Working Notes

Wednesday, January 25, 2023   10:25 PM

Jan 25

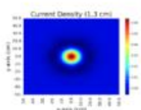
Task list:

- try diff ratios of length and conductivities
- compare actual results to predicted results, quantify error wrt changes in params
- try coding up robust version of optimization
- try hingeplace
- try ROAST

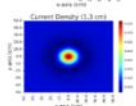
pattern

conds:

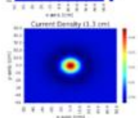
0.5, 1.5, 0.006



0.2, 1.8, 0.01

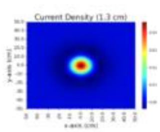


0.33, 2, 0.001

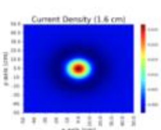


radii

9.2, 0.4, 0.4, 0.1

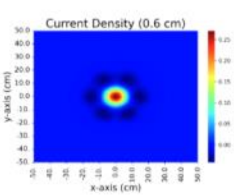


9.5, 0.6, 0.5, 0.1



focus pt not in brain

8.5, 0.4, 0.4, 0.3



Base conductivity vector
(0.33, 1.79, 0.006)

2 → (0.4, 1.79, 0.002)

no change in A above x

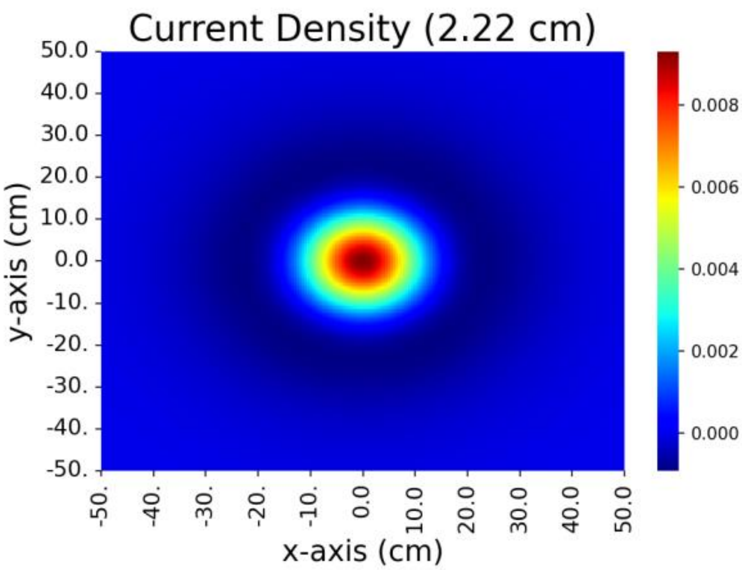
3) cond: 0.4, 1.79, 0.002

radii: 9.5, 0.3, 0.3, 0.2

OG: Area above 80 of the max: [0.52413178 0.02072173 0.] cm^2

3 : Area above 80 of the max: [0.58963182 0.0233113 0.] cm^2

4) perturb each val. in cond/radii by ±10% (choose ± randomly)



← elliptical flattened a bit along x

Area above 80 of the max: [0.809223 0.03199291 0.] cm^2

OG:
sparsePlace.Af
array([[0.02782634, 0.01867485, 0.01871939, 0.01839632, 0.0182149 ,
0.01839795, 0.01872106, 0.00774028, 0.00792383, 0.00784561,
0.00777967, 0.00771063, 0.00764125, 0.00765985, 0.0076417 ,
0.00771141, 0.0077806 , 0.00784642, 0.0079243 , 0.00331592,
0.0033961 , 0.00338746, 0.00337674, 0.00335872, 0.00333304,
0.00331609, 0.00330443, 0.00329883, 0.00328954, 0.00329895,
0.00330465, 0.00331638, 0.00333337, 0.00335906, 0.00337704,
0.00338769, 0.00339622]]

New (10% perturbations)

sparsePlace2.Af
array([[0.07551761, 0.02745501, 0.02729267, 0.02656382, 0.02613097,
0.02656813, 0.02729717, 0.00626817, 0.00636719, 0.00631606,
0.00622396, 0.00615124, 0.0061227 , 0.00608644, 0.00612317,
0.00615205, 0.00622492, 0.00631691, 0.00636768, 0.00210608,
0.00215387, 0.00214771, 0.0021332 , 0.00211993, 0.00211443,
0.00210391, 0.00208351, 0.00208495, 0.00207638, 0.00208502,
0.00208366, 0.00210411, 0.00211466, 0.00212016, 0.00213341,
0.00214787, 0.00215396]]

