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Financial optimization in the Turkish electricity market: Markowitz's mean-variance approach

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ABSTRACT

Electricity constitutes the input into many products that produced by industry and used by people. Hence, it can be considered as a product or service that has vital importance in human life and economy. Since it has such special properties of instantaneous production and consumption obligation and unfeasible storage, electricity market is not like other markets. In a competitive electricity market, generation company faces price risks and delivery risks. So that risk management is an important part of a generation company and can deeply effect companies' profitability. This paper focuses on electricity generation asset allocation between bilateral contracts, such as forward contracts, and daily spot market, considering constraints of generating units and spot price risks. The problem is to find the optimal portfolio based on known electricity generation total costs, bilateral contract prices, it employed Turkish historical balanced market hourly system marginal and day-ahead hourly market prices between of 2006 and 2011. There are limited studies about portfolio optimization in electricity markets in literature and this paper should be considered frontier study taking spot market's hourly prices separately as risky assets. Markowitz mean-variance optimization which is claimed to be the beginning of modern portfolio theory in financial sector is used to demonstrate this approach. Mean-variance optimization has been successfully applied to all cases that modeled for electricity market. Some suggestions for future work are also listed in this paper.

Contents

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1. Introduction

In many countries today, there is a remarkable tendency for deregulation and restructuring of electric power industries. Turkey is one of these countries and it is also in a restructuring period. It is aiming to deregulate all electricity industry until 2015. However, this new market environment brings along some risks. In a competitive environment, generation companies' main objective is to maximize their profit and minimize the associated risks, therefore clear determination of the risks and taking necessary steps are very important. It is essential to create a trading strategy with risk management before bidding into spot markets (high profit with volatile prices) and bilateral contract market (moderate profit with stable prices). At this point, main step is the determination of portfolio weights and optimization of the portfolio with respect to investors' risk aversion.

Various risk management techniques have been applied to electricity markets. Risk management can be divided into two main sections; risk control and risk assessment. In the study of Liu and Wu [1], hedging and portfolio optimization have been defined as risk control techniques. On the other hand risk measurement and asset valuation are described as risk assessment techniques. Hedging is necessary to offset risk of position (generally in spot markets) and to reach this goal investors must buy some other derivatives like forward contracts, future contracts, options and swaps, etc. [2,3]. In electricity markets hedging the risk of spot price with forward contract and futures contract have been investigated [4,5]. Forward contracts, purchased by a consumer are commitment by the suppliers to provide a specified amount of electrical energy at a future time, are particularly useful for coordinating supply and demand side operations decisions under spot pricing [4]. A future contract obligates each party to buy or sell a specified amount of energy at a specified price. But, buyers and sellers of future contracts deal with an exchange, not with each other [6]. Hedging techniques using futures contracts in electricity market appear to have lower standard deviation or risk [5]. Options, Swaps and special electricity contracts (callable forward contract, puttable forward contract) are also determined as other tools for hedging strategies.

Portfolio optimization, listed among risk control techniques, refers to optimally allocation of energy trading tools with the aim of maximizing benefits while minimizing the corresponding risk. Except Monte Carlo Method in reference study of Vehviläinen and Keppo, there are two types of methods can be used to solve portfolio optimization problems: Decision analysis and modern portfolio theory [6,7]. Decision analysis is the determination of all possible events with its consequences and probabilities and the constitution of decision tree with respect to these data and the evaluation of tree [8]. Modern portfolio theory (MPT), which is the subject of this paper, is the other technique that can be used for portfolio optimization [1,2,6–12]. Mean-variance optimization is the essential part of this approach. Only few studies have been done by using MPT in electricity markets. It is very popular in financial literature but it is brand new approach for electricity markets. Risk assessment techniques for portfolio optimization are not scope of this study so it is not necessary to give additional information on them.

Section 2 introduces the background of the electricity market with different trading regimes and pricing system which includes trading environment. Section 2 also introduces current status of Turkish electricity market. Section 3 describes the fundamental theory and methodology (especially mean-variance optimization) to portfolio optimization which can be applied to electricity markets. Section 4 explains data and methodology and includes the application of mean variance to electricity market environment. Section 5 demonstrates results of study in graphically and table representation forms. Finally, Section 6 provides conclusions.

2. Current status of Turkish electricity market

Electricity markets in the world generally offer two types of market structure: spot market and physical market. In addition to these, there are derivative markets too. Spot markets include balance and/or day ahead spot markets while physical markets include bilateral and/or physical forward contracts. There are also derivative markets (for financial instrumentation of electricity with, futures, option, swap and special other derivatives) in some of the electricity trading regimes.

To keep interconnected system in a stable condition, it is necessary to keep frequency value of network in a bound around a specific value.[2] Instantaneous load changes must be compensated by load rejection/acceptation of generation units. Excess power in networks causes increase in frequency while opposite case causes decrease in frequency. Balance markets are founded to respond this need in a short time (generally less than 15 min) so that they are defined as real time markets. As to day-ahead market all energy traded approximately 24 h (in Turkish electricity market between 12 and 36 h before real consumption time) before real consumption time. The common scenario among these markets is that they all involve a centralized auction mechanism, by Independent System Operators (ISOs), Regional Transmission Operators (RTO) or any such organizations, to determine which generation units should be deployed and how much energy should be produced to meet the demand by selected unit [1]. Depends on the countries spot market mechanism preferences, trading intervals to submit bids in spot market can be different. For instance, in NEM of Australia, the time interval for electricity trading is defined as 30 min but all counterparts can give their bids for every 5 min. In Greece, STA has single and separate market for each hour. In England and New Zealand, NETA and NZEM have 48 half hour trading periods for each day. In IMO of Ontario bids and/or offers are submitted for every 5 min. As to Elspot in Nordic market, market participants bid for purchase and sale of power contracts 1-h duration for all 24 h of the next day. Turkish electricity market has been operated for hourly trading interval, too.

Three types of pricing mechanism are common in world's energy markets: Uniform marginal pricing, zonal pricing and locational (or nodal) pricing. In balance market, some other pricing system which is called as pay-as-bid or discriminatory pricing system can also be seen for instance Wales and England market. In markets, where uniform marginal pricing were adopted, only one energy price is used for ex post settlement for each trading interval. Generation companies can make certain of their revenues by signing bilateral contracts with their customers at fixed energy prices. In zonal pricing systems, there is a geographical point of view to

[2] American network system is being operated at 60 Hz frequency and 110 V, European and Turkish network systems are being operated at 50 Hz frequency and 220 V.

