GWP3

Team Member 1

In [ ]:
```
!pip install yfinance &> /dev/null
!pip install arch &> /dev/null
```

In [ ]:
```
import pandas as pd
import numpy as np
import yfinance as yf
import matplotlib.pyplot as plt
import seaborn as sns

from arch.unitroot import ADF, KPSS
from sklearn.preprocessing import StandardScaler
from statsmodels.tsa.api import VAR
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.vector_ar.vecm import VECM, coint_johansen


plt.rcParams["figure.figsize"] = (12, 9)


import warnings
warnings.filterwarnings('ignore')
```

For our Modeling I am going to use 5 years of daily data from 2019-2024 using yahoo finance API, the data we are collecting are from 3 major liquid ETF's SPY (SPDR S&P 500 ETF Trust), QQQ (Invesco QQQ Trust), VGT (Vanguard Information Technology Index Fund ETF Shares).

- Reasons for selcting these ETF's :

> 1. Market represntation
> 2. Liquidity
> 3. Sector Diversification

Overall, selecting these ETF's for analysis provides a balanced representation of the market, ensures data reliability due to high liquidity, and allows for insights into both broad market trends and sector-specific dynamics, especially in the technology sector.

In [ ]:
```
def assets(tickers):
    df = pd.DataFrame()
    for ticker in tickers:
        try:
            data = yf.download(ticker, start='2019-04-01', end='2024-04-01')
            data.dropna(inplace=True)
            data = data[['Adj Close']].reset_index()
            data.rename(columns={'Adj Close': ticker}, inplace=True)
        except Exception as e:
            print(f"Failed to retrieve data for {ticker}: {str(e)}")
        if df.empty:
          df = data
        else:
          df = pd.merge(df, data, on='Date', how='outer')
    df.dropna(inplace=True)
    return df
```

In [ ]:
```
tickers = ['SPY', 'QQQ', 'VGT']
```

In [ ]:
```
T1_df = assets(tickers)
```

```
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
```

In [ ]:
```
T1_df
```

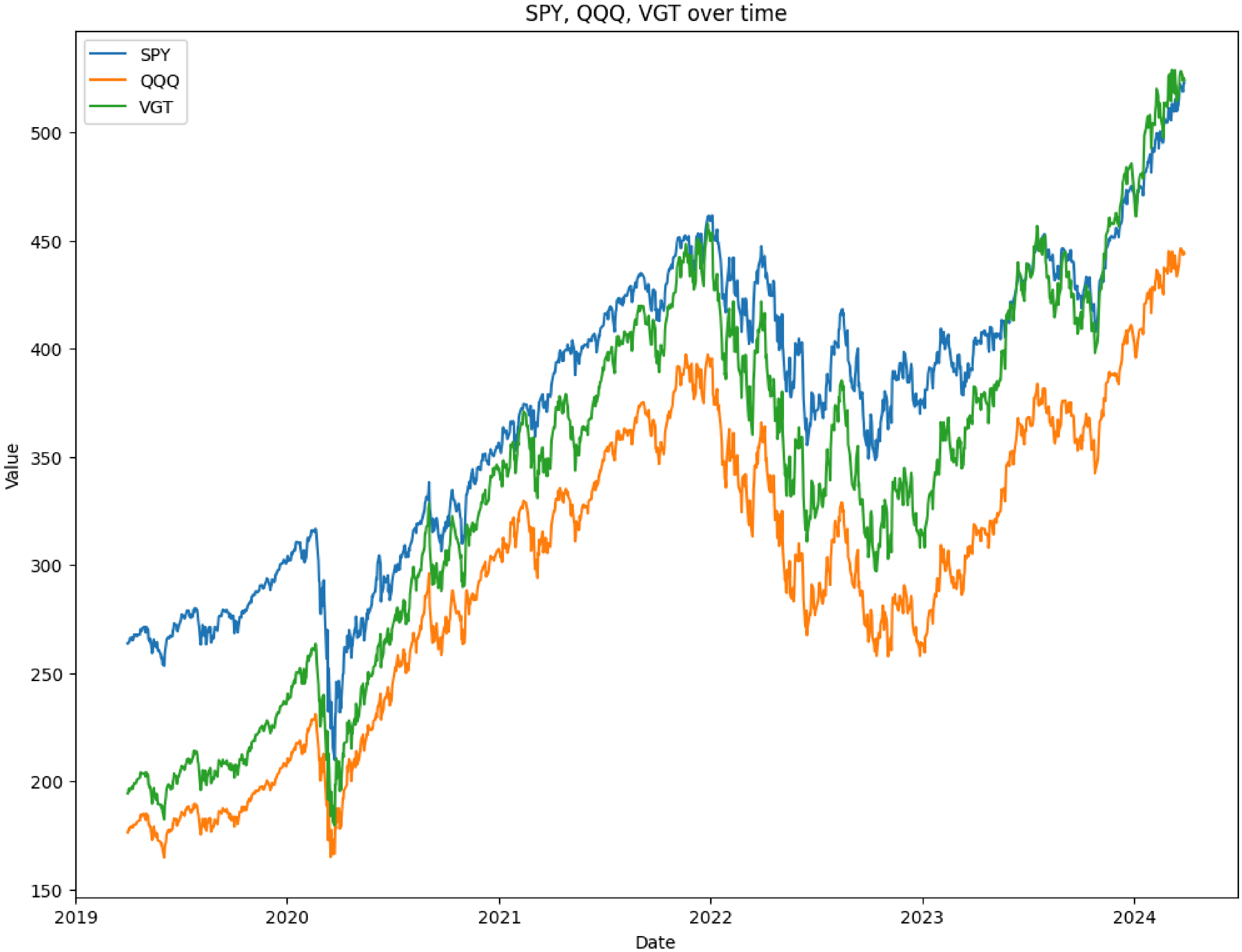
Out[ ]:

	Date	SPY	QQQ	VGT
0	2019-04-01	263.752014	176.232605	194.335678
1	2019-04-02	263.881317	176.900589	194.889709
2	2019-04-03	264.296478	177.917130	196.532715
3	2019-04-04	264.997864	177.849365	195.481964
4	2019-04-05	266.280396	178.769043	196.408539
...	...	...	...	...
1253	2024-03-22	521.210022	446.380005	528.280029
1254	2024-03-25	519.770020	444.760010	526.150024
1255	2024-03-26	518.809998	443.320007	523.840027
1256	2024-03-27	523.169983	444.829987	525.080017
1257	2024-03-28	523.070007	444.010010	524.340027

1258 rows × 4 columns

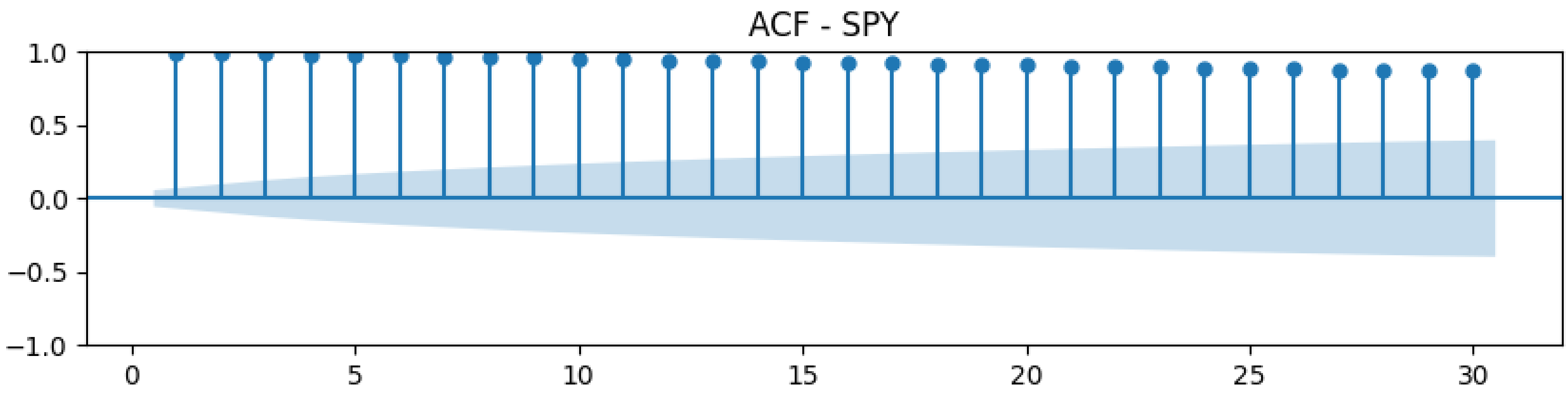
In [ ]:

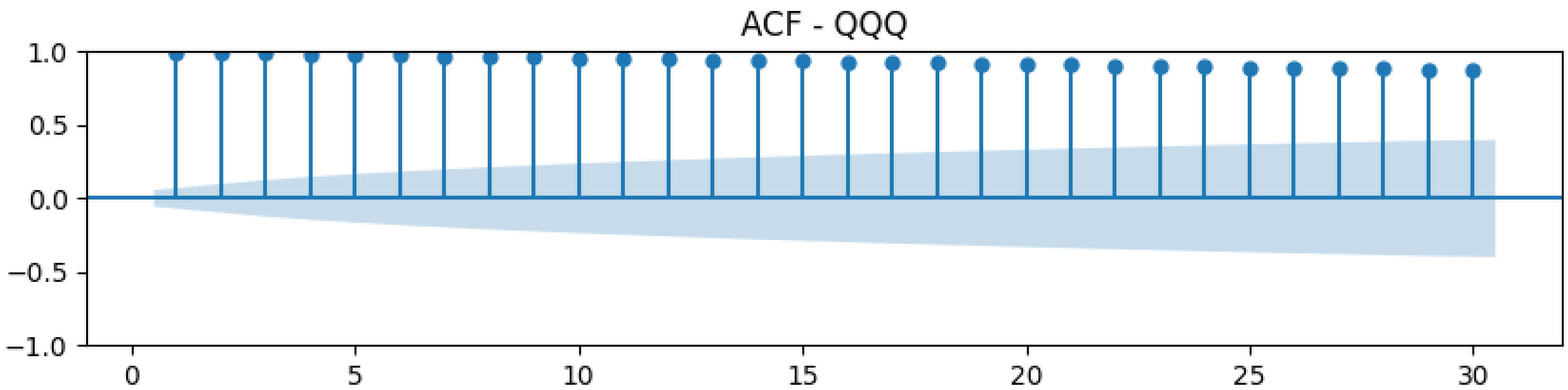
```
# Time series Plot
plt.plot(T1_df['Date'], T1_df['SPY'], label='SPY')
plt.plot(T1_df['Date'], T1_df['QQQ'], label='QQQ')
plt.plot(T1_df['Date'], T1_df['VGT'], label='VGT')
plt.xlabel('Date')
plt.ylabel('Value')
plt.title('SPY, QQQ, VGT over time')
_ = plt.legend()
```

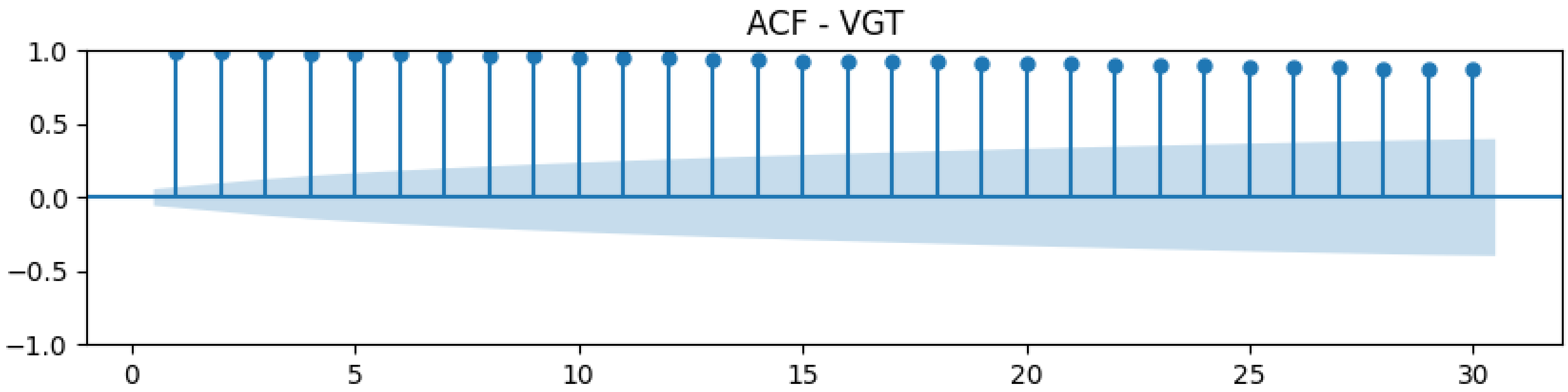

SPY, QQQ, VGT over time
SPY
QQQ
VGT
Value
Date
500
450
400
350
300
250
200
150
2019
2020
2021
2022
2023
2024

As you can see in Figure 1, All the ETF's are moving together in a similar trend. So we will now first do some visualization check for stationarity and after that we will do some econometric test for stationarity.

In [ ]:
```
#ACF Plot
for idx, col in enumerate(T1_df.columns):
    if idx > 0:
        fig, ax = plt.subplots(figsize=(10, 2))
        SIGNIFICANCE_LEVEL = 0.05

        plot_acf(T1_df[col], zero=False, ax=ax,
                 lags=30, alpha=SIGNIFICANCE_LEVEL, title=f'ACF - {col}')

        plt.show()
```







In this Figure 2, we can see the plot of ACF plot for each ETF price. In the ACF diagrams we can see a slow decay suggestion non-stationarity.

Now we will do some Econometrics tests like ADF and KPSS tests to check for stationarity.

In [ ]:
```
# ADF Test Results with 5% Significance Level
spy_adf = ADF(T1_df['SPY'], trend="n", method="bic")
qqq_adf = ADF(T1_df['QQQ'], trend="n", method="bic")
vgt_adf = ADF(T1_df['VGT'], trend="n", method="bic")


pd.DataFrame(
    {
        "SPY Price": (spy_adf.stat,  spy_adf.critical_values["5%"]),
        "| QQQ Price": (qqq_adf.stat,  qqq_adf.critical_values["5%"]),
        "| GLD Price": (vgt_adf.stat, vgt_adf.critical_values["5%"],),
    },
    index=["ADF Test Statistic", "5% Critical Value"],
)
```

Out[ ]:

	SPY Price	| QQQ Price	| GLD Price
ADF Test Statistic	1.700658	1.474284	1.467270
5% Critical Value	-1.941216	-1.941216	-1.941216

In the ADF results you can see none of the ETF's prices has test statistic is greater than 5% critical value so we cannot reject the H_0 (null) Hypothesis and there are unit roots in all three time series.

Now let's do KPSS tests :

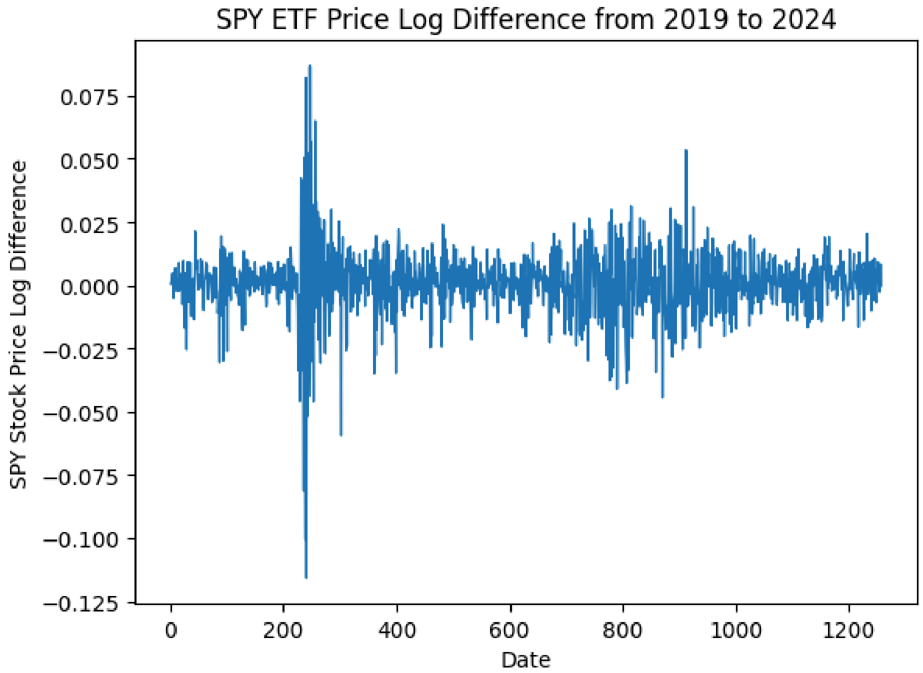
In [ ]:
```
# KPSS Test
print(KPSS(T1_df['SPY'], trend="ct", lags=-1))
print(KPSS(T1_df['QQQ'], trend="ct", lags=-1))
print(KPSS(T1_df['VGT'], trend="ct", lags=-1))
```

