

EVALUATION AND ANALYSIS OF SYMMETRIC COOPERATIVE MIMO CHANNEL

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Abstract- Cooperative communication is virtual Multiple Output communication, as it provides the spatial diversity inspite of the hardware constraints and limitations associated with mobile units. In this paper Space Time coding under Rayleigh fading scenario is considered. PEP (Pair wise error probability) is used to analyze the performance. It is the probability which highlights the ability of the receiver to select an erroneous signal instead of the original signal at the destination node. Though in previous research, PEP has been developed for Block Fading channel, however in this work, dependency of considered PEP on five variables namely Energy per Symbol (E_s), Energy per bit (E_b), Modulated rate (R_m), Coded rate (R_c), Signal to Noise Ratio(SNR) are shown. Symmetric case study have been implementing for this purpose.

Keywords:- Cooperative MIMO; Spatial diversity; Space time codes; Pairwise error probability(PEP); Energy per symbol(E_b); Signal to noise Ratio(SNR); Energy per bit(E_b); Coded Rate(R_c); Modulated Rate(R_m)

I. INTRODUCTION

In communication systems, fading refers to the loss of energy or attenuation caused within the propagation media depending on varied factors like distance, time, radio frequencies etc[1]. This may hamper the signal's SNR (Signal to noise Ratio), error rate. The basis of Diversity lies in transmission of variety of versions of the same signal at the receiver's end. This is assumed for digital communication scenario which helps in stabilizing and improving a links performance and it turn reducing the error rate. Limitations in establishing a wireless communication systems such as transmitting high data rate, information measure limitations, data loses, addition of noise, interference and attenuation also tends make scenario more critical[2,3]. Diversity techniques can combat fading. The principle of diversity is to produce the receiver with multiple versions of an equivalent signal. If these are often created to be affected in several ways that by the signal path, the chance that they're going to all be affected at an equivalent time is significantly reduced. Consequently, diversity helps to stabilize a link and improves performance, reducing error rate. Due to hardware constraints Multiple Input Multiple Output (MIMO) becomes impracticable for various wireless communication applications such as Adhoc Networks, Wireless switching

networks, etc. For such type of scenario cooperative communication or cooperative MIMO is the solution. In cooperative MIMO wireless nodes cooperate among themselves to establish cooperative communication [14]. Various cooperative protocols are there. In this paper Space Time Cooperation Approach has been introduced. In order to achieve power efficient high data rate transmission and wider bandwidth over fading channels, Space-time coding integrates channel coding with multiple transmit and multiple receive antennas [12,15]. In the considered cooperative scenario, spectral efficient cooperative communication protocols are used which includes a source, a relay and a destination [6]. So then, corresponding outage probabilities are approximated with respect to the cooperative scheme in high SNR regimes and thus achieving full diversity order as shown further. Involving more than one relay in the cooperative communication scenario, the achieved diversity gain is related to the number of relays [7]. However, utilizing a large number of relays will significantly decrease the multiplexing gain, thus decreasing the spectral efficiency.

In space-time continuum signal process the time (the natural dimension of digital communication data) is complemented with the abstraction dimension inherent within the use of multiple spatially distributed antennas, i.e. [1] the utilization of multiple antennas set at totally different points. Consequently, MIMO wireless systems are often viewed as a logical extension to the good antennas that are used for several years to boost wireless [9].

PEP(Pair Wise Error Probability) defines the probability which focuses on the ability of a receiver to select an erroneous signal instead of the original signal. PEP performances are widely analyzed in several mode of weakening channels particularly quasi-static and fast attenuation channels for MIMO channels system [4,8,11].

The channel correlation matrix can be designed as the product of the temporal correlation and the spatial correlation matrix which has its influence on the pairwise error probability of space-time coding. The Euclidian

distance between two space-time code word is written in the combined form of independent chi-square random variables and therefore the probability density operate of the Euclidian distance is calculated by Laplace remodel, which provides the pairwise error probability expression for the closed-form. Simulation results verify the validity of our methodology. [10].