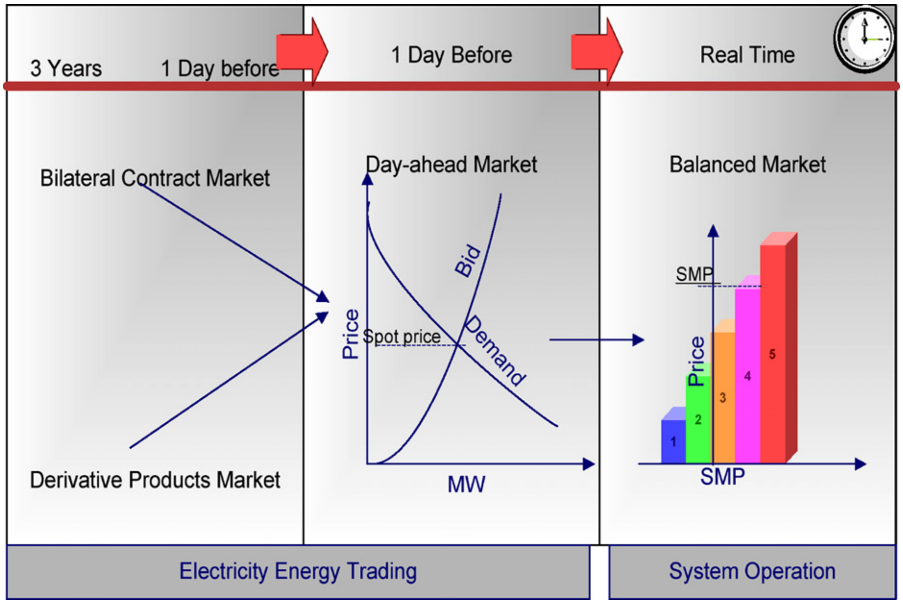

Fig. 1. European market model/multi environment advanced market model.

market structure and all system divided sub systems with respect to pre-defined geographical areas, called zones. When there is no congestion between zones, one uniform market clearing price is used throughout the system. Under congestion conditions all zones separated each other and produce their own market clearing prices, called zonal prices. These pricing systems have congestion risks for bilateral markets because of their uncertain and unpredictable pricing mechanisms. Zonal pricing is still used in Nordic (11 zones) and some other markets [13]. Locational marginal pricing is the determination of prices in each location or node of power system. When there is no congestion, one market clearing price is used in the system (ignoring transmission loses) same as zonal pricing. In zonal and locational pricing regimes, except contract with customers at the same location or zone, all contracts are risky [1]. Theoretical point of view, methodology for each pricing system is the same.

In Turkey, deregulation and reconstruction processes are proceeding. Turkish authorities are planning to reach completely deregulated market environment before 2015. With respect to Turkey Progress Report of European Union 2009, the threshold for eligible consumers was further reduced to 480,000 kWh, equivalent to opening 50% of market [14]. After this report Turkish Energy Market Regulator Authority published new threshold value as 100,000 kWh [15]. Turkey is adopting European Market Model to itself, in that respect Nordic Electricity Market structure is a good example for Turkey. Turkish electricity market consists of two main market environments now: balanced market for balancing load imbalances as a spot market and bilateral contract application. Day-ahead planning is in service to prepare market participants to day-ahead market. In near future (at the end of 2011) day-ahead market will be available for energy traders. In Fig. 1 common details of target model of Turkey can be seen.

3. Portfolio optimization theory

3.1. Modern portfolio theory

In classical portfolio theory, it is thought that you can avoid your investment risk by constructing your portfolio from different industries' shares, treasury bills with different maturities, and different foreign currencies, in other words via diversification. In modern portfolio theory, taking into consideration of relations (correlations, co-movement) of portfolio assets with each other is very important and diversification is necessary but not the only solution of portfolio risk management. Taking these co-movements of securities into account satisfies an ability to construct a portfolio that has the same expected return and less risky portfolio than a portfolio constructed by ignoring the interactions between securities [1].

It is impossible for investors to know exactly what the asset's return and risk will be tomorrow, so they are using some forecasting tools to determine these values. The main problem by which investors must be faced is to figuring out weight percentages of assets being in the portfolio. This problem is called as portfolio selection problem. To find a solution to this problem, Harry M. Markowitz published a paper, which is called as milestone study for portfolio theory and fundamentals of modern portfolio theory, "*Portfolio Selection*" in 1952 [3]. Markowitz, in his famous paper, argued that a process of portfolio selection can be divided into two stages. "*The first stage starts with observation and experience and end with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio.*" His paper is concerned about the second stage [11]. His body of work on portfolio theory resulted in him being awarded Nobel prize in economics in 1990. He shared this prize with Merton H. Miller and William F. Sharpe and they were awarded jointly for their pioneering work in the theory of financial economics. After Markowitz's famous paper, theory was amplified by Sharpe in 1964 and by Linther in 1965 [16,17]. The addition of a risk-free asset by Sharpe and Lintner in the mid 1960s led to the capital market line and the CAPM [18].

3.2. Portfolio selection and determination of utility function

Markowitz's portfolio theory is based on a mean-variance optimization process that searches for efficient portfolios. An efficient portfolio means one that provides minimum risk for a given level of return or maximum return for a given level of risk [19]. It must be done another basic assumption that rational risk-averse investors select their portfolios using only mean-variance criteria. The main assumptions of the mean-variance analysis are based on the following issues [20]:

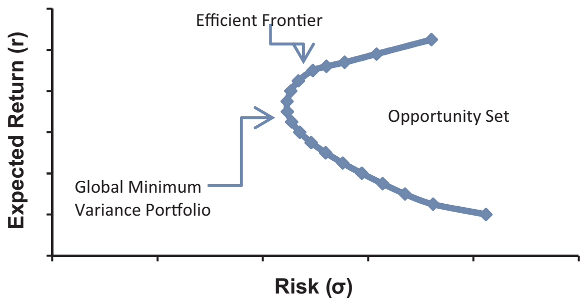

Fig. 2. Efficient frontier and efficient portfolios.

- All investors are risk averse so that they prefer less risk to more for the same level of expected return.
- Investors have the information regarding the expected returns, variances and covariances of all assets.
- Investors need only to know the expected returns, variances and the covariances of returns to determine optimal portfolios.
- And there exist no transaction costs or taxes limitation.

The required inputs necessary for mean-variance optimization model are the expected returns for each asset, the variance of each asset, and the covariances between assets. If the asset's returns obey a normal distribution, then the entire distribution of a portfolio can be described by the mean and variance only [21]. Markowitz mean-variance optimization process produces an efficient frontier (Fig. 2) which consists of efficient portfolios sets on it.

The expected return for a particular portfolio, which includes "N" assets, can be expressed as:

$$E(r_p) = \sum_{i=1}^{N} X_i r_i \tag{1}$$

while "N" describes number of assets that take place in portfolio, "X_i" denotes the proportion (weight percentage) of ith asset in portfolio and "r_i" denotes expected return of ith asset. The variance for a particular can be expressed as:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_{ij} \tag{2}$$

"σ_{ij}" is the covariance between the returns on the ith asset and the jth asset. The covariance σ_{ij} measures how many the returns on two assets move together.

The basic mean-variance optimization model includes the minimization of the portfolio's variance under three fundamental constraints. These constraints are expected return of portfolio must be equal to target return; the sum of the proportions of financial assets that are being in portfolio must be equal to "1" and finally non-negativity condition for assets' proportions. Under above determined conditions it can be set the basic equation as:

$$\text{Min.}\,(\sigma_p^2) = \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_{ij} \tag{3}$$

s.t.

$$\sum_{i=1}^{N} X_i r_i = r_e \tag{4}$$

$$\sum_{i=1}^{N} X_i = 1 \tag{5}$$

$$X_i \geq 0, \quad \forall X_i \in [i = 1, 2, \ldots, N]$$

After solution of this problem set it can be reached the efficient portfolios and efficient frontier as shown in Fig. 2 for N risky assets portfolio.