% diffs:
array([[-171.38894284, -47.01596292, -45.7988948 , -44.39744414,
-43.45930826, -44.40814911, -45.80996188, 19.01889122,
19.64510869, 19.49557205, 19.99712953, 20.22385399,
19.87300866, 20.54094463, 19.87164559, 20.22148866,
19.99434873, 19.49311117, 19.64368619, 36.4856958 ,
36.57804949, 36.59823404, 36.82660024, 36.88290404,
36.56144805, 36.5545215 , 36.94802616, 36.79739976,
36.87953228, 36.7972316 , 36.94771814, 36.55407798,
36.56094266, 36.88241573, 36.8261653 , 36.5979013 ,
36.57787143]]

Af mean difference: .00374
Ac mean difference: .00302

Optimize for: 1) Worst Case

2) Avg case

Define: α chooses random perturbations for input radii/conductivities

$\alpha \in \mathbb{S}^{|radii|+|cond.|}$

Std problem: $\min_x f(x, \alpha)$

1) $\min_x \max_\alpha f$

2) $\min_x \sum_i f(x, \alpha_i) \quad \forall \alpha_i$ (choose n randomly)

Let $\alpha = \{ \alpha_i \in (-.05, .05) : i \in |v| \}$

v = radii vec or cond. vec

$v \leftarrow v(1+\alpha)$

perturbation param

Make a grid in params $\pm$ (Δ) (params)

Feb 15
The hyperrectangle idea likely not gonna work because you'd have to show that
f(x) = f(c) + grad(f) * (x-y), grad(f) > 0 forall

Minmax not convex
Try solving iteratively:

A_{true}, $\tilde{A}$

use $\tilde{A}_1$, solve for I_1

for I_1, use opt or grad descent to get $\tilde{A}_2$ which maximizes error

use $\tilde{A}_2$ to find I_3 . . .

2/21

- try playing w/ opt. settings

- send script to Chaitanya
- save Ac, Af

2/23

making model w/ global

radius-head
focus_depth
conductivities
radii

running sparseplace w/ choices for

per

focus pts
cancel pts

Tdes
Tsafety

Notes old

Wednesday, January 25, 2023 10:40 PM

$TI = E_I,\ T \in \mathbb{R}^{3M \times N},\ E_I \in \mathbb{R}^{3M}$

$\max_{x \in X} \min_{u \in U(x)} f(x,u)$

min worst case I^*

$\min \|A_c I\|_2^2$

$I^* = \arg\min \|A_c I\|_2^2$

$\|I\|_1 \le 2 I_{tot}$

$\|I\|_\infty \le I_{safe}$

$\max_{x \in X, v \in \mathbb{R}} \{v : v \le f(x,u), \forall u \in U(x)\}$

$TI = E$

$\boxed{I^* =} \max_{I} \min_{u} f(I,u)$

$f(x,u) = I^T (T(u))^T \Gamma_c^{-1} f(u) I$

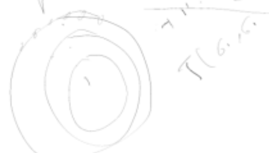
s.t. $\|I\|_1 \le 2 I_{tot}$

$\|I\|_\infty \le I_{safe}$

if we do UB, LB for T deviations,

and decide $T - UB = LB - T$

$\max_{I,u} \{ u \mid u \le f(x,u) \wedge \|I\| \le [2 I_{tot}, I_{safe}]^T \}$



How can we opt. robust to T?

How can we model T?

How do we include info about T's uncertainty?

membrane potential

more general than only ... model

$\frac{V_o}{I} = |K(\omega)|$

$\pm V$

I

Oct / Nov

Started by deriving Tau for N=3

Wrote script for simulating tau under perturbation to conductivities for 3 layers of spherical model

Dec 25

next steps:

- change each conductivity randomly & independently
- try Chaitanya's version of Tau
- find out if there is only a multiplicative factor variation of Tau w.r.t. I or is there mistake in code

Qs

use everything w/ inbuilt conductivity – built this only ... measurements ...

my trend wrt l backwards

good way to see

what's going on as ang. of τ?

change each conductivity independently

change in τ w/ Chris's code?

is there truly only multiplicative factor variation wrt l or mistake?

l → deg of sph harmonic
→ represents spatial frequency
∝ how quickly potential/current changes wrt θ, ϕ @ constant r

$\tau_{l,m}(r)$: (scalp-current density) → (head potential)