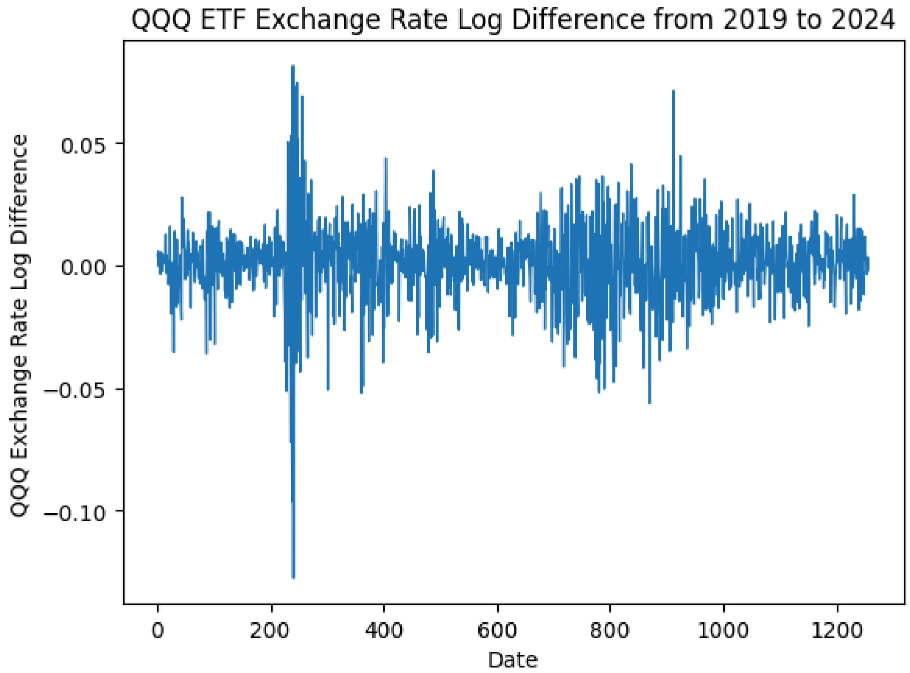
```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.577
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.653
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.595
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

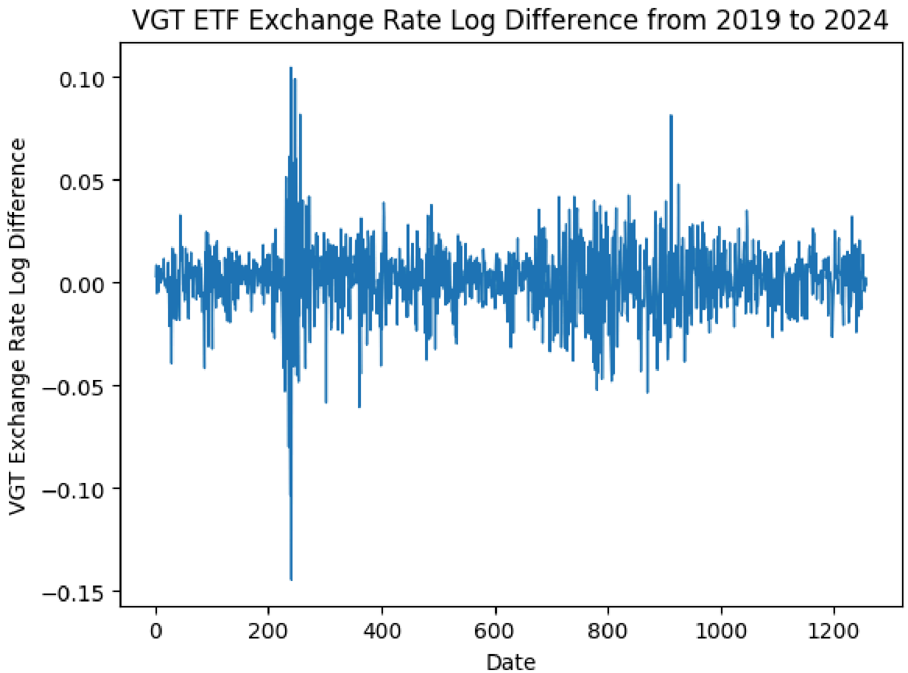
Here in the Result of KPSS test we can see all the time series have p-value lower than 0.05, which clearly reject the H_0 hypothesis, which means all these time series are non stationary and has unit root.

Now to verify our results of non-stationarity we will do differencing of the time series data and again do the visualization and econometric tests to check if the data becomes stationary or not.

In [ ]:
```
# First difference time plot

fig, axs = plt.subplots(2, 2)

d_spy = np.log(T1_df['SPY']).diff().dropna()
d_spy.plot(
    linewidth=1,
    xlabel="Date",
    ylabel="SPY Stock Price Log Difference",
    title="SPY ETF Price Log Difference from 2019 to 2024",
    ax=axs[0, 0],
)

d_qqq = np.log(T1_df['QQQ']).diff().dropna()
d_qqq.plot(
    linewidth=1,
    xlabel="Date",
    ylabel="QQQ Exchange Rate Log Difference",
    title="QQQ ETF Exchange Rate Log Difference from 2019 to 2024",
    ax=axs[0, 1],
)

d_vgt = np.log(T1_df['VGT']).diff().dropna()
d_vgt.plot(
    linewidth=1,
    xlabel="Date",
    ylabel="VGT Exchange Rate Log Difference",
    title="VGT ETF Exchange Rate Log Difference from 2019 to 2024",
    ax=axs[1, 0],
)

axs[1, 1].axis("off")
fig.tight_layout()
plt.show()
```

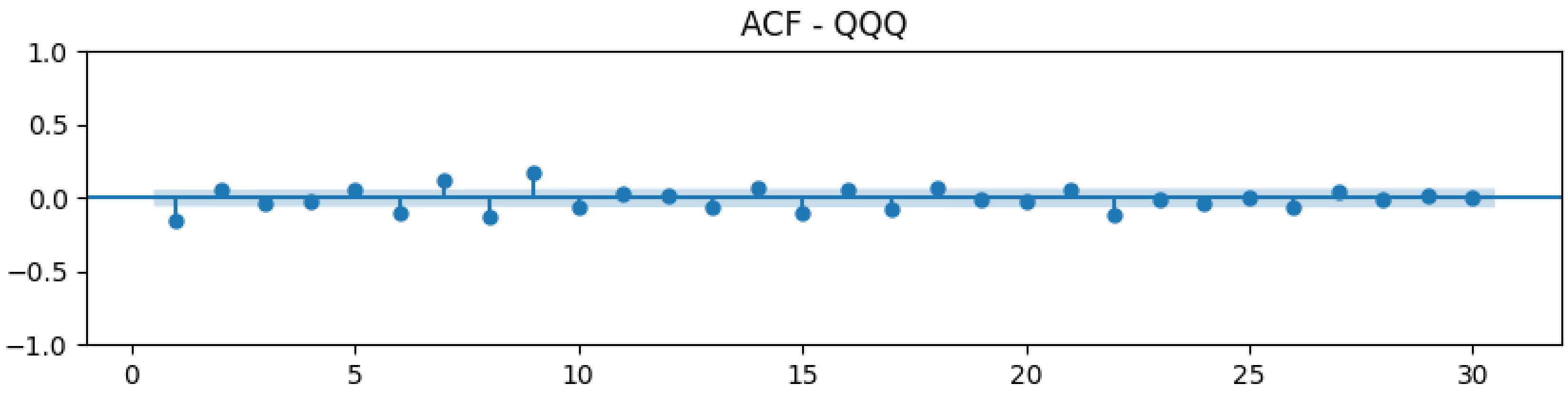

SPY ETF Price Log Difference from 2019 to 2024
SPY Stock Price Log Difference
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−0.125
0
200
400
600
800
1000
1200
Date

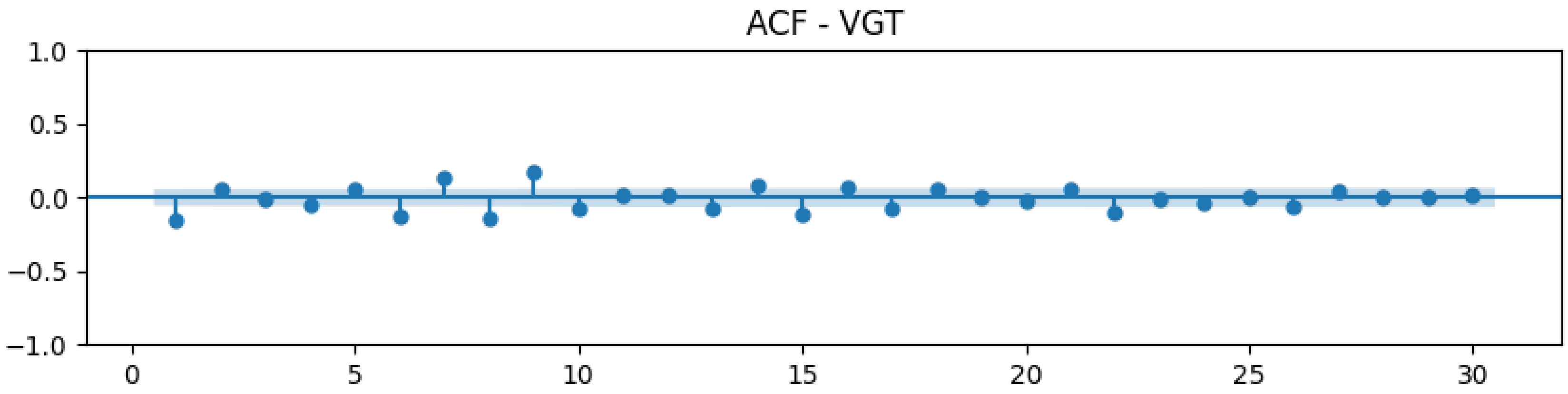

QQQ ETF Exchange Rate Log Difference from 2019 to 2024
QQQ Exchange Rate Log Difference
0.05
0.00
−0.05
−0.10
0
200
400
600
800
1000
1200
Date


VGT ETF Exchange Rate Log Difference from 2019 to 2024
VGT Exchange Rate Log Difference
0.10
0.05
0.00
−0.05
−0.10
−0.15
0
200
400
600
800
1000
1200
Date

In this Figure 3, we can see the time series plot of the differenced data of the ETF prices.

In [ ]:
```
ACF_T1_diff = pd.concat(
    [d_spy, d_qqq, d_vgt], axis=1
)

#ACF Plot
for idx, col in enumerate(ACF_T1_diff.columns):
    if idx > 0:
        fig, ax = plt.subplots(figsize=(10, 2))
        SIGNIFICANCE_LEVEL = 0.05

        plot_acf(ACF_T1_diff[col], zero=False, ax=ax,
                 lags=30, alpha=SIGNIFICANCE_LEVEL, title=f'ACF - {col}')

        plt.show()
```





In this Figure 4, we can see the plot of ACF plot for each ETF differences price. In the ACF diagrams we can see the ACF is dropping to zero quickly suggesting stationarity.

Now we will do some Econometrics tests like ADF and KPSS tests to check for stationarity.

In [ ]:

```
#ADF test for difference
d_spy_adf = ADF(d_spy, trend="n", method="bic")
d_qqq_adf = ADF(d_qqq, trend="n", method="bic")
d_vgt_adf = ADF(d_vgt, trend="n", method="bic")

pd.DataFrame(
    {
        "SPY Price": (d_spy_adf.stat, d_spy_adf.critical_values["5%"]),
        "| QQQ Price": (d_qqq_adf.stat, d_qqq_adf.critical_values["5%"]),
        "| GLD Price": (d_vgt_adf.stat, d_vgt_adf.critical_values["5%"])
    },
    index=["ADF Test Statistic", "5% Critical Value"],
)
```

Out[ ]:

	SPY Price	| QQQ Price	| GLD Price
ADF Test Statistic	-10.413947	-10.742549	-10.972278
5% Critical Value	-1.941217	-1.941217	-1.941217

In the ADF results you can see none of the ETF's prices has test statistic is lower than 5% critical value so we can reject the H_0 (null) Hypothesis and there are no unit roots in all three time series.

Now let's do KPSS tests :

In [ ]:
```
# KPSS Test
print(KPSS(d_spy, trend="ct", lags=-1))
print(KPSS(d_qqq, trend="ct", lags=-1))
print(KPSS(d_vgt, trend="ct", lags=-1))
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.059
P-value                         0.464
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.093
P-value                         0.194
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.075
P-value                         0.306
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

Here in the Result of KPSS test we can see all the time series have p-value higher than 0.05, which cannot reject the H0 hypothesis, which means all these time series are stationary and has no unit root.

