H.S
	$\boxed{1}$	0	$3/7$	$-1/7$	0	$20/7$
	$-2/3$	1	0	$1/3$	0	$1/3$
	-4	0	0	5	1	5

Apply $R_2 \to R_2 + \frac{2}{3} R_1$ and $R_3 \to R_3 + 4R_1$

B.V	u	v	x	y	p	R.H.S
	1	0	$3/7$	$-1/7$	0	$20/7$
	0	1	$2/7$	$\frac{5}{21}$	0	$\frac{47}{21}$
	0	0	$12/7$	$31/7$	1	$\frac{115}{7}$

No negative entry in last row

Minimize $c = \frac{115}{7}$

at $x = 12/7$ and $y = 31/7$

Min. $c = x+y$ subject to $2x + 3y \geq 22$, $2x + y \geq 14$ and $x, y \geq 0$

Sol^n.

Augmented matrix;

$$A = \left(\begin{array}{cc|c} 2 & 3 & 22 \\ 2 & 1 & 14 \\ \hline 1 & 1 & 0 \end{array}\right)$$

$$\text{Transpose}, A^T = \begin{array}{c} \\ \\ \\ p \end{array}\begin{array}{c} \begin{array}{cc|c} u & v & \end{array} \\ \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 3 & 1 & 1 \\ \hline 22 & 14 & 0 \end{array}\right) \end{array}$$

Dual maximize problem.

$$\text{Max}, p = 22u + 14v$$
$$\text{st} : 2u + 2v + x \leq 1$$
$$3u + v + y \leq 1$$

Standard eqn of above LPP.

$$2u + 2v + x = 1$$
$$3u + v + y = 1$$
$$-22u - 14v + p = 0$$

Initial simple table,

B.V	u	v	x	y	p	R.H.S
	2	2	1	0	0	1
	$\boxed{3}$	1	0	1	0	1
	-22 ↑	-14	0	0	1	0

smallest negative number in last row ; -22

$\therefore$ pivot column = u

Now, $\frac{1}{2} = 0.5$ and $\frac{1}{3} = 0.33$

Smallest non negative ratio $\frac{1}{3}$

∴ pivot element 3.

Apply $R_2 \to \frac{1}{3} R_2$

B.V	u	v	x	y	p	R.H.S
	2	2	1	0	0	1
	$\boxed{1}$	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$
	-22	-14	0	0	1	0

$R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 + 22R_2$

B.V	u	v	x	y	p	R.H.S
	0	$\boxed{\frac{4}{3}}$	1	$-\frac{2}{3}$	0	$\frac{1}{3}$
	1	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$
	0	$-\frac{20}{3}$	0	$\frac{22}{3}$	1	$\frac{22}{3}$

Negative entry exist in last row,

∴ pivot column = v

Now $\frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4} = 0.25$, $\frac{\frac{1}{3}}{\frac{1}{3}} = 1$

Smallest non negative ratio is $\frac{\frac{1}{3}}{\frac{4}{3}}$.

∴ pivot element $= \frac{4}{3}$

$R_1 \to \frac{3}{4} R_1$

B.V	u	v	x	y	P	R.H.S
	0	$\boxed{1}$	$3/4$	$-1/2$	0	$1/4$
	1	$1/3$	0	$1/3$	0	$1/3$
	0	$-\frac{20}{3}$	0	$\frac{22}{3}$	1	$22/3$

$R_2 \to R_2 - \frac{1}{3}R_1$ and $R_3 \to R_3 + \frac{20}{3}R_1$

B.V	u	v	x	y	P	R.H.S
	0	1	$3/4$	$-1/2$	0	$1/4$
	1	0	$-1/4$	$1/2$	0	$1/4$
	0	0	5	4	1	9

No negative entry in last row,

$\therefore$ Minimize, $C = 9$

at $x = 5$ and $y = 4$