Transfer fn ∀ Y_{lm}, decay of V wrt radius

$\tau_{l,m} = \frac{\hat{v}(r)}{\hat{i}_{l,m}}$, $\hat{i}$ injected I density @ surface of sphere

$\hat{V}$ radius → potential @ radius in sphere

$i(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \hat{i}_{lm} Y_{lm}(\theta, \phi)$

$v(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \hat{v}_{lm}(r) Y_{lm}(\theta, \phi)$

$$\hat{i}_{l,m} = \int_0^{\pi} \int_0^{2\pi} i(\theta, \phi) Y_{l,m}(\theta, \phi)\, d\theta\, d\phi$$

$$\hat{V}_{l,m} = \tau_{l,m}(r) \hat{i}_{l,m}$$

?

$$\tau(r) = \frac{r_N}{\sigma_N\left(l-(l+1)\gamma_{lm}^{(N)}\right)}\left(1+\gamma_{lm}^{(s)}\left(\frac{r_s}{r}\right)^{2l+1}\right) \times \left(\frac{r}{r_N}\right)^{l} \prod_{i=s+1}^{N}\left(1+\gamma_{lm}^{(i)}\left(\frac{r_i}{r_{i-1}}\right)^{2l+1}\right) \times \left(1+\gamma_{lm}^{(i-1)}\right)^{-1},\ \forall r_{s-1} \le r \le r_s$$
where $s \in \{1, \ldots, N\}, r_0 = 0$, (13)

$$\gamma_{lm}^{(1)} = 0, \quad (14)$$

$$\zeta_{lm}^{(i)} = \frac{\sigma_i}{\sigma_{i+1}}\left(1-\frac{\gamma_{lm}^{(i)}(l+1)}{l}\right)\left(1+\gamma_{lm}^{(i)}\right)^{-1}, \quad (15)$$

$$\gamma_{lm}^{(i+1)} = \left(\frac{r_i}{r_{i+1}}\right)^{2l+1}\left(1-\zeta_{lm}^{(i)}\right)\left(\frac{l+1}{l}+\zeta_{lm}^{(i)}\right)^{-1}. \quad (16)$$

ex: $\sigma_{scalp} = 1 \frac{S}{m} = \sigma_{brain}$

$\sigma_{skull} = \frac{1}{80} \frac{S}{m}$

$$\tau = \frac{r_N}{\sigma_N\left(l-(l+1)\gamma_{lm}^{(N)}\right)}\left[\ldots\right], \ N = 3$$

$\gamma' = 0$

7%

ex: $\sigma_{scalp} = 1 \frac{S}{m} = \sigma_{brain}$

$\sigma_{skull} = \frac{1}{80} \frac{S}{m}$

$$\tau = \frac{r_N}{\sigma_N\left(l-(l+1)\gamma^{(N)}_{lm}\right)}\left[\ \ldots\ \right], \quad N=3$$

$\gamma^1 = 0$

$$\gamma^2 = \left(\frac{r_1}{r_2}\right)^{2l+1}\frac{\left(1-\delta^{(1)}\right)}{\left(\frac{l+1}{l}+\delta^1\right)}, \quad \delta^1 = \frac{\sigma_1}{\sigma_2}\frac{\left(1-\gamma^{(1)}(\ldots)\right)}{\left(1+\gamma^{(1)}\right)} = \frac{\sigma_1}{\sigma_2}$$

$$\gamma^2 = \left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}$$

$l=1$

$$\gamma^2 = \left(\frac{r_1}{r_2}\right)^3\frac{1-\frac{\sigma_1}{\sigma_2}}{2+\frac{\sigma_1}{\sigma_2}}$$

$l \to \infty$

$\gamma^2 \to \infty$

large l

$$\gamma^s \simeq \frac{\sigma_s - \sigma_{s-1}}{\sigma_s + \sigma_{s-1}}\left(\frac{r_{s-1}}{r_s}\right)^{2l+1}$$

$$\frac{1-\frac{\sigma_{s-1}}{\sigma_s}}{1+\frac{\sigma_{s-1}}{\sigma_s}}$$

$$\gamma^3 = \left(\frac{r_2}{r_3}\right)^{2l+1}\frac{\left(1-\delta^2\right)}{\left(\frac{l+1}{l}+\delta^2\right)}$$

$$\delta^2 = \frac{\sigma_2}{\sigma_3}\frac{\left(1-\frac{\gamma^2(l+1)}{l}\right)}{1+\gamma^2} = \frac{\sigma_2}{\sigma_3}\frac{\left(1-\left[\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}\right]\frac{l+1}{l}\right)}{1+\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}}$$

