Exponential Series

Exponential Series is a series which is used to find the value of $\mathbf{e^x}$.
The formula used to express the e^x as Exponential Series is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Expanding the above notation, the formula of Exponential Series is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

For example,

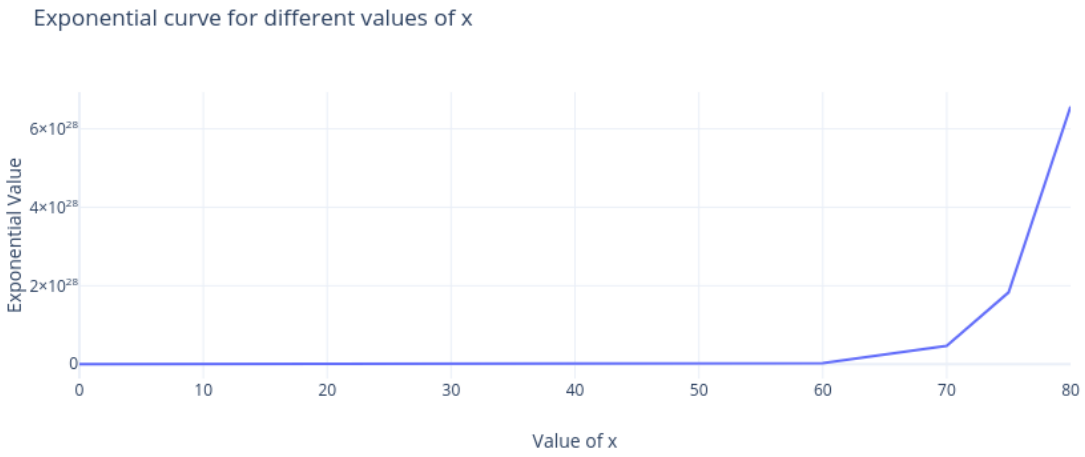
Let the value of **x** be **3**.

$$e^3 = 1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \cdots$$

So, the value of **e3** is 20.0855

Analysis 1: Calculation of exponential function for different values of x

S.no.	x	e^x (Value calculated by Keil)	e^x (Expected value)	Iterations
1	0	1	1	15
2	1	2.71828	2.718282	15
3	5	148.745	148.4132	15
4	10	22611	22026.47	15
5	15	2.43E+08	3269017	15
6	20	1.77E+10	4.85E+08	15
7	25	4.95E+11	7.2E+10	15
8	30	7.54E+12	1.07E+13	15
9	50	1.57E+16	5.18E+21	15
10	60	2.22E+26	1.14E+26	15
11	70	4.68E+27	2.52E+30	20
12	75	1.83E+28	3.73E+32	20
13	80	6.57E+28	5.54E+34	20