In this paper symmetric case has been analyzed. In symmetric case of PEP (repeated poles case of), all the mentioned products are equal. Therefore in this case, new random variables are defined to arrive at the PEP expression in a much easier fashion and so the averaged PEP is directly computed with respect to the newly defined random variables. Also, unlike the PEP expressions given in [4] and [6], this exact form of PEP can be easily utilized with the code's transfer function to obtain an upper bound on the bit per frame error probability and distance spectrum of trellis codes.

The main contribution of this paper is that

- It shows dependency of PEP on E_s (Symbol energy) for $E_s=0$ to $E_s=10$.
- It shows dependency of PEP on R_c (Coded rate). It is shown that PEP decays and becomes equal to zero as coded rate is varied.
- It shows dependency of PEP on SNR (signal to Noise Ratio). It is shown that PEP tends to increase gradually with SNR.
- It shows dependency of PEP on R_m (Modulated Rate) for $R_m=-0.5$ to $R_m=0.5$
- It shows dependency of PEP on E_b (Bit energy) where PEP increases simultaneously with E_b .

The further paper is organized as follows:

Section 2 lays emphasis on system model, section 3 describes Performance evaluation and numerical results, section 4 highlights the Simulation Results and finally section 5 concludes the paper.

II. SYSTEM MODEL

For expressing equations of Multiple Input Multiple Output (MIMO) pairwise error probability, M block fading channels, T_x antennas at transmitting side and R_x antennas at receiving side is considered. Where $x=1, 2, 3 \dots M$. The numbers of antennas used at receiving side in regular block fading channels, totally depends on number of antennas used at transmitter side. If a block uses single antenna at transmitter side, so single antenna would be used at the receiver side. The code words which are

transmitted by different blocks of users have independent fading towards the end in a space time cooperative system, so that T_x antennas may be different for variant number of transmitting antennas by different block of users and hence, all R_x becomes equal to R . Each block could have different SNR's and for different blocks of users we have different values of SNR's. Representing total received SNR often expressed in decibels in block X as $\{(E_{s,x})/(N_0)\}$ where E_s denotes symbol energy and N_0 is the Gaussian noise variance. The fading value given as $(\alpha_{t,x})$. Where fading values are observing from transmitting antenna end of "t" to receiving at another end of antenna "r" in block x . The observed fading values are i.i.d Gaussian random variables which are complex in nature. Lamda (Λ) represents eigen value which is 4.

III. PERFORMANCE EVALUATION AND NUMERICAL RESULTS

In order to analyze the above system model, it is desired to define the erroneous input code of the transmitter as C_{tx}^n , output code word of the receiver as e_{tx}^n and Delta (Δ) as the difference between C_{tx}^n and e_{tx}^n . The total time slots are given by N which are multiple of x without losing any generality. Then the change Δ_{tx}^n is given as :

$$\Delta_{tx}^n = C_{tx}^n - e_{tx}^n \quad (1)$$

Source code is manipulated in such a way that it reduces probability of error at the end of the R_x (Receiving antenna). N defines the ability of correction depending on value of n . Larger the value of n , higher the correction rate. Expression for signal difference by matrix D^x of elements size $(T_x * N/M)$ is derived where T_x is the transmitting antenna, N are the time slots and M are no of block fading channels.

$$D = \begin{pmatrix} \Delta_{1x}^1 & \Delta_{1x}^2 & \dots & \Delta_{1x}^{N/M} \\ \Delta_{2x}^1 & \Delta_{2x}^2 & \dots & \Delta_{2x}^{N/M} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{Tx_x}^1 & \Delta_{Tx_x}^2 & \dots & \Delta_{Tx_x}^{N/M} \end{pmatrix} \quad (2)$$

A newly introduced matrix called Hermitian matrix denoted by N^i , is composed of signal difference matrix and its transpose.

$$N^i = (D^x) * (D^x)^H \quad (3)$$

Where H represents complex conjugate of signal difference matrix.

The considered PEP is derived by calculating Euclidian distance 'd²' between the code words c_{tx}^n and e_{tx}^n over N time slots. On the basis of i.i.d (identically distributed)

Gaussian random variables, PEP is considered for symmetric case[13].