The main issue for the basic mean-variance optimization model is the determination of optimal proportional allocation "X_i" to the ith financial asset. After determination of efficient portfolios and efficient frontier, it is important the determination of investor's utility functions that represent investor's risk aversion level. It is assumed that each investor can assign a utility score to competing investment portfolios based on expected return and risk of those portfolios. Combining two, we can define the objective or utility function for an investor in terms of expected return $E(r)$ and variance of returns σ^2 as follows [1,2,6,22,23]:

$$U = E(r) - \frac{1}{2}A\sigma^2 \tag{6}$$

where U is the utility value and "A" is an index of investor's risk aversion. The factor of ½ is just a scaling convention. In financial approximations, $A = 3$ is taken for representation of average risk aversion, $A > 3$ means more risk aversion and $A < 3$ means less risk aversion. In the light of above information, now it can be found optimum portfolio for an investor that has level "A" risk aversion, after the solution of below equations set:

$$U = E(r_p) - \frac{1}{2}A\sigma_p^2 \tag{7}$$

s.t.

$$\sum_{i=1}^{N} X_i = 1 \tag{5}$$

$$X_i \geq 0, \quad \forall X_i \in [i = 1, 2, \ldots, N]$$

where

$$E(r_p) = \sum_{i=1}^{N} X_i r_i \tag{1}$$

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_{ij} \tag{2}$$

4. Data and methodology

4.1. Determination of risky and risk-free assets

To apply mean-variance optimization to electricity market, first of all it must be determined risky assets and to enlarge this application also we need a risk-free asset. In Turkish electricity market, we have system marginal prices and day-ahead market prices since August 2006. By knowing electricity generation costs with respect to their types (Hydraulic Power Plant, Lignite fired Thermal Power Plant, Natural Gas Combined Cycle Power Plant), we will use these data to represent risky assets. Bilateral contract prices with fixed values (under the guarantee of clearing authority) will be used to represent risk-free asset. Turkish electricity market's daily average spot prices frequencies between August 2006 and April 2011 can be seen in Fig. 3. We assumed that price frequency has a normal distribution around mean. In calculations, yearly average (US Dollar selling) currency prices have been used [24].

Spot market electricity prices and the amount of electricity consumptions (demand for electricity) have volatile values but they move together with high correlation. Both of these two index can importantly change with respect to different hours of

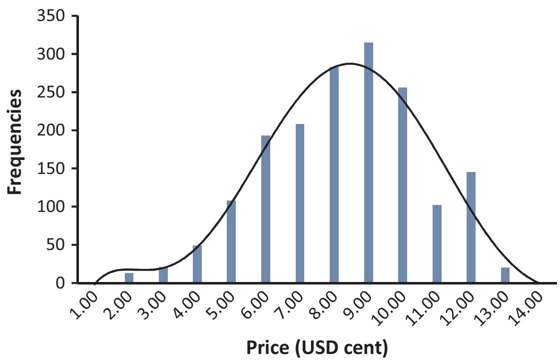

Fig. 3. System marginal prices and day-ahead prices frequency distribution between August 2006 and April 2011.

day, weekdays, weekend, months, seasons, years, and also change supply-demand balance and system congestions but each of them follows a characteristic way. The most well-known characteristic is hourly demand characteristics of consumers. Starting from this point that consumers have similar consumption characteristics in the same hours of a day, by using spot prices of 24 h of day, 24 risky assets have been determined. Above mentioned assumptions have been taken to support this approach:

Daily load curves that have chosen randomly for different years, seasons, months, and weekdays can be seen from Fig. 4. In spite of the all differences, consumers have similar consumption profiles for five different sample days. In such a way that load curves generally decreases after 10 p.m. and increases after 7 a.m. The amount of electricity consumption is low for the first 8 h of a day. In all curves it can be seen two or three head/shoulder formation shapes. Except extreme conditions like public holidays, religious holidays, very hot and cold days, network congestions and trip of generation units etc. load curves profile does not change so much. Statistical point of view, correlation constants for these five days are shown in Table 1:

As can be seen in Table 1 that load curve correlations between these days are very high in positive direction and have average 0.96 value.

The same characteristic structure of load curve that is placed in Fig. 4 can be seen in hourly spot prices as well. The spot prices for electricity are generally low for the first 8 h of the day. There are dramatical increase in prices after 7 a.m. while decrease in prices after 10 p.m. like the amount of electricity consumed by customers that has been previously shown in Fig. 4.

In our case, because of above mentioned reasons, instead of choosing one spot price to represent risky asset in electricity market, it has been chosen 24 h of day's district spot prices to represent 24 risky assets in electricity spot market. For analysis, hourly

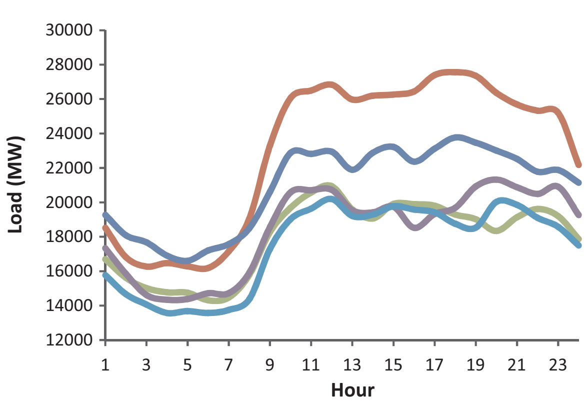

Fig. 4. Daily load curve belongs to different weekdays between August of 2006 and December of 2009.

system marginal prices which had been formed between August of 2006 and December of 2009 and hourly day-ahead spot prices which had been formed between December of 2009 and April of 2011 are used as data sets. This time interval spreads 57 months and includes 1713 prices' data for every hour of a day. That means 41,112 prices' data had been used in analysis.

$$\vec{f}_1 = \begin{bmatrix} a_{1,1} \\ a_{1,2} \\ \vdots \\ \vdots \\ a_{1,1713} \end{bmatrix}, \quad \vec{f}_2 = \begin{bmatrix} a_{2,1} \\ a_{2,2} \\ \vdots \\ \vdots \\ a_{2,1713} \end{bmatrix}, \ldots, \quad \vec{f}_{24} = \begin{bmatrix} a_{24,1} \\ a_{24,2} \\ \vdots \\ \vdots \\ a_{24,1713} \end{bmatrix} \tag{8}$$

Price vectors are formed as seen in Eq. (8). Where $\vec{f}_n$ $[n = 1, 2, \ldots, 24]$ represent price vector for each hour of day and $a_{n,m}$ $(n = 1, 2, \ldots, 24)$, $(m = 1, 2, \ldots, 1713)$ represent hourly system marginal and day-ahead prices.