$$\gamma^3 = \left(\frac{r_2}{r_3}\right)^{2l+1}\frac{1-\left[\frac{\frac{\sigma_2}{\sigma_3}\left(1-\left[\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}\right]\frac{l+1}{l}\right)}{1+\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}}\right]}{\frac{l+1}{l}+\left[\frac{\frac{\sigma_2}{\sigma_3}\left(1-\left[\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}\right]\frac{l+1}{l}\right)}{1+\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}}\right]}$$

$$\tau(r) = \frac{r_3}{\sigma_3\left(l-(1+l)\gamma^3\right)}\left(1+\gamma^s\left(\frac{r_3}{r}\right)^{2l+1}\right)\left(\frac{r}{r_3}\right)^l\left(\prod_{j=s+1}^{3}\frac{\left(1+\gamma^j\left(\frac{r_j}{r_{j-1}}\right)^{2l+1}\right)}{1+\gamma^{j-1}}\right)$$

$\underset{brain}{\sigma_1} = 1, \quad \underset{skull}{\sigma_2} = \frac{1}{80}, \quad \underset{scalp}{\sigma_3} = 1$

in brain, $r_0 \le r \le r_1$

$0 \le r \le r_1$

$$\tau(r) = \frac{r_3}{\sigma_3\left(l-(1+l)\gamma^3\right)}\left(1+\gamma^3\left(\frac{r_1}{r}\right)^{2l+1}\right)\left(\frac{r}{r_1}\right)^l\left(\frac{1+\gamma^2\left(\frac{r_2}{r_1}\right)^{2l+1}}{1+\gamma^1}\right)\left(\frac{1+\gamma^3\left(\frac{r_3}{r_2}\right)^{2l+1}}{1+\gamma^2}\right)$$

$$\gamma^3 = \left(\frac{r_2}{r_3}\right)^{2l+1} \frac{1 - \left[\frac{\frac{\sigma_2}{\sigma_3}\left(1 - \left[\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}\right]\frac{l+1}{l}\right)}{1+\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}}\right]}{\frac{l+1}{l} + \left[\frac{\frac{\sigma_2}{\sigma_3}\left(1 - \left[\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}\right]\frac{l+1}{l}\right)}{1+\left(\frac{r_1}{r_2}\right)^{2l+1}\frac{1-\frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l}+\frac{\sigma_1}{\sigma_2}}}\right]}$$

$$\gamma' = 0$$

d) $$\tau(r) = \frac{r_3}{\sigma_3(l - (1-l)\gamma^3)} \left(1 + \gamma^3\left(\frac{r_1}{r}\right)^{2l+1}\right)\left(\frac{r}{r_1}\right)^{l} \left(1 + \gamma^2\left(\frac{r_2}{r_1}\right)^{2l+1}\right)\left(\frac{1 + \gamma^3\left(\frac{r_3}{r_2}\right)^{2l+1}}{1+\gamma^2}\right)$$

a) $$\gamma^2 = \left(\frac{r_1}{r_2}\right)^{2l+1} \frac{1 - \frac{\sigma_1}{\sigma_2}}{\frac{l+1}{l} + \frac{\sigma_1}{\sigma_2}}$$

b) $$\delta^2 = \frac{\frac{\sigma_2}{\sigma_3}\left(1 - \frac{\gamma^2(l+1)}{l}\right)}{1+\gamma^2}$$

c) $$\gamma^3 = \frac{\left(\frac{r_2}{r_3}\right)^{2l+1}\left(1 - \delta^2\right)}{\frac{l+1}{l} + \delta^2}$$

10/29

diff vals between my/their versions
w/ order of magnitude but same trend/
shape w.r.t. l for both

maybe because of radius units?
not fully

$I = J A_{\text{circle}}$

good pattern
to find I

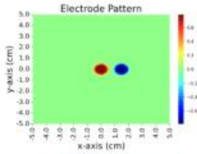
center: 1 w/A
outsides: 1/? w/A

https://xiongpengnus.github.io/rsome/ro_rso
xiongpengnus.github.io

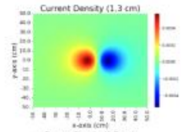
provides robust opt.
for only regions that
are 2^{nd} order cone or
exponential cone representable

1/18/23

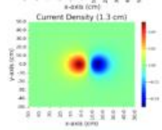
pos = [9.2-1.3, np.pi/2.0, 0]
J[0] = 1, J[[1]] = -1



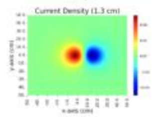
OG (.33, 1.79, 0.006):