In order to make calculations simpler, we consider symmetric case which has been discussed earlier, the variable a_m^i 's where a_m^i is the mentioned product at the receivers end are assumed to be equal to a . It means that all $a_m^i = \frac{E_{s,i} \lambda_m^y}{4M_i N_o}$ equals to $a = (E_s \lambda) / (4MN_o)$, then we can easily obtain the PEP for symmetric case [5], where E_s is the symbol energy, λ represents constant eigen value, M are number of block fading channels, T is the number of antennas on transmitting side, R is the number of antennas on receiving side and N_o defines the Gaussian noise variance.

$$PEP(c \rightarrow e) = \frac{(1 - \frac{1}{p})^\Lambda}{2^\Lambda} \sum_{k=0}^{\Lambda-1} 2^{-k} \binom{\Lambda-1+k}{k} \left(1 + \frac{1}{p}\right)^k$$

$$p = \sqrt{1 + \frac{1}{a}} = \sqrt{1 + \frac{1}{\frac{E_s \lambda}{4TN_o}}}$$

$$\Lambda = MTR \quad (4)$$

Now different parameters namely like Energy per Symbol (E_s), Energy per bit (E_b), Modulated rate (R_m), Coded rate (R_c), Signal to Noise Ratio(SNR) are considered and the dependency of pairwise error probability(PEP) on these five variables is analyzed.

SNR provide data about noise level of unwanted signal. If the ratio of SNR is higher than 1:1, it means signal strength is more than noise level. Modulating rate (R_m) is referred as signaling rate i.e. these are represented as no. of bits per second or no. of words per minute. To find the signaling rate of a signal in the terms of bauds, only divide that term by number 1 which is shortest unit of the signal interval.

Noise spectral density (N_o) is defined in terms of watts per Hz or milli watts per Hz, E_b/N_o measured value are observed only at receiving end side at input terminals. This measured value is unaffected from whatever components and filters. Where E_s and E_b are in joules. One symbol may include many bits of information.

Now conversational relation between symbol energy and bit energy given by

$$\frac{E_b}{N_o} = \frac{E_s}{R_m R_c N_o} \quad (5)$$

Where,

E_b = Energy per bit of information

Now on further simplification,

$$E_s = R_c * E_b * R_m \quad (6)$$

Where,

$R_m = \log_2 M$ (We take different values of M in case of BPSK, QPSK, 16QAM as $M=2, 4$ and 16 respectively).

Now substituting equation (6) in equation (4) and replacing term values of ' a ' accordingly-

$$PEP(c \rightarrow e) = \frac{(1 - \frac{1}{p})^\Lambda}{2^\Lambda} \sum_{k=0}^{\Lambda-1} 2^{-k} \binom{\Lambda-1+k}{k} \left(1 + \frac{1}{p}\right)^k$$

$$\text{where } p = \sqrt{1 + \frac{1}{a}} = \sqrt{1 + \frac{1}{\frac{R_c * E_b * R_m}{4TN_o}}}$$

$$(7)$$

Where,

$$a = \left[1 + \left(\frac{4MN_o \lambda}{R_c * E_b * R_m}\right)\right]^{1/2}$$

IV. SIMULATION RESULTS

To evaluate the performance of the MIMO system, we consider the number of transmitters as 3, number of receivers as 2 and value of λ as 4. As mentioned in the previous section, the criteria for performance evaluation depends on Energy per Symbol (E_s), Energy per bit (E_b), Modulated rate (R_m), Coded rate (R_c), Signal to Noise Ratio(SNR). Each performance criteria is then evaluated by varying different parameters and PEP under these circumstances is studied.

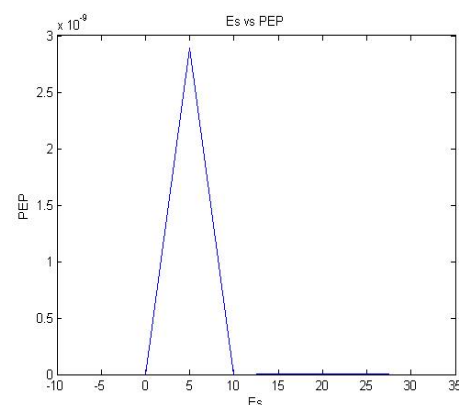


Fig. 1

Figure 1 indicates the effect of Energy per Symbol (E_s) on Pair wise Error Probability (PEP) for symmetric cooperative MIMO block fading channels. It is shown that as E_s has a direct effect on PEP. Initially with the variation of E_s from 0 to 5 joules. The PEP increases and then suddenly there is an abrupt decrease from E_s 5 to 10 after which PEP remains 0.