In financial literature rate of return of an asset can be determined as the percentage change of investor's wealth from the beginning to the end of a term. With electricity marketing point of view this approach is not true; we cannot economically store electricity or hold like other securities for a while to sell later and this product must be generated and consumed instantly. So it must be taken into consideration generation cost of electricity. So Min Liu and Felix Wu had determined rate of return for electricity spot market as; "*rate of return = (spot prices − generation cost)/generation cost*" [25]. Depends on your assumptions this generation cost parameter can be taken as dynamic or passive. In dynamic approach $C^* = \sum_{k=1}^{M}(a + bp_k + cpk^2)\,t\omega_k^F$ power of generation unit P_k and ω_k^F fuel prices are used as a parameter [24]. Besides $C^* = S_k + D_k p_k$ equation can also be used with constant cost S_k, variable costs D_k and power of unit p_k to represent cost term of calculation. In passive approach like in this study it can be taken into consideration real average total electricity generation costs for a generation unit as a constant term. So that all price vectors in Eq. (8) are transferred to the rate of return vectors in Eq. (10) by applying the calculations in Eq. (9).

$$r_{n,m} = \frac{(a_{n,m} - C^*)}{C^*}, \quad (n = 1, 2, \ldots, 24), \ (m = 1, 2, \ldots, 1713) \tag{9}$$

$$\vec{r}_1 = \begin{bmatrix} r_{1,1} \\ r_{1,2} \\ \vdots \\ \vdots \\ r_{1,1713} \end{bmatrix}, \quad \vec{r}_2 = \begin{bmatrix} r_{2,1} \\ r_{2,2} \\ \vdots \\ \vdots \\ r_{2,1713} \end{bmatrix}, \ldots, \quad \vec{r}_{24} = \begin{bmatrix} r_{24,1} \\ r_{24,2} \\ \vdots \\ \vdots \\ r_{24,1713} \end{bmatrix} \tag{10}$$

where C^* represents marginal electricity generation costs and "*" index on it represents generation type (H: Hydro Power, T: Thermal Power with lignite and N: Natural Gas Combined Cycle Table 2). In this study EUAŞ's real average electricity generation cost values has been used as a generation cost data set. $r_{n,m}$ $(n = 1, 2, \ldots, 24)$, $(m = 1, 2, \ldots, 1713)$ represent rate of returns and $\vec{r}_n$ is rate of return vector. After determination of these vectors, the average rate of return and variances of each set are easily calculated by using Eqs. (11) and (12):

$$\bar{r}_n = \frac{1}{1713}\left(\sum_{m=1}^{1713} r_{n,m}\right) \tag{11}$$

$$\sigma_n = \sqrt{\frac{1}{1713}\sum_{m=1}^{1713}(r_{n,m} - \bar{r}_n)^2} \tag{12}$$

Table 1
Correlation matrix for five independent days.

Dates	21.09.06	20.03.07	08.06.07	07.01.08	18.11.09
21.09.06	1	0.9514011	0.9703129	0.9706092	0.9684779
20.03.07		1	0.9251289	0.9361949	0.9470435
08.06.07			1	0.9605889	0.9486646
07.01.08				1	0.9853352
18.11.09					1

4.2. Construction of three base models

Before developing base form of Markowitz mean-variance optimization model, constitution of Variance–Covariance Matrix is necessary. By using Eq. (13), all elements of Variance–Covariance Matrix in Eq. (14) are determined as follows:

$$\sigma_{xy} = \frac{1}{1713}\sum_{m=1}^{1713}(r_{x,m} - \bar{r}_x)(r_{y,m} - \bar{r}_y) \tag{13}$$

$$\begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,24} \\ \sigma_{2,1} & \sigma_2^2 & & \\ \vdots & & \ddots & \\ \sigma_{24,1} & & & \sigma_{24}^2 \end{bmatrix} \tag{14}$$

These results will be used three main model approach.

4.2.1. Standard mean-variance optimization model

In this case electricity generators have only opportunity to sell their generations to spot electricity markets. That means it is constructed and used optimization model with 24 risky selling alternatives. It is called as standard mean-variance optimization approach and the mathematical programming formulations are formed as follows:

$$\text{Min.}\,(\sigma_p^2) = \sum_{i=1}^{24}\sum_{j=1}^{24} X_i X_j \sigma_{ij} \tag{15}$$

s.t.

$$r_p = \sum_{n=1}^{24} X_n \bar{r}_n = r_e \tag{16}$$

$$\sum_{n=1}^{24} X_n = 1 \tag{17}$$

$$0 \le X_n \le 1, \quad [n = 1, 2, \ldots, 24]$$

In above mentioned equation set electricity generators can sell, whatever amount of electricity they want to sell, in every hour of day without limitation. Firstly Eqs. (15)–(17) are used for formation of possible efficient portfolio alternatives and drawing of efficient frontier. Then, in second step is here finding the optimal portfolio with in efficient portfolios alternatives with respect to electricity generators' risk aversion characteristic. To find the optimal portfolio, utility function (U) as mentioned before in Eq. (7) is used and optimization equation set is formed as follows:

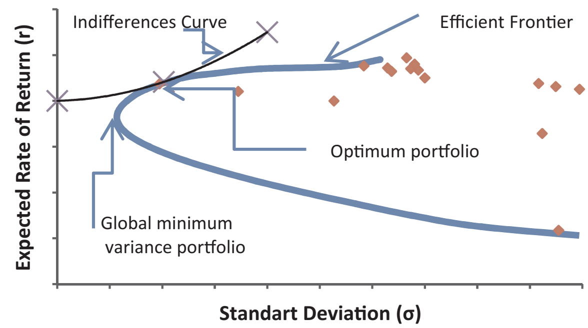

Fig. 5. Portfolio optimization results for risky assets.

$$\text{Max}_{X_n} U = E(r_p) - \frac{1}{2}A\sigma_p^2 \tag{18}$$

s.t.

$$\sum_{n=1}^{24} X_n = 1 \quad 0 \le X_n \le 1, \quad \forall X_n \in [n = 1, 2, \ldots, 24]$$

where

$$E(r_p) = \sum_{n=1}^{24} X_n \bar{r}_n \tag{19}$$

$$\sigma_p^2 = \sum_{i=1}^{24}\sum_{j=1}^{24} X_i X_j \sigma_{ij} \tag{20}$$

To represent average risk-aversion level, "A" is taken as "3" [2,25]. Some researchers in financial literature has been tried to guess the value of "A" for average risk aversion. In spite of there is no strict value determined for "A" it is accepted has a value between 2 and 4 for average risk aversion. After solution of above determined equations set it is reached optimal weight percentage of risky assets in the optimum portfolio for an electricity generator that is only taking into consideration of 24 risky alternatives in spot market to sell electricity. To reach the minimum global variance portfolio it is necessary the solution of Eq. (21). After solutions of all equations in this case it is reached result graphic as shown in Fig. 5:

$$\lim_{A\to\infty}\left\{\text{Max}_{X_n} U = E(r_P) - \frac{1}{2}A\sigma_p^2\right\} \tag{21}$$

Table 2
EUAS average electricity generation costs according to generation type 2010 [26].