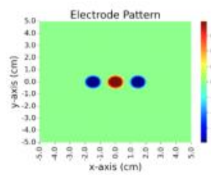
Conductivities very slightly higher
(.007, .34, 1.8, 0.31):



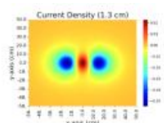
Conductivities lower (0.9 * OG)



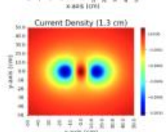
J[0] = 1, J[[1]] = -1.0, J[[2]] = -1.0



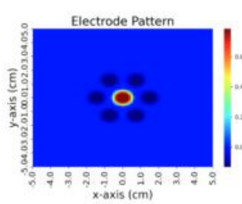
OG:



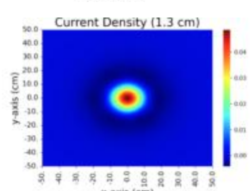
Brain conductivity (0.006 --> 0.01):



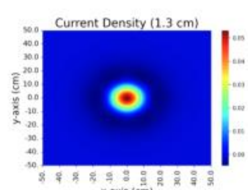
J[0] = 1, J[[1:6]] = -1



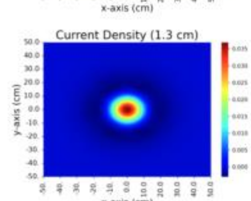
OG:



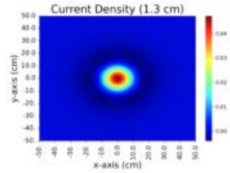
Brain 0.4:



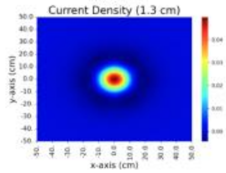
Brain 0.2:



CSF 1.9


Current Density (1.3 cm)
y-axis (cm)
x-axis (cm)

CSF 1.7


Current Density (1.3 cm)
y-axis (cm)
x-axis (cm)

Sources and Stuff

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HingePlace paper
https://ieeexplore.ieee.org/document/9630436

Model T: neuronal dynamics in some way that is simple enough to optimize with
hodgkn huxley
https://neuronaldynamics.epfl.ch/online/Ch1.S3.html (leaky integrate and fire equation)

whole ass book: https://books.google.com/books?hl=en&lr=&id=D4j2AwAAQBAJ&oi=fnd&pg=PR9&dq=neuronal+dynamics&ots=-E0pq1yU0b&sig=GFoAEVi3L1YW1m6VaWq6KUrKpgM#v=onepage&q=neuronal%20dynamics&f=false

or https://www.frontiersin.org/articles/10.3389/fninf.2018.00088/full

https://www.frontiersin.org/articles/10.3389/fnhum.2021.652393/full
https://brain.ieee.org/newsletter/2020-issue-2/bayesian-optimization-for-automated-neurostimulation-future-directions-and-challenges/

Effect of skull thickness and conductivity on current propagation for noninvasively injected currents
https://iopscience.iop.org/article/10.1088/1741-2552/abebc3/pdf

General formulation for Robust Opt models
https://xiongpengnus.github.io/rsome/ro_rsome

Romodel: modeling robust optimization problems in Pyomo
https://link.springer.com/article/10.1007/s11081-021-09703-2

ROAST: Realistic vOlumetric Approach to Simulate Transcranial electric stimulation
Open source tool to model stim
https://www.parralab.org/roast/

Robust Optimization for Non-Convex Objectives
https://arxiv.org/abs/1707.01047

NL OPT
https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/

Intro Notes

Wednesday, January 25, 2023 10:35 PM

Intro:

https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=9630436

Hingeplace can help by exploiting the threshold activation property of neurons to find more optimal
electrode placements and currents which does not hold the electric field to 0 outside the 'focus' region where activation is wanted, but holding the electric field to below some activation threshold.

As expected, the algorithm performs better than the state-of-the-art, which optimizes with a constraint
which holds field outside focus region ('cancel' region) to 0 when the threshold is higher.

Questions:
- how are neural activation thresholds determined? is it something we have knowledge about and if so, could we formulate the
problem w/ this in mind?
- what are the downsides to smaller currents that don't end up activating neurons?
- waveforms?
- how does DCM produce configurations s.t.

I will need to do some background reading on how we can produce more concentrated-looking current, such as which the DCM placement solutions produce, without bleeding out significantly outside of a certain region. I'm guessing that the fields produced by the electrodes can cancel out.