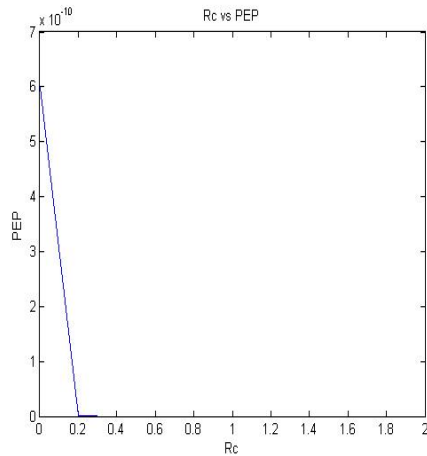


Fig. 2

Figure 2 indicates the effect of Coded rate (R_c) on PEP. It is shown that as bits per symbol increases, PEP tends to decrease to a certain and then remains constant at 0.

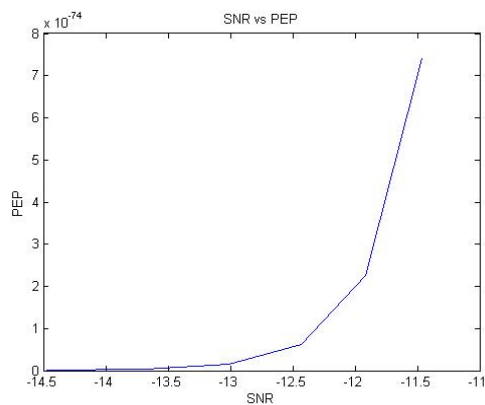


Fig. 3

Figure 3 indicates the effect of SNR on PEP. Initially SNR is ineffective on PEP. After certain range PEP tends to increase with the increase in SNR.

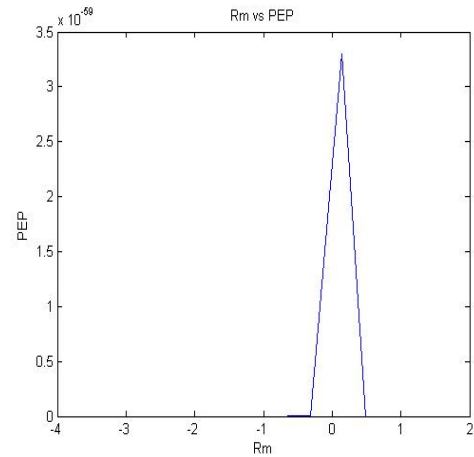


Fig. 4

Figure 4 shows the effect of R_m on PEP. It shows that PEP suddenly shoots as the value of R_m reaches to 0.5 and then tend to decrease suddenly after $R_m=1$.

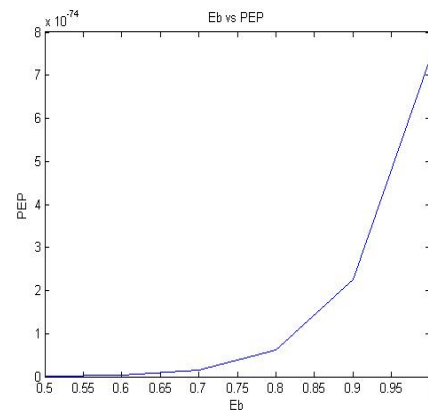


Fig. 5

Figure 5 demonstrates the effect of Energy per Bit (E_b) on PEP. This indicates there is no change in PEP till $E_b=0.6$, after which it increases slightly till $E_b= 0.8$ and lately it increases abruptly and attains a maximum value of 7.5×10^{-34} .