Type of power plant	Generation (kWh)	Industrial cost (cent/kWh)	Commercial cost (cent/kWh)
Natural gas combined cycle	20,107,683,691	10.15	10.31
Lignite fired thermal power	26,429,805,243	7.61	8.50
Hydraulic power plants	39,914,183,522	0.75	0.99

Electricity Generation Co. Inc. (EUAS) 2010 real average electricity generation cost values. EUAS is a state owned company with the biggest market share (approximately % 45 in generation and %50 in installed capacity) in Turkish electricity market.
Commercial cost includes industrial cost and all other costs like administrative expenses, marketing, operating and maintenance costs.

4.2.2. Mean-variance optimization model with upper investment constraints

Unlike the standard Markowitz mean-variance approximation, in this model, electricity generators cannot sell all energy in one of spot market hour they want. Instead of this, there are some constraints for electricity generators for selling energy. This model basically is more realistic than standard model. In real electricity market environment generators (strategies can be change with respect to type of generators; hydraulic, coal, natural gas, nuclear, etc.) generally prefer less risky and stable conditions. Selling all of their energy to spot market without limitation is not a realistic case. So that it is constructed and used optimization model with 24 risky selling alternatives with upper constraints for selling energy to each hours and equation sets are formed as follows:

$$\text{Min.}\,(\sigma_p^2) = \sum_{i=1}^{24}\sum_{j=1}^{24} X_i X_j \sigma_{ij} \tag{22}$$

s.t.

$$r_p = \sum_{n=1}^{24} X_n \bar{r}_n = r_e \tag{23}$$

$$\sum_{n=1}^{24} X_n = 1 \tag{24}$$

$$X_n \le \theta_n, \quad [n = 1, 2, \ldots, 24] \tag{25}$$

$$0 \le X_n, \quad [n = 1, 2, \ldots, 24]$$

In above equation sets it is used same standard equations with standard model (Eqs. (15)–(17)) we talked about in case a but there are limitations $X_n \le \theta_n$ for selling electricity in spot market hours in this model. These limitations can be gotten different values also but in our model all of them are taken same and equal to θ_n. After the solution of Eqs. (22)–(25) efficient portfolios and efficient frontier are obtained like in previous case. Second step is here finding optimal portfolio with in efficient portfolios alternatives with respect to electricity generators' risk aversion characteristic. To find the optimal portfolio, utility function as mentioned before in Eq. (7) is used and optimization equation set is given by as follows:

$$\text{Max}_{X_n} U = E(r_p) - \frac{1}{2} A \sigma_p^2 \tag{26}$$

s.t.

$$\sum_{n=1}^{24} X_n = 1$$

$$X_n \le \theta_n, \quad [n = 1, 2, \ldots, 24]$$

$$0 \le X_n, \quad \forall X_n \in [n = 1, 2, \ldots, 24]$$

where

$$E(r_p) = \sum_{n=1}^{24} X_n \bar{r}_n \tag{27}$$

$$\sigma_p^2 = \sum_{i=1}^{24}\sum_{j=1}^{24} X_i X_j \sigma_{ij} \tag{28}$$

To represent average risk-aversion level "A" is taken as "3". After solution of above determined equation sets it is reached optimal weight percentages of risky assets in the optimum portfolio for an electricity generator that is taking into consideration of 24 risky alternatives in spot market to sell electricity with θ_n limitation for each hour. Reaching to minimum global variance portfolio it is also necessary solution of Eq. (21). After solutions of all equations placed above it is reached results graphic as shown in Fig. 5 same as previous case.

4.2.3. Mean-variance optimization with one risk-free asset

This case includes 24 risky assets and additional one risk-free asset for electricity generators. Bilateral contracts under guarantee of clearing authorities assumed as risk-free assets with their fixed prices for a fixed period of time. There are no limitations for electricity generators to sell their electricity in risky spot market hours or via bilateral contracts. That means it is constructed and used optimization model with one risk-free and 24 risky selling alternative. It is called as mean-variance optimization with one risk-free asset approach and related equation sets are formed as follows:

$$\text{Min.}(\sigma_p^2) = \sum_{i=1}^{24}\sum_{j=1}^{24} X_i X_j \sigma_{ij} \tag{29}$$

s.t.

$$r_c = \sum_{n=1}^{24} X_n \bar{r}_n + X_{r_f} r_f = r_e \tag{30}$$

$$\sum_{n=1}^{24} X_n + X_{r_f} = 1 \tag{31}$$

$$0 \le X_{r_f} \le 1$$

$$0 \le X_n \le 1, \quad \forall X_n \in [n = 1, 2, \ldots, 24]$$

By solution of these equation sets for every expected rate of return it is obtained efficient portfolios and efficient frontier. Eq. (31) shows total weight percentage of portfolio assets including risk-free asset and this value cannot exceed 1. There are no limitations for electricity generators like in case a but in this case extra one risk free asset (bilateral contract option) for electricity generators. To prevent from risk, more risk averse generators can prefer selling their generations into bilateral market instead of spot market. To find the optimal selling portfolio for electricity generators with average risk-aversion, value of "A" is taken 3 again and Eq. (7) is used for utility function (U) and the optimization constraints as follows:

$$\text{Max}_{X_n, X_{r_f}} U = E(r_c) - \frac{1}{2} A \sigma_c^2 \tag{32}$$

s.t.

$$\sum_{n=1}^{24} X_n + X_{r_f} = 1$$

$$0 \le X_n \le 1, \quad \forall X_n \in [n = 1, 2, \ldots, 24]$$

$$0 \le X_{r_f} \le 1$$

where

$$E(r_c) = X_{r_f} r_f + (1 - X_{r_f}) E(r_p) \tag{33}$$

$$\sigma_c^2 = (1 - X_{r_f})^2 \sigma_p^2 \tag{34}$$

$$E(r_p) = \sum_{n=1}^{24} X_n \bar{r}_n \tag{35}$$

Table 3
Risky assets for hydraulic power plants.

H	Expected rate of return $\bar{r}_n$ (%)	Standard deviation σ_n (%)	H	Expected rate of return $\bar{r}_n$ (%)	Standard deviation σ_n (%)
1	891.68	239.52	13	982.25	275.93
2	768.03	300.31	14	997.67	277.90
3	624.91	328.41	15	1019.46	309.23
4	518.24	324.42	16	995.17	302.51
5	472.82	308.34	17	987.67	288.57
6	451.67	291.21	18	971.91	357.12
7	412.64	302.78	19	932.62	315.31
8	598.28	366.44	20	917.25	319.01
9	865.04	342.74	21	929.85	310.05
10	962.99	291.16	22	912.71	322.65
11	1011.41	262.98	23	970.53	195.29
12	1044.88	275.02	24	930.89	220. 84

Table 4
Risky assets for lignite fired thermal power plants.