Now as we can see in the non-differenced data all our test suggests that our dataset is non-stationary but after differencing that data we can see all the tests are suggesting that our data is stationary. this verifies our tests as well as all the assumptions to belive that our data set is non-stationary.

As we have confirmed that our data is non-stationary we will now do some regression modeling before that we need to decide the number of lags in the VEC (regression) model before testing it for cointegration.

So we will run a VAR model to select number of lags for VEC model.

In [ ]:
```
# Selection of Number of Lags for VEC Model
vecm_T1_df = pd.concat(
    [T1_df['SPY'], T1_df['QQQ'],T1_df['VGT']], axis=1
)

# Fit VAR model and run lag selection tool
model = VAR(vecm_T1_df)
x = model.select_order(maxlags=12, trend="c")
x.summary()
```

Out[ ]:

VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	17.79	17.80	5.333e+07	17.80
1	4.216*	4.265*	67.77*	4.235*
2	4.217	4.303	67.83	4.250
3	4.217	4.341	67.83	4.263
4	4.225	4.386	68.39	4.286
5	4.233	4.430	68.91	4.307
6	4.240	4.475	69.43	4.329
7	4.245	4.516	69.74	4.347
8	4.232	4.540	68.84	4.348
9	4.233	4.579	68.91	4.363
10	4.221	4.604	68.13	4.365
11	4.227	4.647	68.50	4.385
12	4.236	4.693	69.15	4.408

In the above results we can see all the information criteria suggest we should select lag 1 for the level of the VAR model.

Now as we have decided number of lag, we will now check for cointegration using Johansen Trace test :

In [ ]:
```
# Johansen Trace Test Result for ETFs

jtest = coint_johansen(vecm_T1_df, det_order=0, k_ar_diff=1)

# Print the results
print(f"Eigenvalues of VECM coefficient matrix : {jtest.eig}\n")

pd.DataFrame(
    {
        "Test statistic": jtest.trace_stat,
        "Critical values (90%)": jtest.trace_stat_crit_vals[:, 0],
        "Critical values (95%)": jtest.trace_stat_crit_vals[:, 1],
        "Critical values (99%)": jtest.trace_stat_crit_vals[:, 2],
    },
    index=["rank=0", "rank<=1", "rank<=2"],
)
```

```
Eigenvalues of VECM coefficient matrix : [0.01479121 0.00389053 0.00087087]
```

Out[ ]:

	Test statistic	Critical values (90%)	Critical values (95%)	Critical values (99%)
rank=0	24.706843	27.0669	29.7961	35.4628
rank<=1	5.990324	13.4294	15.4943	19.9349
rank<=2	1.094289	2.7055	3.8415	6.6349

In [ ]:
```
# VECM model
vecm_model = VECM(endog=vecm_T1_df, k_ar_diff=1, deterministic="ci").fit()
print(vecm_model.summary())
```

```
Det. terms outside the coint. relation & lagged endog. parameters for equation SPY
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.SPY        -0.1901      0.075     -2.546      0.011      -0.336      -0.044
L1.QQQ        -0.0101      0.141     -0.071      0.943      -0.286       0.266
L1.VGT         0.0968      0.113      0.856      0.392      -0.125       0.318
Det. terms outside the coint. relation & lagged endog. parameters for equation QQQ
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.SPY        -0.1887      0.075     -2.502      0.012      -0.336      -0.041
L1.QQQ        -0.0295      0.142     -0.207      0.836      -0.308       0.249
L1.VGT         0.1035      0.114      0.907      0.365      -0.120       0.327
Det. terms outside the coint. relation & lagged endog. parameters for equation VGT
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.SPY        -0.2532      0.093     -2.721      0.007      -0.436      -0.071
L1.QQQ         0.0116      0.176      0.066      0.947      -0.333       0.356
L1.VGT         0.1066      0.141      0.756      0.449      -0.170       0.383
                Loading coefficients (alpha) for equation SPY
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0279      0.010     -2.908      0.004      -0.047      -0.009
                Loading coefficients (alpha) for equation QQQ
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0266      0.010     -2.749      0.006      -0.046      -0.008
                Loading coefficients (alpha) for equation VGT
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0246      0.012     -2.061      0.039      -0.048      -0.001
          Cointegration relations for loading-coefficients-column 1
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
beta.1         1.0000          0          0      0.000       1.000       1.000
beta.2         1.2276      0.401      3.060      0.002       0.441       2.014
beta.3        -1.7958      0.332     -5.408      0.000      -2.447      -1.145
```

```
const        -127.9210      13.418      -9.533      0.000     -154.220     -101.621
==============================================================================
```

From the above johnsen trace test, we can see three tests: H_0: rank = 0, H_1 : rank = 1, H_2 : rank = 2. We will use 5% as our decision point.

- For H_0: rank = 0, we can see the test statistic is 24.7068 and the 5% critical value is 29.7961 which means the test statistic is ledd that the critical value so we do not reject the null hypothesis that there are zero cointegration relationships.
- For H_1: rank = 1, we can see the test statistic is 5.9903 and the 5% critical value is 15.4943 which means the test statistic is ledd that the critical value so we do not reject the null hypothesis that there are at most one cointegration relationships.
- For H_2: rank = 2, we can see the test statistic is 1.0942 and the 5% critical value is 3.8415 which means the test statistic is ledd that the critical value so we do not reject the null hypothesis that there are most two cointegration relationships.

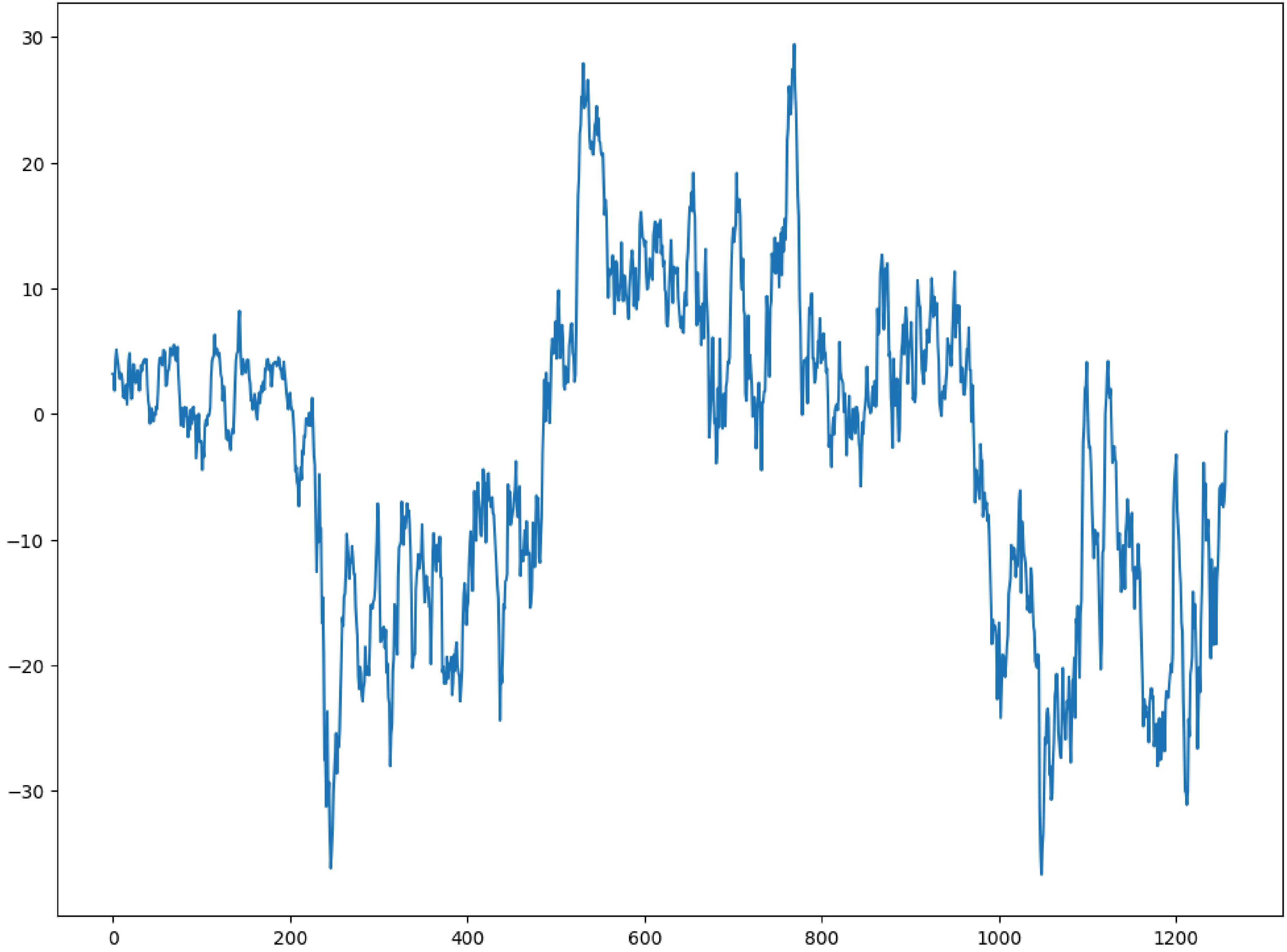
Overall the Johansen Trace Test suggested that there are at most two cointegrating relationships among the variables. This means that some linear combination of the variables is stationary, indicating a long-term equilibrium relationship.