Description:
Investigating feasibility of incorporating robust optimization into previous work regarding formulization for optimization of
electrode placement for neurostimulation

Questions:
1 background understanding
2 given Ac / Af have errors, how to deal w/ them?
T has error - not perfect. For TI = E, how can we optimize robustly?
Error within a range.

Working Notes 2

Thursday, March 2, 2023 10:48 PM

```
A_new = cvx.Variable(Ac.shape[1])
if J_return is None:
    constraints = [cvx.sum(I) == 0, cvx.atoms.norm_inf(I)<=Jsafety, cvx.atoms.norm1(I)<=beta]
else:
    ones = np.zeros((1, Af.shape[1]))
    ones[len(ones) - 1] = 1
    constraints = [ones@I==J_return, cvx.sum(I) == 0,
                   cvx.atoms.norm_inf(I) <= Jsafety,
                   cvx.atoms.norm1(cvx.multiply(I, area_elec)) <= beta]
obj = cvx.Maximize(cvx.atoms.norm(A_new@I_new - Jdes)) #+beta*cvx.atoms.norm1(I)
prob = cvx.Problem(obj, constraints)
prob.solve(verbose=True, solver=cvx.ECOS)
```

$$\max_{A\_new} \left\| (A\_new)(I) - J_{des} \right\|_2^2$$

$$\text{s.t. } \left\| A' - A \right\|_2^2 \le tol$$

(* later, replace w/ element-wise)
(* later, replace w/ fn of params)

design:

1 - solve for I w/ A

2 - loop (max n times)

a - solve max problem for A' w/ I

b - solve for I' w/ A'

c - if $(I' \approx I) \rightarrow$ END

else $I \leftarrow I'$

for tol,

use $\gamma \|A\|_2^2$

assuming if $A' = \gamma A$,

$$\gamma \|A\|_2^2 \approx \|A - A'\|_2^2$$

setting $\gamma = 0.05$ for initial testing

Original try:

Normal spherical sparseplace
array([43.88879141, -10.93569609, -11.45279649, -11.72112075,
-11.89664465, -11.70143956, -11.48286624, 1.6846836 ,
3.43227551, 3.60177558, 2.50821954, 5.35055902,
2.2215138 , 5.71837827, 2.21700326, 5.31305811,
2.54175593, 3.60308225, 3.48718842, 12.51705527,
-9.27534241, 4.07932857, -4.06221926, 1.71729924,
-0.77038679, -4.79496162, 1.98391992, -2.19613274,
-2.15556403, -2.14982236, 1.91043968, -4.63452095,
-0.90362621, 1.77186567, -4.11955688, 4.11044786,
-9.40594387])

Robust:

Robust with no constraints on A:
array([6.71790799e-01, 5.84650931e-01, -9.12987675e-01, 2.96315166e-01,
-8.53255070e-01, 3.08338931e-01, -9.03046187e-01, 2.96201962e+00,
-3.74351450e+00, 4.72132231e+00, -3.33295499e+00, 3.34114704e+00,
-3.92897656e+00, 5.55456204e+00, -4.18383679e+00, 3.74553520e+00,
-3.83566303e+00, 5.04181143e+00, -3.89616464e+00, 3.93533938e+01,
-4.19024310e+01, 5.25048649e+01, -5.80896222e+01, 4.67640141e+01,
-2.79390650e+01, 1.03353076e+01, -2.40461629e+00, 5.65961215e+00,
-1.11445730e+01, 4.87033756e+00, 4.98891396e-02, 6.45132937e+00,
-2.42243722e+01, 4.46619999e+01, -5.68898346e+01, 5.14300389e+01,
-4.11233670e+01])

Constraint:
```
[cvx.atoms.norm(Af - A_new) <= tol * cvx.atoms.norm(Af)]
Tol = 0.05
```

[43.88879375 -10.93520736 -11.45318656 -11.72092645 -11.89683305
-11.70123918 -11.48326288 1.68202329 3.43366163 3.60224331

2.50730286 5.35202813 2.21982818 5.72039257 2.21533765
5.31448215 2.54087585 3.60352923 3.48859972 12.54557862
-9.29744073 4.0947104 -4.07607633 1.72750502 -0.77424276
-4.79677254 1.98581082 -2.1943578 -2.16001289 -2.14798208
1.91212673 -4.63599146 -0.90779415 1.78229266 -4.13360291
4.12601306 -9.42820654]

Constraint:

```
[cvx.atoms.norm(Af - A_new) <= tol * cvx.atoms.norm(Af)]
Tol = 0.1
```