H	Expected rate of return $\bar{r}_n$ (%)	Standard deviation σ_n (%)	H	Expected rate of return $\bar{r}_n$ (%)	Standard deviation σ_n (%)
1	15.50	27.90	13	26.05	32.14
2	1.10	34.98	14	27.85	32.37
3	−15.57	38.25	15	30.38	36.02
4	−27.99	37.79	16	27.55	35.23
5	−33.28	35.91	17	26.68	33.61
6	−35.75	33.92	18	24.85	41.59
7	−40.29	35.26	19	20.27	36.72
8	−18.67	42.68	20	18.48	37.16
9	12.40	39.92	21	19.95	36.11
10	23.81	33.91	22	17.95	37.58
11	29.45	30.63	23	24.69	22.75
12	33.34	32.03	24	20.07	25.72

Table 5
Risky assets for natural gas combined power plants.

H	Expected rate of return $\bar{r}_n$ (%)	Standard deviation σ_n (%)	H	Expected rate of return $\bar{r}_n$ (%)	Standard deviation σ_n (%)
1	−4.48	23.00	13	3.92	26.50
2	−16.65	28.84	14	5.40	26.69
3	−30.39	31.54	15	7.49	29.69
4	−40.63	31.15	16	5.16	29.05
5	−45.00	29.61	17	4.44	27.71
6	−47.03	27.96	18	2.93	34.29
7	−50.77	29.07	19	−0.84	30.28
8	−32.95	35.19	20	−2.32	30.63
9	−7.33	32.91	21	−1.11	29.77
10	2.07	27.96	22	−2.76	30.98
11	6.72	25.25	23	2.80	18.75
12	9.93	26.41	24	−1.01	21.21

$$\sigma_p^2 = \sum_{i=1}^{24}\sum_{j=1}^{24} X_i X_j \sigma_{ij} \tag{36}$$

where X_{rf} is the weight percentage for risk-free asset (bilateral contract), $E(r_c)$ is the expected rate of return for mixed portfolio which constitutes of risky and risk-free assets, and σ_c^2 is the variance of mixed portfolio.

By solution of above equation sets it can be obtained weight percentages of optimal portfolio in a risky and risk-free market environment. To represent average risk-aversion level, "A" is taken as 3 again. Reaching to minimum global variance portfolio it is also necessary solution of Eq. (21). After solutions of all Eqs. (29)–(36) it is reached results graphic as shown in Fig. 5 same as previous cases.

5. Results

All cases which have been determined in previous sections are applied to three specific examples; Hydraulic Power Plants with C^H = 0.99 cent/kWh, Lignite Fired Thermal Power Plants with C^T = 8.50 cent/kWh, and Natural Gas Combined Cycle Power Plants with C^N = 10.31 cent/kWh. They have been calculated means and variances of each hour for three special cases and shown in the following tables (Tables 3–5).

In all cases while finding optimum portfolios, "A" is taken 3 for average risk aversion and 1000 for global minimum variance portfolios. Actually with respect to Eq. (16) "A" must be taken infinity to find global variance portfolios but in practice this is not applicable situation and it has been seen some error propagations had occurred when taken "A" with high values. So "A" has been taken 1000 to find global variance portfolios.

5.1. Efficient frontier and optimum portfolio for hydraulic power plants

Case a. Standard mean-variance optimization: there are 24 risky assets for electricity generators and no limitation for selling of electricity in spot market's hours. "OP" represents optimum portfolio and "GMVP" represents global minimum variance portfolio in Fig. 6 and next graphics.

Case b. Mean-variance optimization model with upper investment constraints: there are 24 risky assets for electricity generators

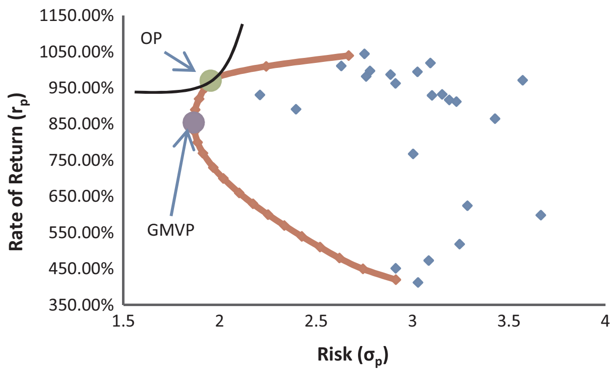

Fig. 6. Portfolio optimization for hydraulic power plants with 24 risky assets.

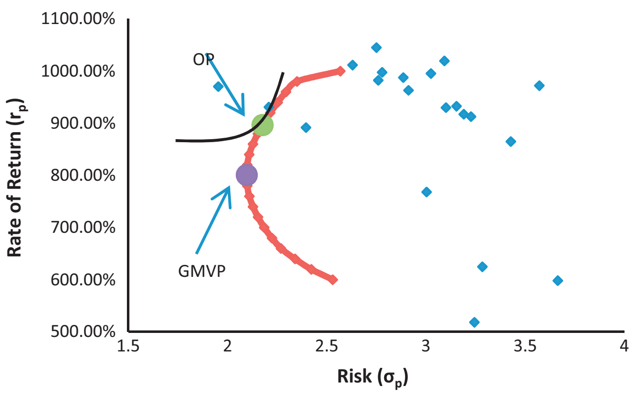

Fig. 7. Portfolio optimization for hydraulic power plants with 24 risky assets and upper investment constraints.

but there are also upper limit determined for electricity trading for each hour. When this limit is taken as 12.63% for each hour it has been reached results in Fig. 7.

Case c. Mean-variance optimization model with one risk-free asset: in this case beside the 24 risky assets there is also one risk-free asset (bilateral contract). Rate of return for bilateral contract is assumed as 750% (*8.415 cent/kWh, this is because of low generation costs of hydraulic power plants*) (Fig. 8).

5.2. *Efficient frontier and optimum portfolio for lignite fired thermal power plants*

Case a. Standard mean-variance optimization: there are 24 risky assets for electricity generators and no limitation for selling of electricity in spot market's hours (Fig. 9).

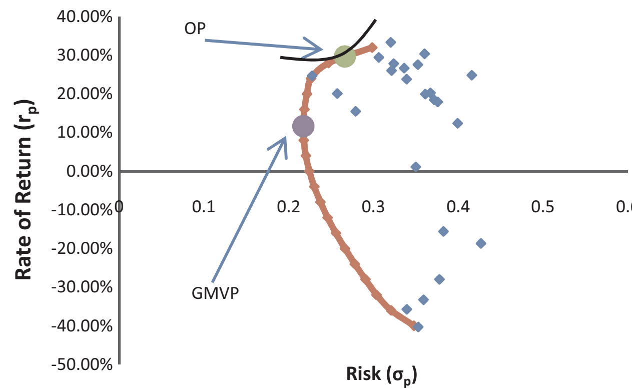

Fig. 9. Portfolio optimization for lignite fired thermal power plants with 24 risky assets.

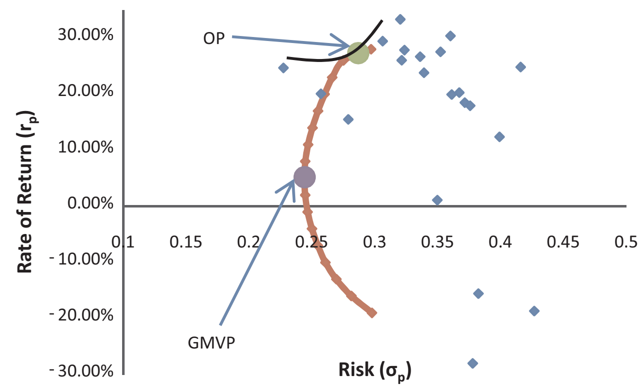

Fig. 10. Portfolio optimization for lignite fired thermal power plants with 24 risky assets and upper investment constraints.