So then we ran a VEC model and using the "*Cointegration relations for loading-coefficients-column 1*" we can write the linear combination as follows :

> S = -127.9209 + 1 . SPY + 1.2276 . QQQ - 1.7958 . VGT

The above equation is the devation from the long-term equilibrium of the three time series. Let's check out the plot to see if this deviation is stationary.

In [ ]:
```python
# Time Plot for Deviation from Long-Term Equilibrium
"""
The precise coefficients of the linear combination are in:
vecm_model.const_coint and vecm_model.beta
S = -127.9209 + 1*SPY + 1.2276*QQQ - 1.7958*VGT
"""
S = (
    vecm_model.const_coint[0][0]
    + vecm_model.beta[0][0] * vecm_T1_df.SPY
    + vecm_model.beta[1][0] * vecm_T1_df.QQQ
    + vecm_model.beta[2][0] * vecm_T1_df.VGT
)
plt.plot(S)
plt.show()
```


30
20
10
0
−10
−20
−30
0
200
400
600
800
1000
1200

In this Figure 5 we can see a long-term equilibrium is moving within a channel. Let's take a look at the ADF test for the deviation.

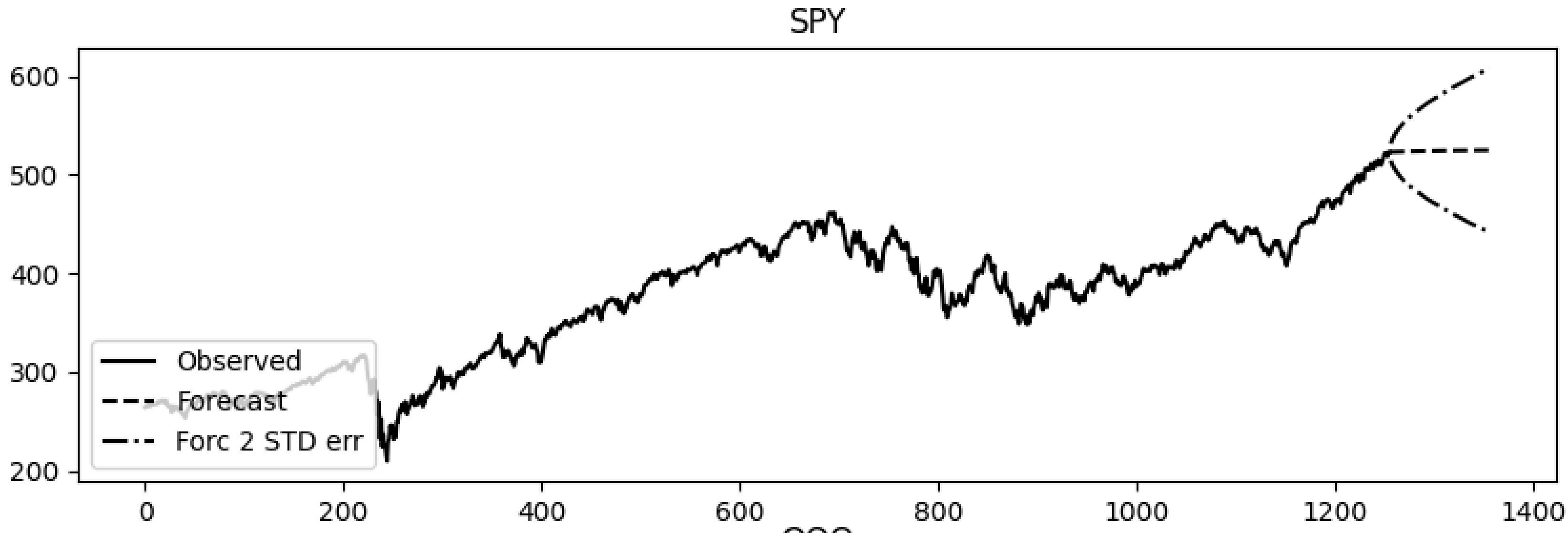
In [ ]:
```
# ADF Test Result for Deviation from Long-Term Equilibrium
S_adf = ADF(S, trend="n", method="bic")
print("Augmented Dickey-Fuller Unit Root Test\n", S_adf.regression.summary())
print("\nTest statistics and critical values: \n", S_adf)
```

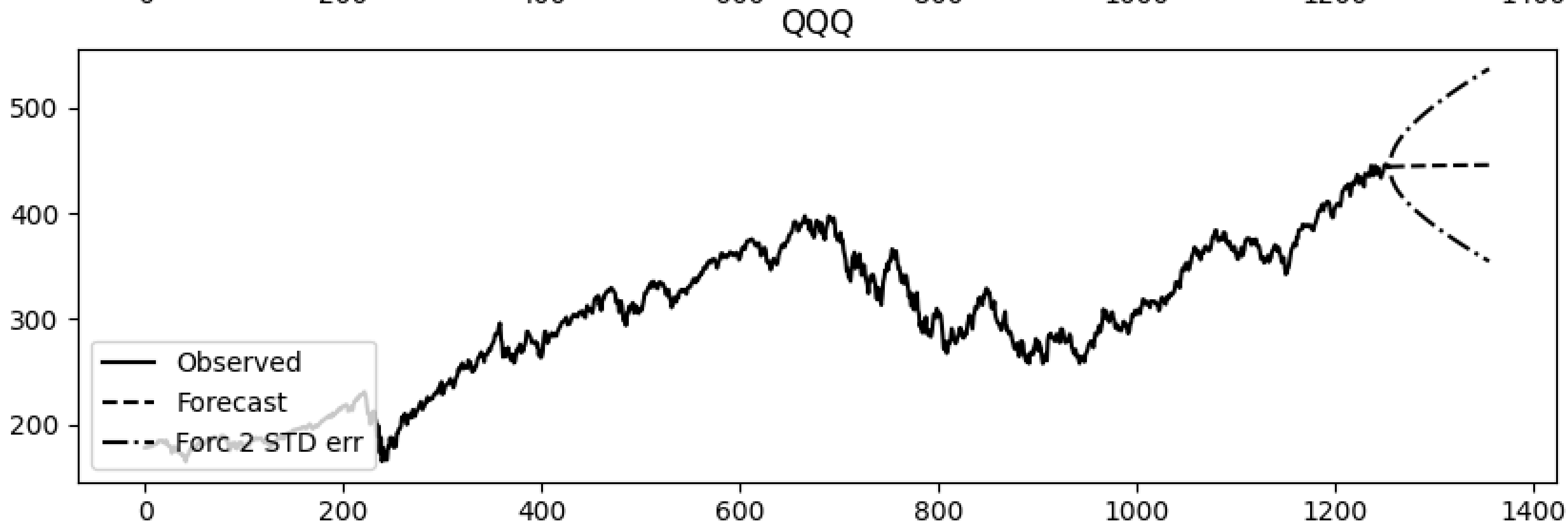
```
Augmented Dickey-Fuller Unit Root Test
                                  OLS Regression Results
=======================================================================================
Dep. Variable:                      y   R-squared (uncentered):                   0.008
Model:                            OLS   Adj. R-squared (uncentered):              0.007
Method:                 Least Squares   F-statistic:                              10.29
Date:                Mon, 08 Apr 2024   Prob (F-statistic):                     0.00137
Time:                        07:21:58   Log-Likelihood:                         -2837.2
No. Observations:                1257   AIC:                                      5676.
Df Residuals:                    1256   BIC:                                      5682.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Level.L1      -0.0162      0.005     -3.207      0.001      -0.026      -0.006
==============================================================================
Omnibus:                       50.461   Durbin-Watson:                   1.997
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              143.461
Skew:                          -0.074   Prob(JB):                     7.05e-32
Kurtosis:                       4.648   Cond. No.                         1.00
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Test statistics and critical values:
    Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -3.207
P-value                         0.001
Lags                                0
-------------------------------------

Trend: No Trend
Critical Values: -2.57 (1%), -1.94 (5%), -1.62 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

Here in this ADF results we can see the P-value is less thatn 0.05 and the Test statistic is also lower than critical value of 5% so we can easily reject the null hypothesis and that the deviation has a unit root, which makes this devation stationary.

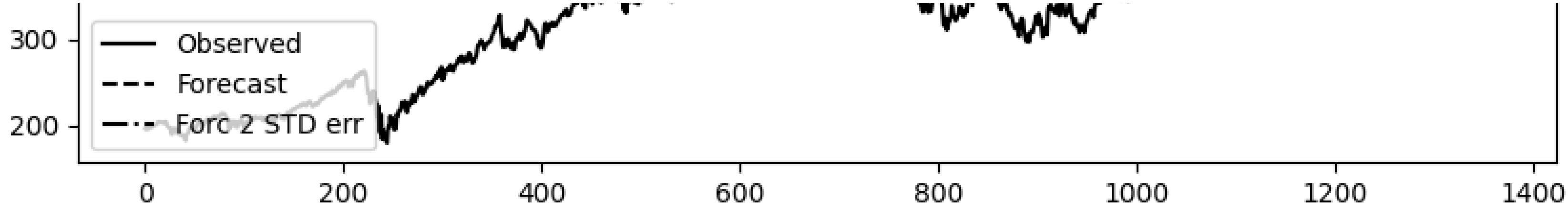
At last now we will plot the forecast of our model :

In [ ]:
```
# VEC Model Forecast of the SPY, QQQ, VGT.