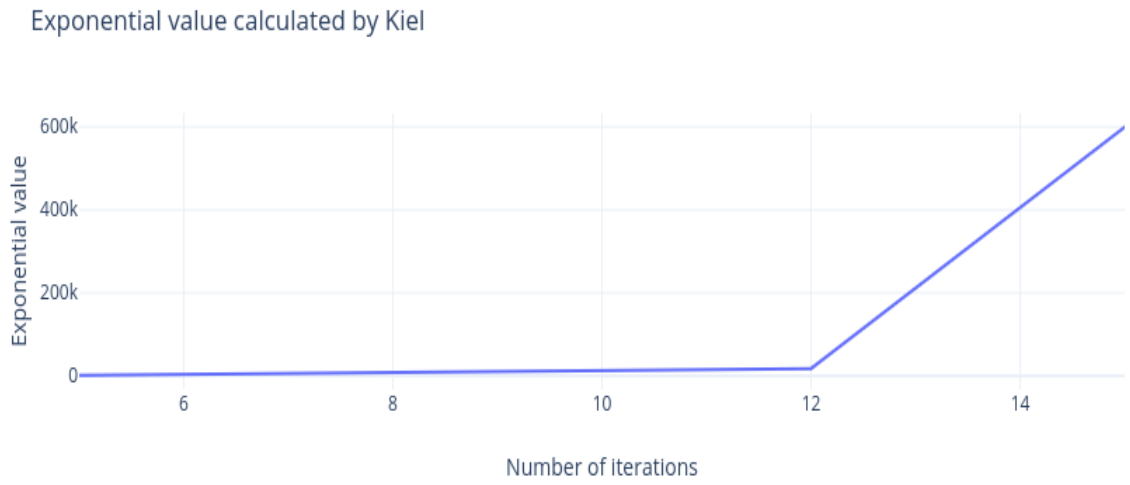


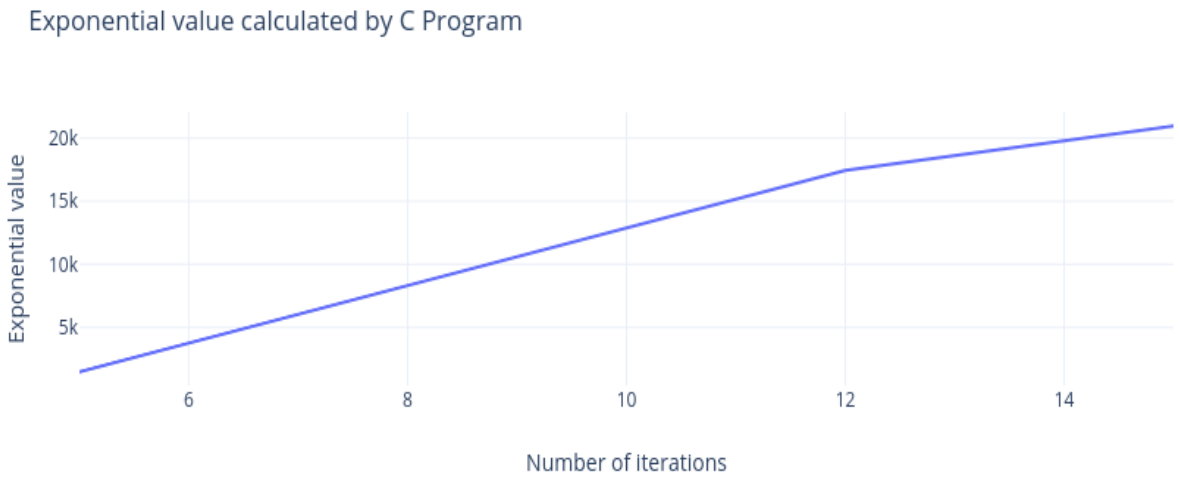
In the above curve, value of exponential function for different values of x has been calculated according the program compiled in Kiel. The curve more or less follows the exponential nature of the function.

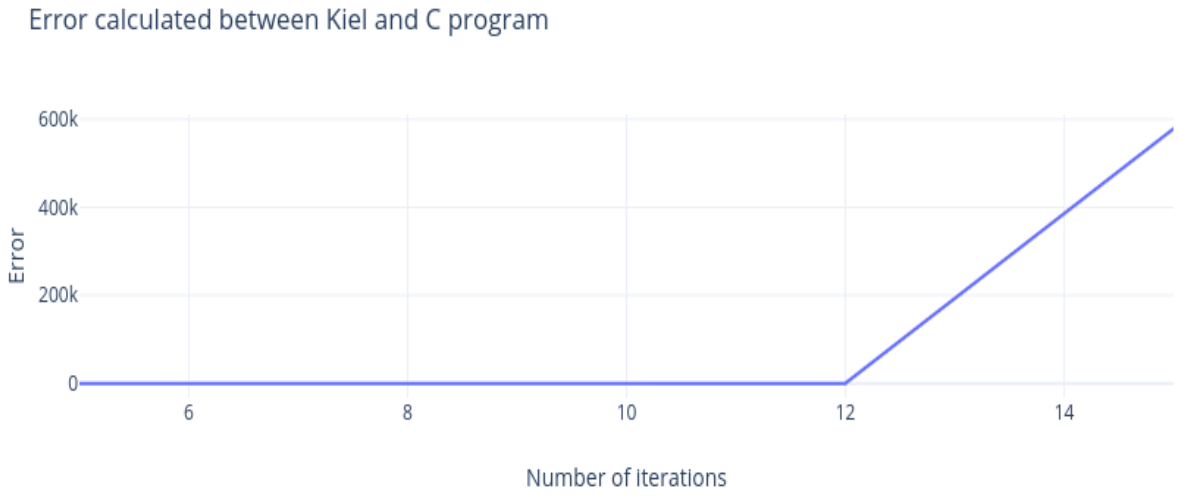
Analysis 2: Deviation of curve from the actual one for different values of iteration

No. of iterations	Value by Kiel	Value by C	Error
5	1477.67	1477.6667	0.0033
10	12842.3	12842.3066	0.0066
12	17435.2	17435.1934	0.0066
15	599725	20952.8887	578772.1

For x=10







The above curves show the deviation of curve from the expected values as the number of iterations varies. The gap is most likely to widen when the number of iterations are going beyond a limit (15 in this case).