CONCLUSION

In this paper Symmetric Cooperative MIMO block fading channel is analyzed and its performance is evaluated on the basis of Pairwise error probability. This probability helps the receiver to select an erroneous signal instead of the original signal. The analytical PEP expression fully

accounts for computing the average PEP with respect to the newly defined variables for symmetric case. . It is mathematically shown the dependency of PEP for the considered channel upon Energy per Symbol (E_s), Energy per bit (E_b), Modulated rate (R_m), Coded rate (R_c), Signal to Noise Ratio(SNR). In practice this technique is helpful in dense areas to cater a large number of users and also in extreme areas where setting up of radio towers is quite a challenging task. Also this helps to attain a high signal strength and good communication quality Lower the PEP better is the performance of the system. By varying these parameters PEP can be controlled and desired performance can be achieved.

REFERENCES

- [1] Bernard Sklar "Rayleigh Fading Channels in Mobile Digital Communication Systems Part I: Characterization", IEEE Journals & Magazines, vol. 44, pp 90-100, 1997.
- [2] V.Tarokh, N.Seshadri, A.R.Calderbank, "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction", IEEE Trans. on Info. Theory, Vol.44, No.2, pp.744-765, March 1998.
- [3] A.Naguib, V.Tarokh, N.Seshadri, A.R. Calderbank, "A Space-Time Coding Modem for High-Data-Rate Wireless Communications", IEEE J. Select. Areas on Communication, Vol.16, No.8, pp. 1461-1471, October 1998.
- [4] M. Uysal and C. N. Georgiades, "Error performance analysis of space-time codes over Rayleigh fading channels", Journal of Communication and Network, vol 2., no.4, pp. 351-355, Dec. 2000.
- [5] Zinan Lin, Elza Erkip and Andrej Stefanov, "Exact Pairwise Error Probability for the MIMO Block Fading Channel", ISITA2004, pp. 1-3, Oct. 2004.
- [6] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," IEEE Trans. Inf. Theory, vol. 50, pp. 3062-3080, Dec. 2004.
- [7] Ali A. Haghighi, and Keivan Navaie, "Outage Analysis and Diversity-Multiplexing Tradeoff Bounds for Opportunistic Relaying Coded Cooperation and Distributed Space-Time Coding Coded Cooperation", IEEE Transactions on wireless communications, Vol. 9, No. 3, March 2010.
- [8] M. K. Simon and M. Alouini, "An approach to the performance analysis of digital communication over generalized fading channels, "Proc. of the IEEE, vol. 86, no. 9, pp. 1861 – 1876, Sept. 1998.
- [9] Z. Lin, E. Erkip and A. Stefanov, "An asymptotic analysis on the performance of coded cooperation systems," to appear in Proc. of IEEE Vehicular Technology Conference-Fall, pp. 1-4, 2004.
- [10] QuXin-bo ; He Zhao-xiang ; Wan Zheng ; Li Min, "Performance analysis of the pairwise error probability of space-time coding over correlated Rayleigh-fading channels", Wireless Communications and Signal Processing (WCSP), pp 1-4, Oct. 2010.
- [11] H. Lu, Y. Wang, P. V. Kumar and K. M. Chugg, "Remarks on space-time codes including a new lower bound and an improved code", IEEE Trans. Inform. Theory, vol. 49, no. 10, Oct. 2003
- [12] Tharaka A. Lamahewa, Marvin K. Simon, Thushara D. Abhayapala, and Rodney A. Kennedy, "Exact pairwise error probability analysis of space-time codes in spatially correlated fading channels" Journal of telecommunication and information technology, VOL. 1, pp. 1-8, 2006.
- [13] Bruno Clerckx, Luc Vandendorpe, Danielle Vanhoenacker-Janvie, Paulraj, A.J. Paulraj, "Robust Space-Time Codes for Spatially Correlated MIMO Channels" IEEE Trans Communication Society, Vol 1, no.10, pp.453 – 457, Nov 2004.
- [14] Zijian Mo ; Weifeng Su ; Batalama, S. ; Matyjas, J.D., "Cooperative Communication Protocol Designs Based on Optimum Power and Time Allocation", IEEE Journals & Magazines, Vol. 13, pp. 4283-4296, Aug. 2014.
- [15] Hua-An Zhao ; Marye, Y.W., "Adaptive modulation for cooperative wireless communication systems", IEEE Conference Publications, pp 102-105, Aug. 2014.