vecm_model.plot_forecast(steps=100, alpha=0.05, plot_conf_int=True, n_last_obs=None)
plt.show()
```


SPY
600
500
400
300
200
0
200
400
600
800
1000
1200
1400
Observed
Forecast
Forc 2 STD err


QQQ
500
400
300
200
0
200
400
600
800
1000
1200
1400
Observed
Forecast
Forc 2 STD err


VGT
600
500
400



In this last figure 6, we can see time series forecast in which SPY and QQQ are slightly up but VGT remains flat in the forecast.

In [ ]:
```
predictions = vecm_model.predict(steps=len(T1_df))

# Extract the predicted values for 'SPY', 'QQQ', and 'VGT'
predicted_spy = predictions[:, 0]
predicted_qqq = predictions[:, 1]
predicted_vgt = predictions[:, 2]
```

In [ ]:
```
plt.figure(figsize=(10, 6))

# Plot actual values
plt.plot(T1_df.index, T1_df['SPY'], label='Actual SPY', color='blue')
plt.plot(T1_df.index, T1_df['QQQ'], label='Actual QQQ', color='green')
plt.plot(T1_df.index, T1_df['VGT'], label='Actual VGT', color='red')

# Plot predicted values
plt.plot(T1_df.index, predicted_spy, label='Predicted SPY', linestyle='--', color='blue')
plt.plot(T1_df.index, predicted_qqq, label='Predicted QQQ', linestyle='--', color='green')
plt.plot(T1_df.index, predicted_vgt, label='Predicted VGT', linestyle='--', color='red')

plt.xlabel('Time')
plt.ylabel('Value')
plt.title('Actual vs. Predicted Values')
plt.legend()
plt.show()
```

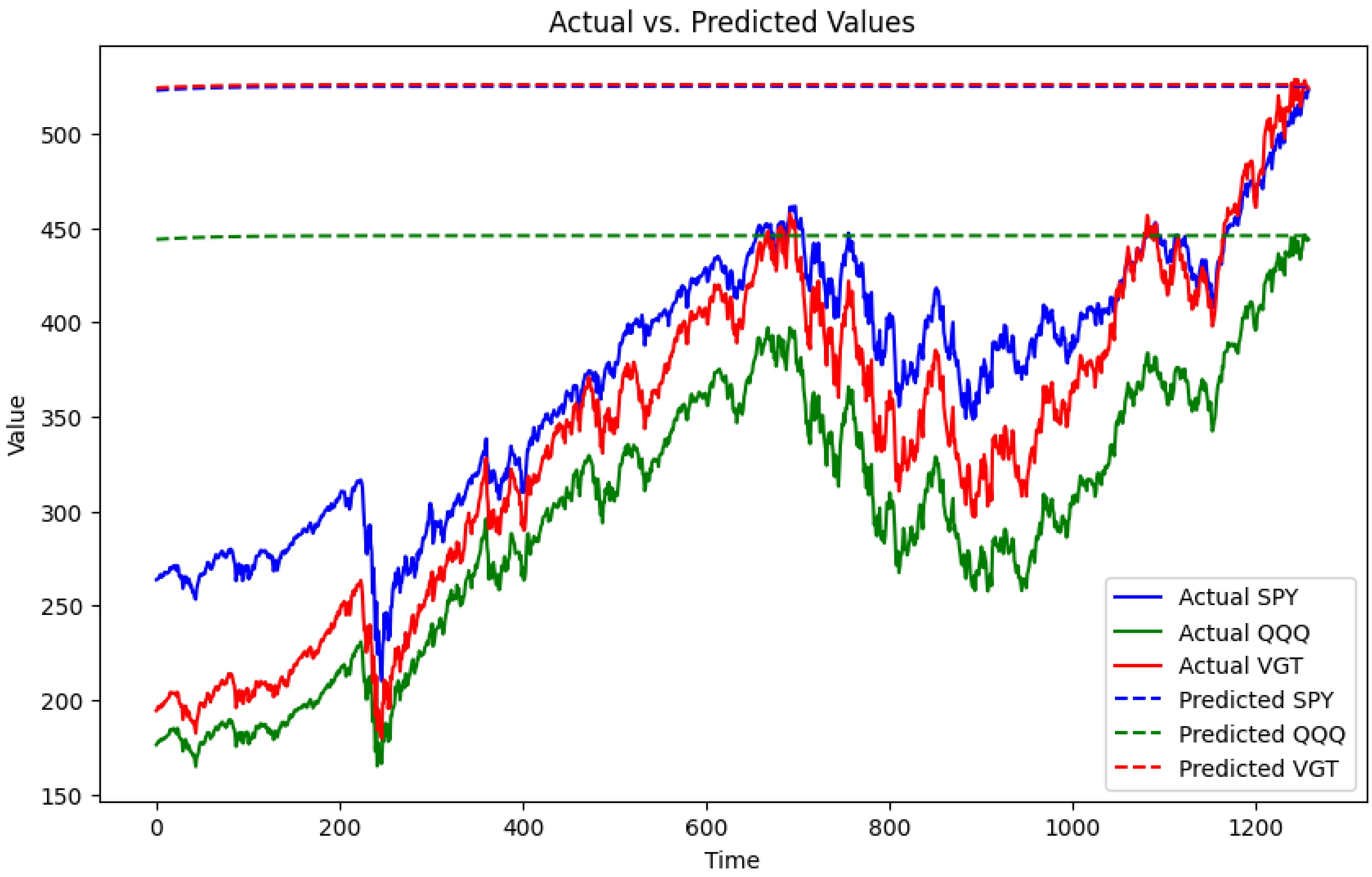


Figure 7 also confirms our analysis of Figure 6.

Team Member 2

For the modeling, I am going to use 5 years of daily data from 2019-2024 for Nifty 50, Infosys, Nifty IT Index data fetched from Yahoo Finance API.

In [ ]:
```
tickers = ['^NSEI', 'INFY.NS', '^CNXIT']
```

In [ ]:
```
T2_df = assets(tickers)
T2_df.rename(columns={"^NSEI": "NSE", "INFY.NS" : "INFY","^CNXIT": "IT"}, inplace=True)
```

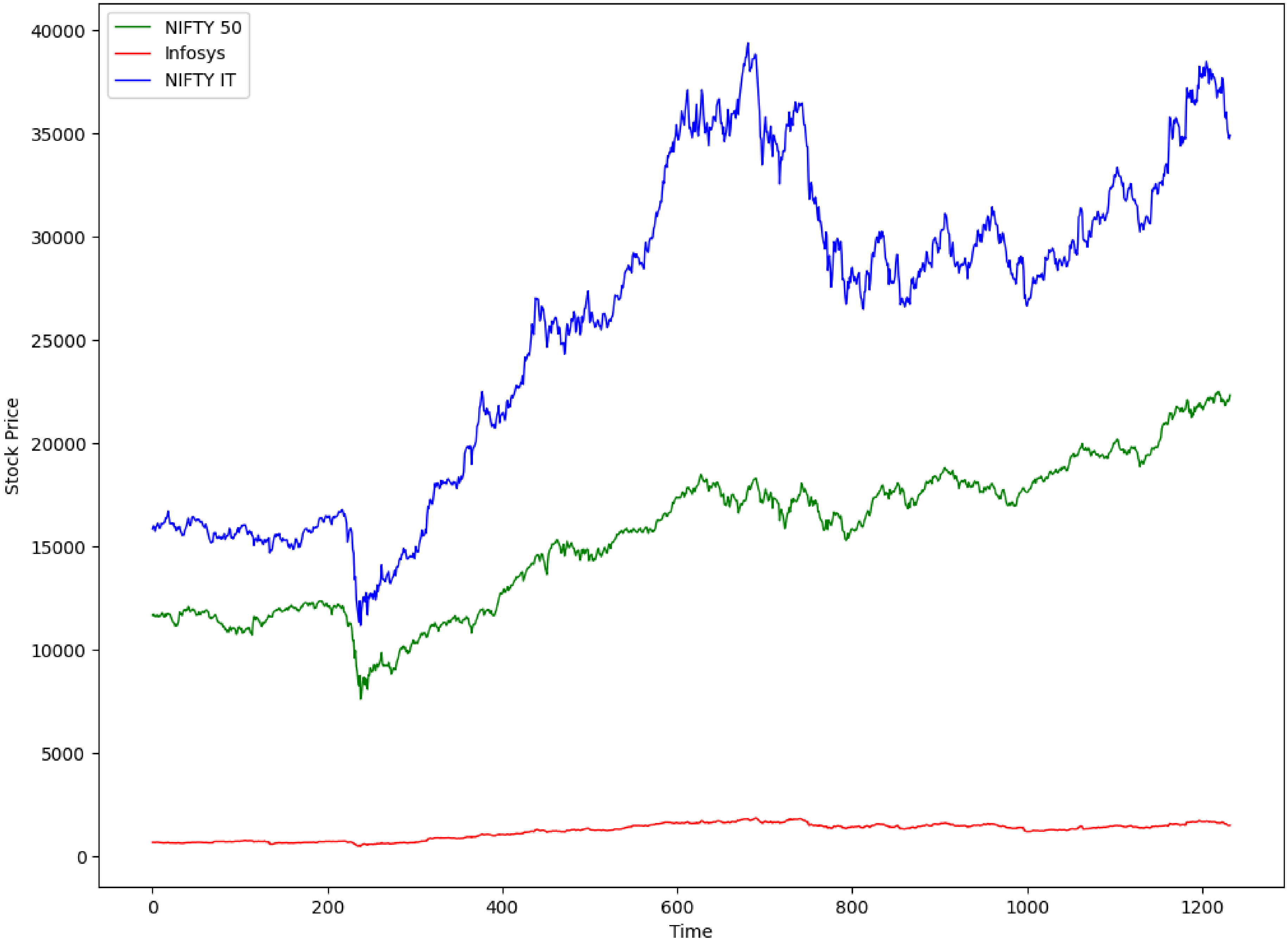
```
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
```

In [ ]:
```
T2_df.head()
```

Out[ ]:

	Date	NSE	INFY	IT
0	2019-04-01	11669.150391	671.111816	15840.650391
1	2019-04-02	11713.200195	674.933533	15988.400391
2	2019-04-03	11643.950195	669.511963	15962.950195
3	2019-04-04	11598.000000	664.712585	15744.799805
4	2019-04-05	11665.950195	674.844666	15923.049805

In [ ]:
```
plt.plot(T2_df["NSE"], linewidth=1, c="g", label="NIFTY 50")
plt.plot(T2_df["INFY"], linewidth=1, c="r", label="Infosys")
plt.plot(T2_df["IT"], linewidth=1, c="b", label="NIFTY IT")
plt.xlabel("Time")
plt.ylabel("Stock Price")
plt.legend()
plt.show()
```


NIFTY 50
Infosys
NIFTY IT
Stock Price
Time
40000
35000
30000
25000
20000
15000
10000
5000
0
0
200
400
600
800
1000
1200

Let's perform ADF Test to check for stationarity and unit roots existence:

In [ ]:
```
# ADF Test Results with 5% Significance Level for NSE, INFY,IT
nifty_adf = ADF(T2_df["NSE"], trend="n", method="bic")
infy_adf = ADF(T2_df["INFY"], trend="n", method="bic")
niftyit_adf = ADF(T2_df["IT"], trend="n", method="bic")