Tan(x) series

Given two integers **N** and **X**. The task is to find the sum of **tan(x)** series up to **N** terms.
The series:
x + x3/3 + 2x5/15 + 17x7/315 + 62x9/2835……..

Examples:
Input: N = 6, X = 1
Output: The value from the expansion is 1.55137626113259
Input: N = 4, X = 2
Output: The value from the expansion is 1.52063492063426

In our ARM program, implementation of tan(x) series is done by dividing sine(x) and cosine(x) to achieve the same accurate results with more simplicity.

Sine(x) series by Taylor's expansion is given by:

$$\sin x = \sum_{x=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Expanding the above notation, the formula of Sine Series is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

Cosine(x) series by Taylor's expansion is given by:

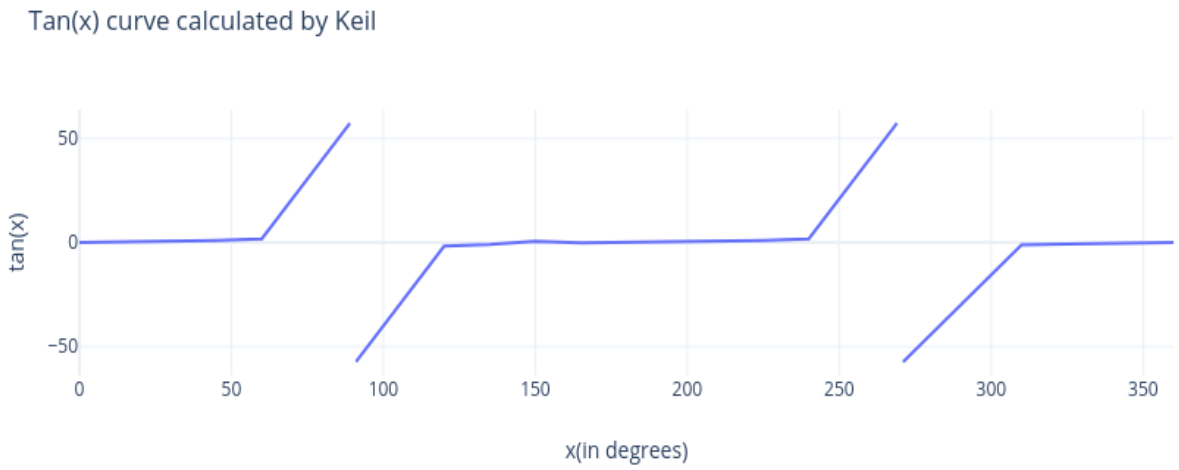
$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

Expanding the above notation, the formula of Cosine Series is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

Analysis 1: Calculation of tangent function for different values of x

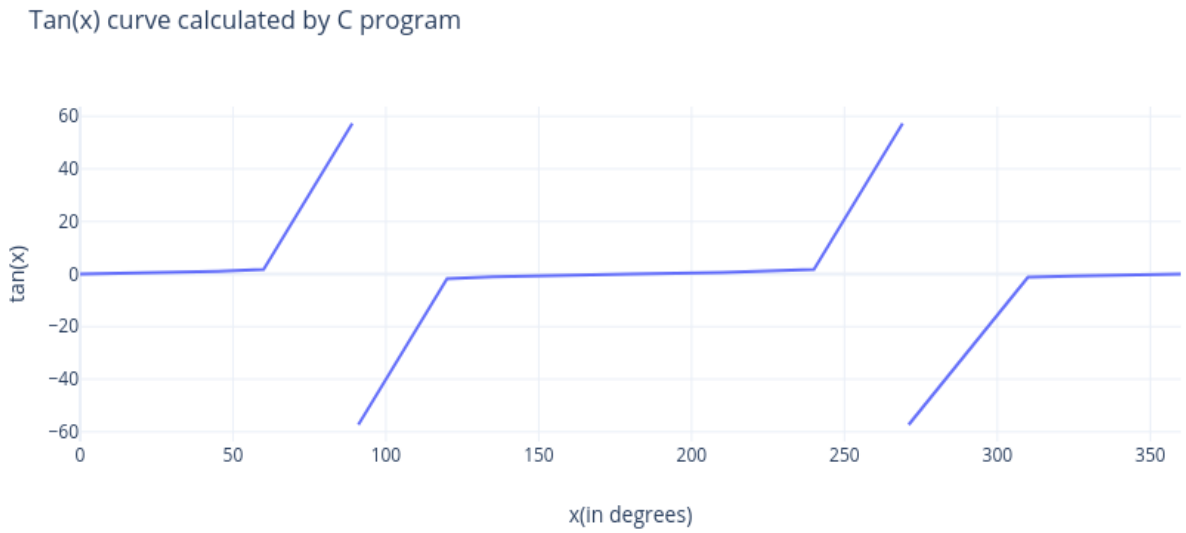
S.no.	x(in degrees)	tan(x) (Value calculated by Keil)
1	0	0
2	30	0.57735
3	45	1
4	60	1.73205
5	89	5.73E+01
6	90	***NaN***
7	91	-5.73E+01
8	120	-1.73E+00
9	135	-1.00E+00
10	150	5.77E-01
11	165	-2.68E-01
12	180	1.36E-06
13	210	0.577352
14	225	1
15	240	1.73206
16	269	57.2995
17	270	***NaN***
18	271	-57.2827
19	310	-1.19172
20	325	-0.700169
21	340	-0.363912
22	360	8.72E-05