[43.8887959 -10.9347781 -11.45352929 -11.72075581 -11.89699856
-11.70106325 -11.48361133 1.6796861   3.43487947 3.60265426
2.50649752 5.35331887 2.2183473   5.72216208 2.21387455
5.31573312 2.54010294 3.60392162 3.48983982 12.5706354
-9.31685305 4.10822208 -4.08824872 1.73646986 -0.77762915
-4.79836456 1.98747284 -2.19279905 -2.16392054 -2.14636599
1.91360915 -4.63728367 -0.91145503 1.79145125 -4.14594011
4.13968519 -9.44776313]

```
Very very small differences because tolerance is quite strict
(||Af-Anew||) = 0.07, ||Af|| = 57
```

```
[cvx.atoms.norm(Af - A_new) <= tol * cvx.atoms.norm(Af)]
Tol = 100
```

[4.40145401e+01 -1.06615188e+01 -1.18321235e+01 -1.16640673e+01
-1.20903568e+01 -1.16353496e+01 -1.18716198e+01 6.61705807e-01
3.54917507e+00 5.48000510e+00 1.56997168e-02 8.56744180e+00
-9.49898306e-01 9.17046648e+00 -9.21431632e-01 8.45845296e+00
1.05685599e-01 5.45189717e+00 3.63730665e+00 4.40145400e+01
-3.62498785e+01 2.72961645e+01 -2.66437161e+01 1.73193036e+01
-3.84740455e+00 -1.12756762e+01 7.12511331e+00 3.79050501e-02
-8.84293313e+00 1.92901913e-01 6.71530076e+00 -1.05625386e+01
-4.48245025e+00 1.77285571e+01 -2.70033335e+01 2.76111672e+01
-3.66190335e+01]

With some corrections and tol = 50
Robust:
[43.89686219 -10.68119285 -11.69708982 -11.53775387 -12.08618299
-11.51653752 -11.72908446 0.87673083 3.74799172 3.97183828
2.25125883 5.54107987 1.81079183 6.40880344 1.81325738
5.48985089 2.2953756   3.96813081 3.80943298 24.13106773
-18.83353797 11.97438176 -11.91542124 8.15227541 -5.43465398
-1.32111009 -1.1653954   1.78878461 -7.1330866   1.86117151
-1.31468206 -1.03880621 -5.68414597 8.2995288 -12.05798523
12.08402767 -19.0259759]

Change perturbation constraint on
A to element-wise:

$$|A'_{i,j} - A_{i,j}| \leq tol_{i,j}$$

$\forall\, i,j \in [m] \times [n]$
for $A \in \mathbb{R}^{m \times n}$

$$tol_{i,j} = \gamma A_{i,j}$$

using $\gamma = 0.05$

Convergence Criteria:

$$|I'_{ij} - I_{ij}| = \alpha |I_{ij}|$$

using $\alpha = 0.01$

normal res: $A @ I = 0.579$

$tol = 0.5 \rightarrow A@I = 0.17$

$= 0.05 \rightarrow A@I = 0.557$

Working Notes 3

Tuesday, March 14, 2023 4:13 PM

3/14

add obj. of og problem to obj. of reverse opt.

og objective

$$\beta \cdot \|A_f I - J_{des}\|_2^2 + f(A_c)$$

use $\beta = |A_c|$ to scale ~~terms~~ approximately to each other

later, try pso

- each particle will need to solve diff forward model

try:

https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/

want iterative quadratic approximation during optimization

- not cvxpy anymore but prob convex I think

```
foc_obj_term = -(cvx.atoms.norm(A_new@I_curr) - Jdes)
canc_obj_term = cvx.atoms.norm(Ac@I_curr)
scale = Af.size / Ac.size
obj = cvx.Maximize(foc_obj_term - scale * canc_obj_term)
```

focus term: $\| A_f I \| - J_{des}$ negative if not enough

J @ focus

$FT = -(\|A_f I\| - J_{des})$

positive if not enough

maximizing $(J_{des} - \|A_f I\|)$

aims to decrease J @ focus below desired

cancel term: $\|A_c I\|$

maximizing aims to increase J in cancel region

Presentation Outline

Thursday, March 23, 2023 11:50 PM

Outline (M.O.M.)