pd.DataFrame(
    {
        "NSE": (nifty_adf.stat,  nifty_adf.critical_values["5%"]),
        "| INFY": (infy_adf.stat,  infy_adf.critical_values["5%"]),
        "| IT": (niftyit_adf.stat, niftyit_adf.critical_values["5%"])
    },
    index=["ADF Test Statistic", "5% Critical Value"],
)
```

Out[ ]:

	NSE	| INFY	| IT
ADF Test Statistic	1.892990	0.696645	1.145281
5% Critical Value	-1.941221	-1.941221	-1.941221

We can see that for all the stock and indices the Test Statistic > 5% Critical Value, thus, we can not reject the Null Hypotheses, and it has unit roots. Hence, as per the ADF test all the series are non-stationary.

In [ ]:
```
print(KPSS(T2_df['NSE'], trend="ct", lags=-1))
print(KPSS(T2_df['INFY'], trend="ct", lags=-1))
print(KPSS(T2_df['IT'], trend="ct", lags=-1))
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.276
P-value                         0.003
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.930
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.711
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

As from the KPSS Test, we can see p-value < 0.05, thus, we can reject the null hypotheses, and thus thus the time series has unit root.Both tests conclude that the series is not stationary.

In [ ]:

```python
# Selection of Number of Lags for VAR Model
vecm_data = pd.concat(
    [T2_df['NSE'],T2_df['INFY'],T2_df['IT']], axis=1
)

# Fit VAR model and run lag selection tool
```

```
model = VAR(vecm_data)
x = model.select_order(maxlags=7, trend="c")
x.summary()
```

Out[ ]:

VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	40.82	40.84	5.362e+17	40.83
1	25.92	25.97*	1.806e+11	25.94*
2	25.91*	26.00	1.797e+11*	25.95
3	25.92	26.05	1.811e+11	25.97
4	25.93	26.10	1.829e+11	25.99
5	25.94	26.14	1.839e+11	26.01
6	25.94	26.18	1.845e+11	26.03
7	25.94	26.22	1.850e+11	26.05

From the above results, BIC and HQIC suggests lag = 1.

In [ ]:

```
# Johansen Trace Test Result
jtest = coint_johansen(vecm_data, det_order=0, k_ar_diff=1)
print(f"Eigenvalues of VECM coefficient matrix : {jtest.eig}\n")

pd.DataFrame(
    {
        "Test statistic": jtest.trace_stat,
        "Critical values (90%)": jtest.trace_stat_crit_vals[:, 0],
        "Critical values (95%)": jtest.trace_stat_crit_vals[:, 1],
        "Critical values (99%)": jtest.trace_stat_crit_vals[:, 2],
    },
    index=["rank=0", "rank<=1", "rank<=2"]
)
```

```
Eigenvalues of VECM coefficient matrix : [0.00494967 0.00278789 0.0001     ]
```

Out[ ]:

	Test statistic	Critical values (90%)	Critical values (95%)	Critical values (99%)
rank=0	9.620843	27.0669	29.7961	35.4628
rank<=1	3.542438	13.4294	15.4943	19.9349
rank<=2	0.122508	2.7055	3.8415	6.6349

In [ ]:

```
# VECM model
vecm_model = VECM(endog=vecm_data, k_ar_diff=1, deterministic="ci").fit()
print(vecm_model.summary())
```

```
Det. terms outside the coint. relation & lagged endog. parameters for equation NSE
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.NSE         0.0128      0.037      0.344      0.731      -0.060       0.086
L1.INFY        0.5969      0.482      1.240      0.215      -0.347       1.541
L1.IT         -0.0388      0.029     -1.335      0.182      -0.096       0.018
Det. terms outside the coint. relation & lagged endog. parameters for equation INFY
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.NSE        -0.0125      0.005     -2.593      0.010      -0.022      -0.003
L1.INFY        0.0104      0.062      0.167      0.868      -0.112       0.133
L1.IT          0.0057      0.004      1.512      0.131      -0.002       0.013
Det. terms outside the coint. relation & lagged endog. parameters for equation IT
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.NSE        -0.0912      0.087     -1.050      0.294      -0.261       0.079
L1.INFY        0.3945      1.124      0.351      0.726      -1.809       2.598
L1.IT          0.0509      0.068      0.751      0.453      -0.082       0.184
                Loading coefficients (alpha) for equation NSE
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1            0.0015      0.002      0.866      0.386      -0.002       0.005
                Loading coefficients (alpha) for equation INFY
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0001      0.000     -0.449      0.654      -0.001       0.000
                 Loading coefficients (alpha) for equation IT
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1            0.0030      0.004      0.716      0.474      -0.005       0.011
          Cointegration relations for loading-coefficients-column 1
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
beta.1         1.0000          0          0      0.000       1.000       1.000
beta.2        31.5261     13.897      2.268      0.023       4.288      58.764
beta.3        -1.8506      0.687     -2.693      0.007      -3.197      -0.504
```

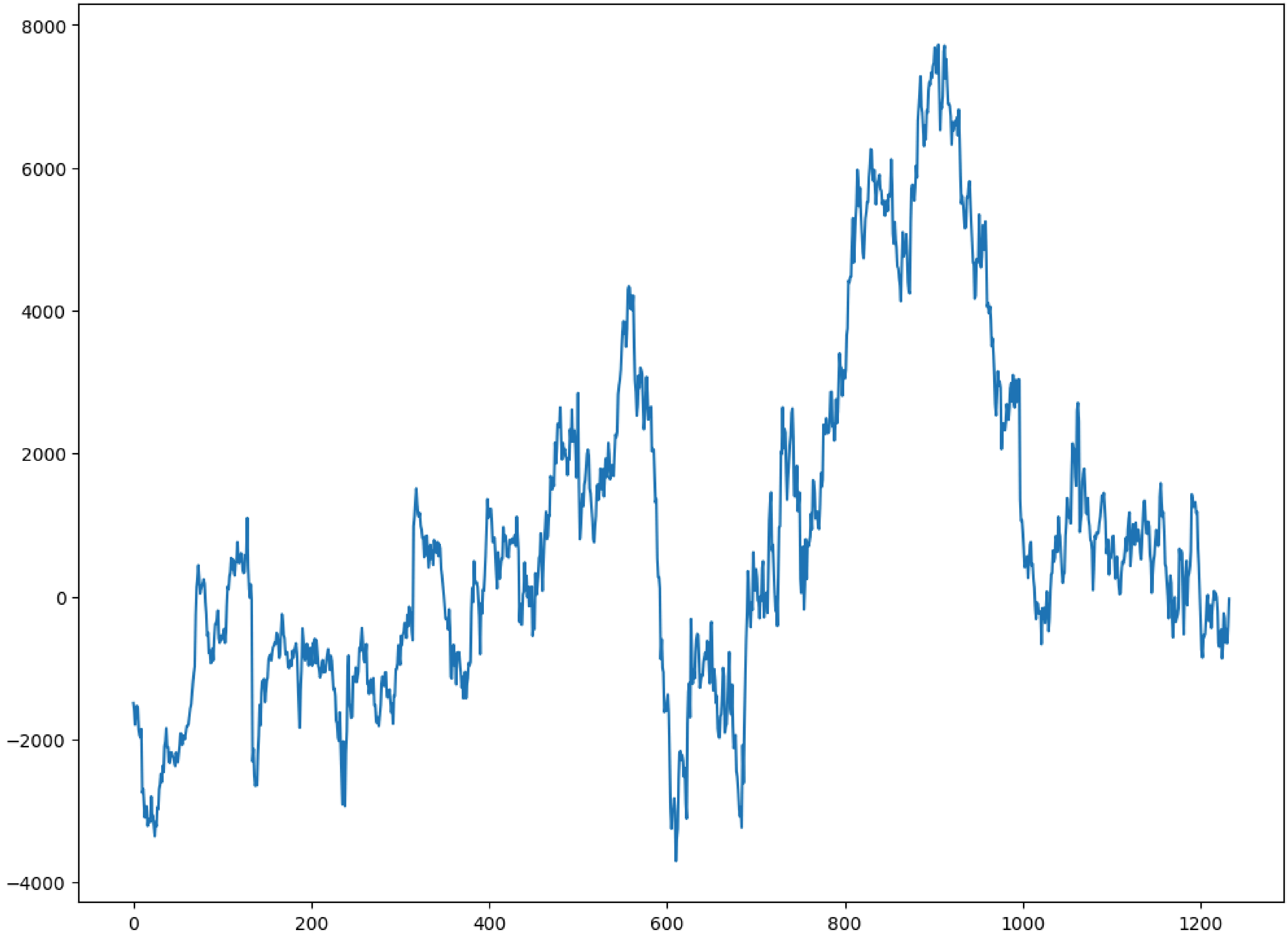
```
const        -5006.0496    3594.299      -1.393       0.164    -1.21e+04     2038.647
==============================================================================
```