Case b. Mean-variance optimization model with upper investment constraints: There are 24 risky assets for electricity generators but there is also upper limit (12.63%) determined for electricity trading for each hour (Fig. 10).

Case c. Mean-variance optimization model with one risk-free asset: In this case beside the 24 risky assets there is also one risk-free asset (bilateral contract). Rate of return for bilateral contract is assumed as 8% (*9.18 cent/kWh, because of high generation costs*) (Fig. 11).

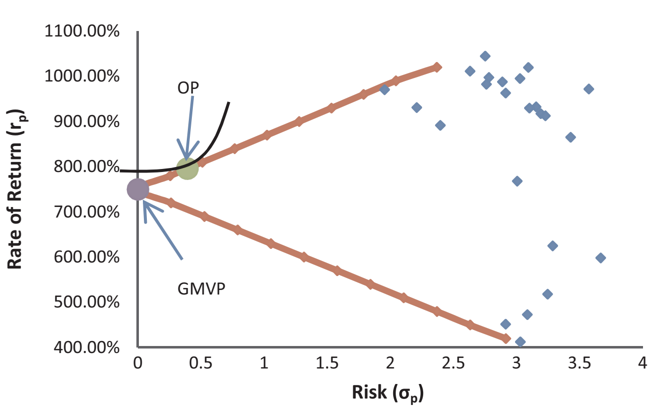

Fig. 8. Portfolio optimization for hydraulic power plants with 24 risky and one risk-free asset.

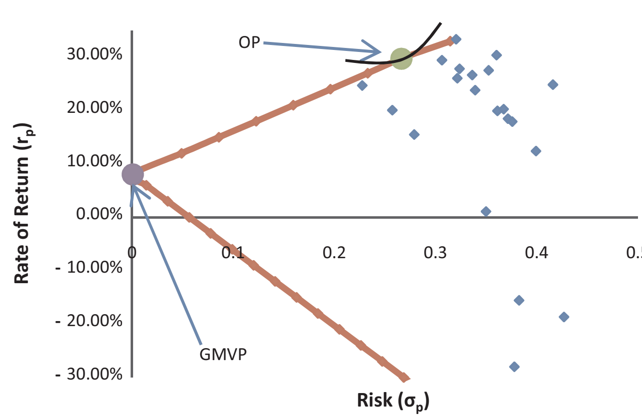

Fig. 11. Portfolio optimization for lignite fired thermal power plants with 24 risky and one risk-free asset.

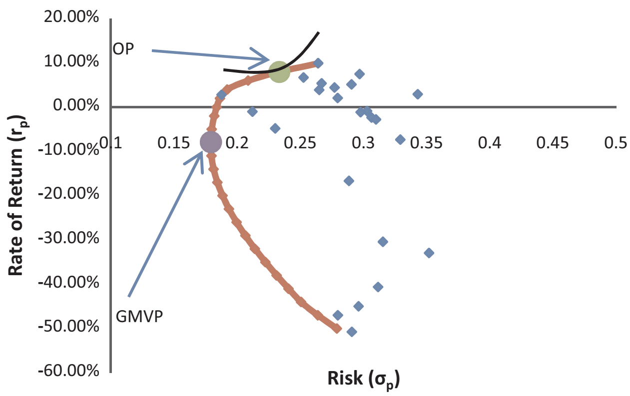

Fig. 12. Portfolio optimization for natural gas combined cycle power plants with 24 risky assets.

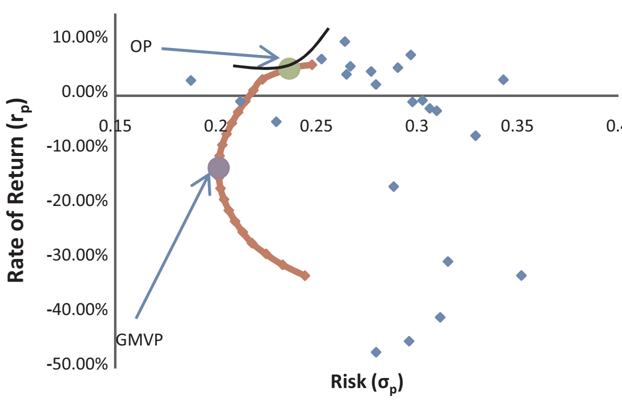

Fig. 13. Portfolio optimization for natural gas combined cycle power plants with 24 risky assets and upper investment constraints.

5.3. *Efficient frontier and optimum portfolio for natural gas combined cycle power plants*

Case a. Standard mean-variance optimization: there are 24 risky assets for electricity generators and no limitation for selling of electricity in spot market's hours (Fig. 12).

Case b. Mean-variance optimization model with upper investment constraints: there are 24 risky assets for electricity generators but there are also upper limits (12.63%) determined for electricity trading for each hour (Fig. 13).

Case c. Mean-variance optimization model with one risk-free asset: In this case beside the 24 risky assets there is also one risk-free asset (bilateral contract). Rate of return for bilateral contract is determined as 4% (10.7224 cent/kWh, because of very high generation costs) (Fig. 14).

5.4. *Analysis of results*

In case a, similar results have been obtained for all electricity generators with different weight percentages for same hours.

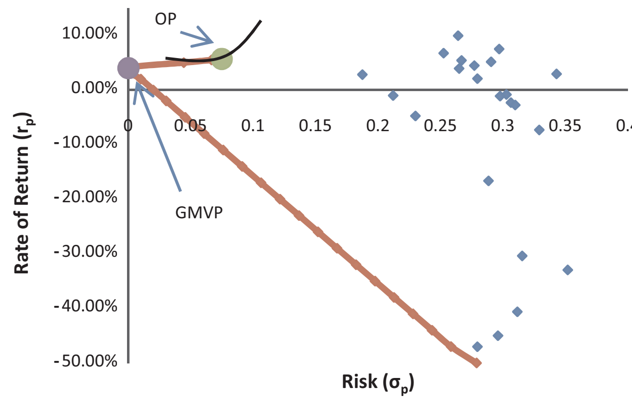

Fig. 14. Portfolio optimization for natural gas combined cycle power plants with 24 risky and one risk-free asset.

As can be seen in Table 6, weight percentages of hour-12 in optimum portfolios for electricity generators increase with increasing electricity generation costs. Optimum portfolios for lignite and natural gas fired electricity generators are same but have different weight percentages for hour-12 and hour-23. In hydraulic there is only one option with 100% of hour-23. Under normal conditions for all three types of electricity generators, most profitable spot market hour (highest average spot prices) is hour-12 but weight percentage of this hour for electricity generator that has hydraulic power plant is only 0%. The reasons of results must be evaluated according to risk and return concept. In our application standard Markowitz mean-variance optimization model in case a has been successfully applied to all electricity generators and has generated meaningful results. Assuming that expected rate of returns have normal distribution, z tests have been applied to results that are placed in Table 6. Possibilities of getting positive rate of return for optimum portfolio preferences of each electricity generators have been obtained. These values are approximately 100% for hydraulic power plants, 87.9% for lignite fired thermal power plants and 63.8% for natural gas combined cycle power plants. The advantage of low generation cost has been easily seen from the results.