1. Introduce problem
2. Review common approach
3. Review state of art
4. Touch on relevance of work
5. params → conductivity/eqn diffs
   ↳ results
   ↳ interpretation
6. initial steps
   - manual perturbations and corresponding observations
   - plots?
7. steps toward optimization
   - intuition for problem (hyperrectangle)
8. iterative for first A → I
   - diff between results robust and og

- why this isn't so useful

9. iterative for params ⇒ I
   w/ PSO
   - results

10. future plans/recommendations

Working Notes 4

Friday, March 24, 2023 4:10 PM

To get results:

compare I_{og} on $A_{worstcase}$

to I_{wc} on A_{og}

Generate many A using random sampling on radii, conductivities

- Show worst-case does better than random/OG on avg of randoms

Do PSO by:

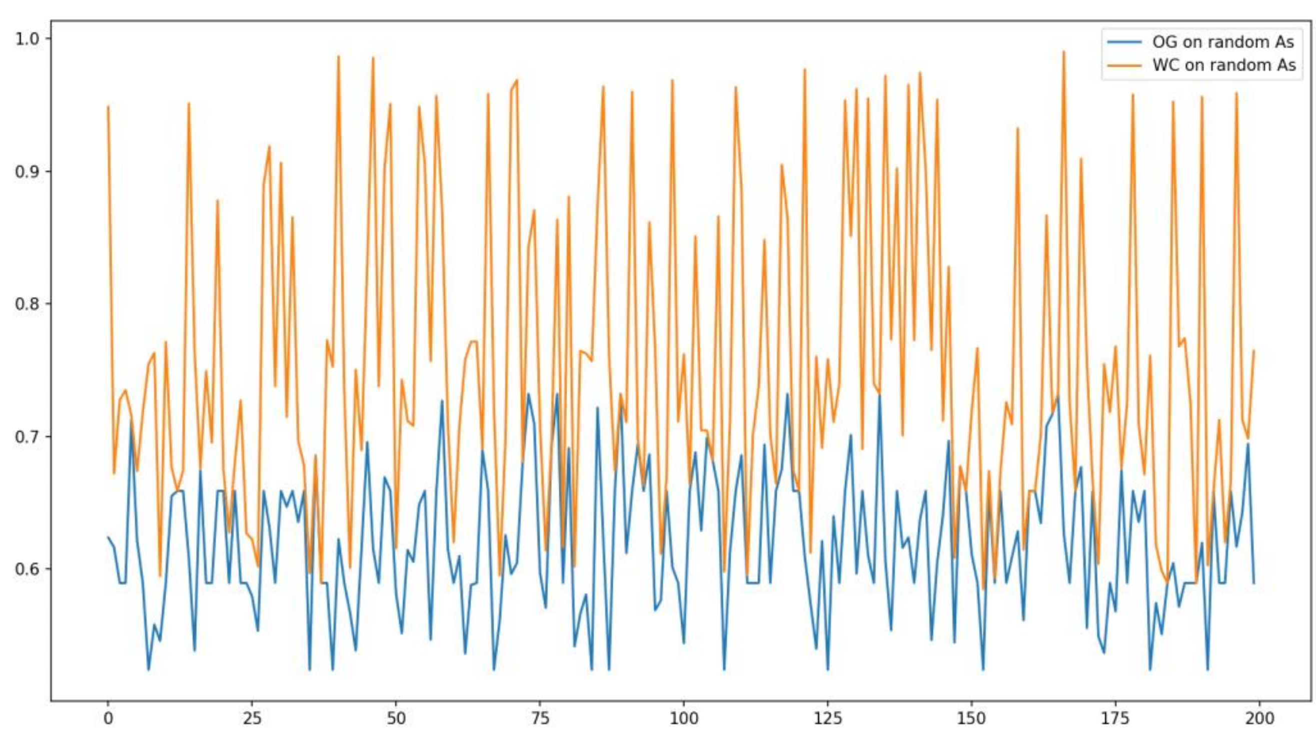
making diff instances of

SparsePlaceSpherical model

& running forward mom

Look at

- tuning WC
- look at I vs. Worst-Case A performance
- add og constraints to WC?
- compare WC performance log vs lwc



0.5241317765208503 0.732178139335467 0.6198054814916838
0.5848717986490183 0.9902005644498272 0.7558363335828241 →

```
(np.min(aaxSPrand), np.max(aaxSPrand), np.mean(aaxSPrand)
(np.min(aaxWCrand), np.max(aaxWCrand), np.mean(aaxWCrand)
```

on A_{wc}, $A_c @ I$

I_{og} gives → 0.6549

I_{wc} gives → 0.6589

w/ more iters to find I_{wc},

I_{og} → 0.6589

I_{wc} → 0.6722

WC by PSO for A
w.r.t. conductivities
(as fn of)

$$\sum |A_c @ I|$$

for WC A_c,

for I_{og} : 85.792

I_{wc} : 75.079