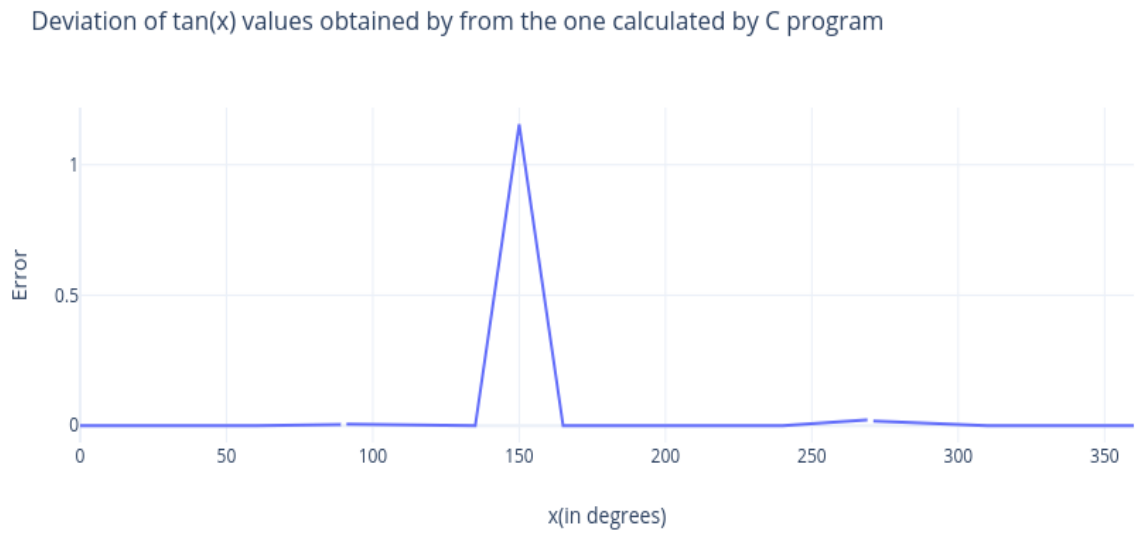


In the above curve, we are observing that the curve follows the ideal tan(x) curve as per the nature of wave shape and values at different angles are to be considered.

Analysis 2: Deviation of curve from the one obtained by C program

S.no.	x(in degrees)	tan(x) (Value calculated by Keil)	tan(x)(Value calculated by C program)	Error
1	0	0	0	0
2	30	0.57735	0.5773	5E-05
3	45	1	1	0
4	60	1.73205	1.732	5E-05
5	89	5.73E+01	57.2858	0.0052
6	90	***NaN***	***NaN***	***NaN***
7	91	-5.73E+01	-57.2945	0.0066
8	120	-1.73E+00	-1.7321	5E-05
9	135	-1.00E+00	-1	2E-06
10	150	5.77E-01	-0.5774	1.154749
11	165	-2.68E-01	-0.268	5.2E-05
12	180	1.36E-06	0	1.36E-06
13	210	0.577352	0.5773	5.2E-05
14	225	1	1	0
15	240	1.73206	1.732	6E-05
16	269	57.2995	57.2771	0.0224
17	270	***NaN***	***NaN***	***NaN***
18	271	-57.2827	-57.301	0.0183
19	310	-1.19172	-1.1917	2E-05
20	325	-0.700169	-0.7002	3.1E-05
21	340	-0.363912	-0.3639	1.2E-05
22	360	8.72E-05	0.0001	1.28E-05





As the figures are self-explanatory, the error obtained is not that significant. The values of tan(x) obtained by Keil much likely follows the actual values which were obtained by C program.