In case b, different results have been obtained for all electricity generators (Table 7).

In this model called as case b, upper investment constraints have been applied (selling limit for spot market hours) to all electricity generators. Results that have been obtained for trading hours 11, 12, 14, 23, 24 are common with 12.63% weight percentages. This case is more realistic than previous case a. Because in real market environments it cannot be sold all available energy to one market hour, there must be some constraints for selling electricity in spot market to real illustration of electricity market environment. It is also necessary to be aware of characteristics of different types of power plants. For example, a hydraulic power plant has a chance to wait for a while before selling electricity because it has a reservoir holding water in it. Same case is not valid for thermal power plants. Start-up and shut-down processes are very expensive and it needs time for smooth cooling and warming to prevent thermal power plant from thermal shocks. An average start-up time required for a lignite power plant is between 8 and 12 h for hot start-up and

Table 6
Optimum portfolio results of standard mean-variance optimization model according to types of electricity generators.

Optimum portfolio	Hydraulic	Lignite fired	Natural gas
Electricity generation cost	0.99 cent/kWh	8.50 cent/kWh	10.31 cent/kWh
Hour 12	–	58.08%	72.49%
Hour 23	100%	41.92%	27.51%
Expected rate of return	970.53%	29.71%	7.97%
Standard deviation	1.952	0.266	0.233

Table 7
Optimum portfolio results of mean-variance optimization model with upper investment constraints according to types of electricity generators.

Optimum Portfolio	Hydraulic	Lignite Fired	Natural Gas
Electricity generation cost	0.99 cent/kWh	8.50 cent/kWh	10.31 cent/kWh
Hour 1	12.63%	–	–
Hour 2	0.16%	–	–
Hour 6	3.99%	–	–
Hour 7	10.14%	–	–
Hour 10	1.97%	–	–
Hour 11	12.63%	12.63%	12.63%
Hour 12	12.63%	12.63%	12.63%
Hour 13	6.94%	11.59%	11.59%
Hour 14	12.63%	12.63%	12.63%
Hour 15	–	12.63%	12.63%
Hour 17	1.02%	12.63%	12.63%
Hour 23	12.63%	12.63%	12.63%
Hour 24	12.63%	12.63%	12.63%
Expected rate of return	896.78%	27.33%	4.97%
Standard deviation	2.176	0.287	0.236

Table 8
Optimum portfolio results of mean-variance optimization model with one risk-free asset according to types of electricity generators.

Optimum portfolio	Hydraulic	Lignite fired	Natural gas
Electricity generation cost	0.99 cent/kWh	8.50 cent/kWh	10.31 cent/kWh
Bilateral contract price	8.42 cent/kWh	9.18 cent/kWh	10.72 cent/kWh
Hour 12	6.08%	58.08%	28.38%
Hour 23	12.73%	41.92%	–
Bilateral contract	81.20%	–	71.62%
Expected rate of return	795.99%	29.71%	5.68%
Standard deviation	0.392	0.266	0.075

maybe larger than 1 day for cold start-up. Natural Gas Combined Cycle Power Plants need less time with respect to lignite fired thermal power plants but still it is expensive process for natural gas combined power plant, too.

In this application mean-variance optimization model with upper investment constraints (case b) has been successfully applied to all electricity generators and has been generated meaningful results. Assuming that expected rate of returns have normal distribution, z tests have been applied to results that are placed in Table 7. Possibilities of getting positive rate of return for optimum portfolio preferences of each electricity generators have been obtained. These values are approximately 100% for hydraulic power plants, 82.89% for lignite fired thermal power plants and 58.32% for natural gas combined cycle power plants.

In case c; in the final case to reach more realistic results, one risk-free asset has been taken into consideration besides 24 risky assets.

Bilateral contract prices have been generated according to reasonable profit margin. These prices have been assumed as 8.42 cent/kWh for hydraulic power plants, 9.18 cent/kWh for lignite fired thermal power plants and 10.72 cent/kWh for natural gas combined power plants. Case c has also been applied to all three electricity generators. Results placed above in Table 8 have been obtained. Results have similarities with case a except bilateral contract weight in optimum portfolio but for lignite fired thermal power plants same results have been obtained in case a and case c. Determination of profit margin is very important part of this application case. Because profit margin directly affects the weight percentages result of optimum portfolios. High electricity generation costs and low profit margins cause to reduction of spot market share in optimum portfolio to reach acceptable level of rate of return and risk. Hydraulic power plants with their low generation cost and generation program which strongly depends on water regime do not prefer bilateral contract so much. Average capacity factor for hydraulic power plants are assumed as 40% and can change from country to country because of geographical reasons. But the situation is different for lignite power plants and natural gas combined cycle power plants. To reduce non-generation risk they need bilateral contracts in their selling strategies and in our analysis also shows that to reach acceptable rate of return and risk levels, bilateral contract's share must be increased in optimum portfolios.

6. Conclusions

This study provides an investigation on the significance of the mean-variance optimization technique for electricity markets in Turkey. It formulates the general portfolio optimization problem for Turkish day-ahead electricity market. Three cases were established with different approaches like in financial literature. It was shown that the solutions of portfolio optimization problems for "n risky assets", "n risky assets with upper investments constraints" and "n risky assets with one risk-free asset" for hydraulic, lignite fired thermal, natural gas combined cycle power plants of EUAS which is the biggest electricity generation company of Turkey.

As a result, mean-variance optimization methodology with three different cases can be applied to energy allocation between spot market and bilateral contracts in a market where there is no transmission congestion in the system. In Turkey, there are no local, zonal or nodal pricing systems so that spot market price is used as a one price everywhere in market. Therefore, it has not been taken into consideration risks because of congestion charge. Turkish balanced market and day-ahead planning market data are used to illustrate the day-ahead Turkish electricity market to apply mean-variance optimization. Price occurred every hour is thought as one risky asset so that it can be determined 24 risky selling alternatives for electricity generators and this is a new assumption such a kind of study. Electricity generators depends on the constraints (production program, maintenance program, fuel prices, network congestions, etc.) of course and by taking into consideration risk and rate of return preferences can prefer to sell their electrical energies in different hours or via bilateral contracts.

The methodology showed here includes mean-variance optimization with three cases for three different electricity generators. This methodology can also be applied to more sophisticated and

detailed problems. Fuel prices, transmission congestions, maintenance program, trips, transaction costs and financial markets (option, swap, forward and futures) are be used as a factor for improvement of this approach. Other risk management techniques hedging, risk measurement and asset valuation can also be applied to Turkish electricity market. These are determined as prospective studies. Consequently, it has been proved the applicability of mean-variance optimization, which is famous in finance literature, to Turkish electricity market.

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