The above results of the Johansen trace test shows the result for three tests: $H0$: rank = 0 , $H0$: rank = 1 , and $H0$: rank = 2 . Let's use 5% as our decision point. For $H0$: rank = 0 we can see the test statistic is 9.620845 and the 5% critical value is 29.7961. We can not reject $H0$ and say the rank is 0 or a cointegration relationship would exist. There is one linear combination of three time series variables that is stationary. The coefficients of the linear combination are in the second part under 'Cointegration relations for loading-coefficients-column 1' heading. We can write the linear combination as follows:

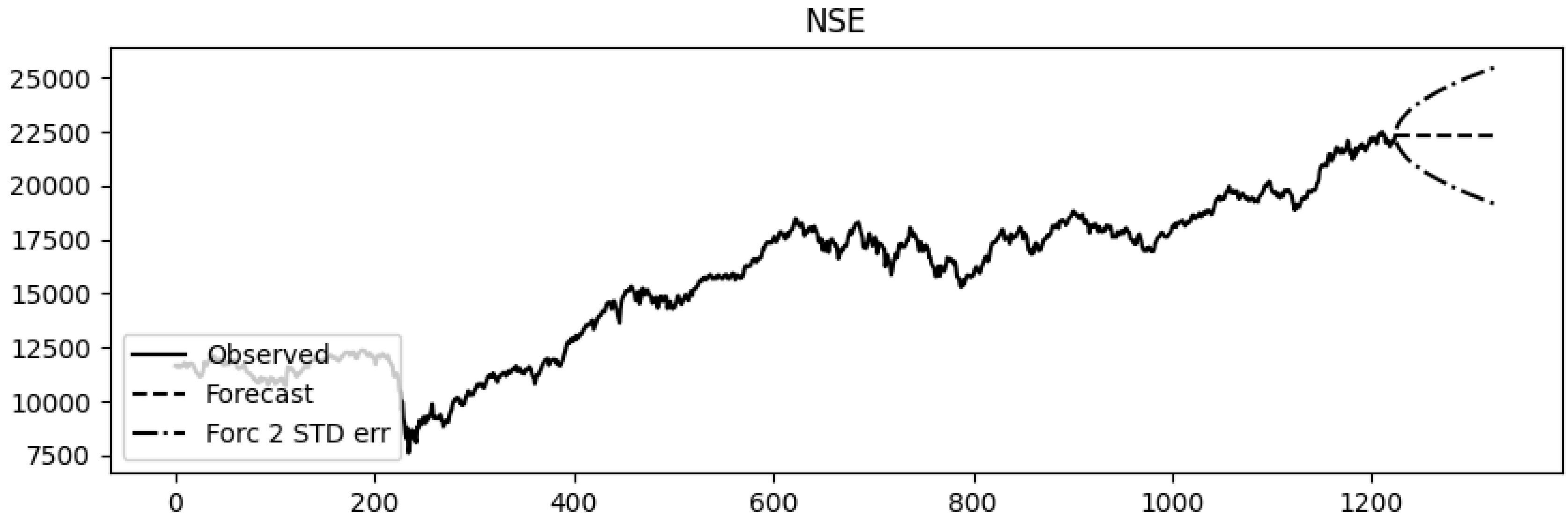
S= -5006.0528 + 1·NSE + 31.5261·INFY - 1.8506·IT

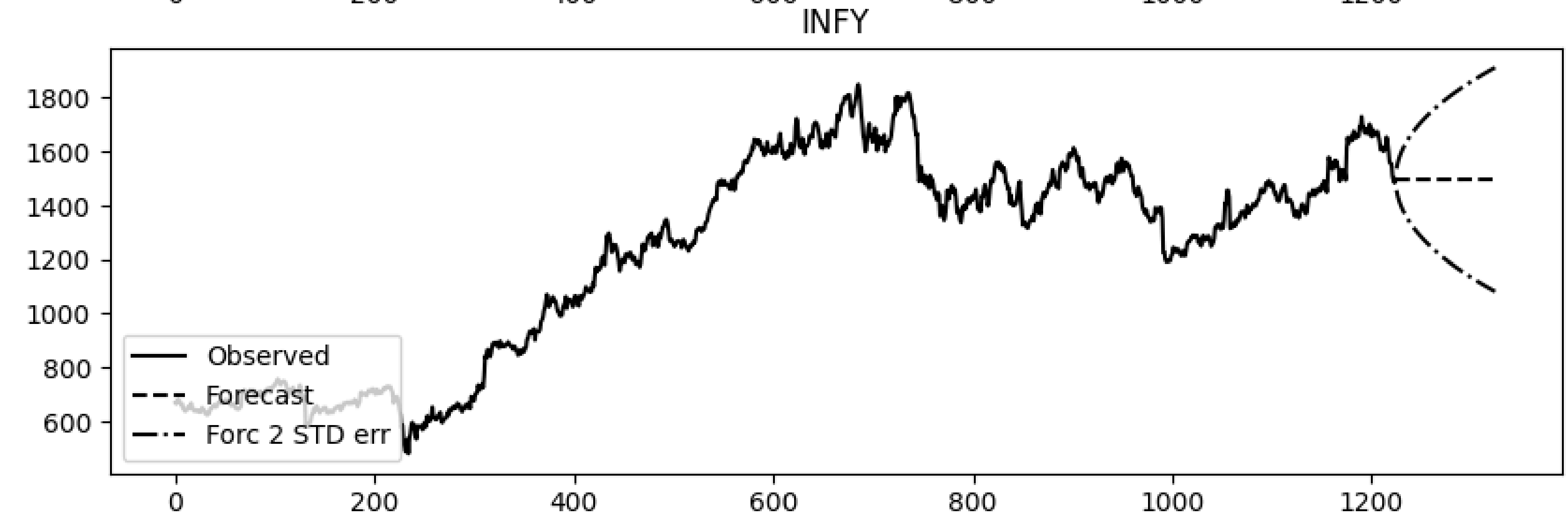
The above equation is the deviation from the long-term equilibrium of the three time series. Let's check out the plot to see if this deviation is stationary.

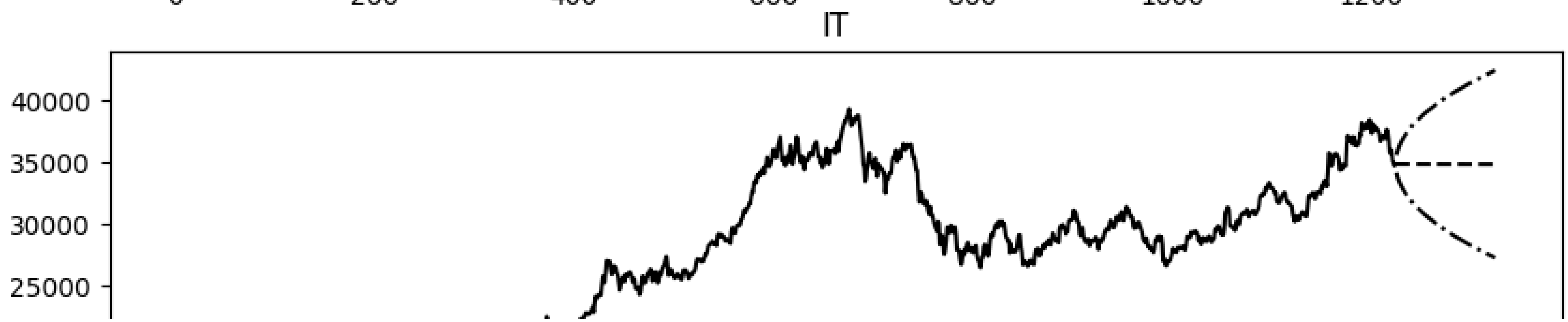
```
In [ ]: # Time Plot for Deviation from Long-Term Equilibrium
"""
The precise coefficients of the linear combination are in:
vecm_model.const_coint and vecm_model.beta
S= -5006.0528 + 1·NSE + 31.5261·INFY - 1.8506·IT
"""
S = (
    vecm_model.const_coint[0][0]
    + vecm_model.beta[0][0] * vecm_data.NSE
    + vecm_model.beta[1][0] * vecm_data.INFY
    + vecm_model.beta[2][0] * vecm_data.IT
)
plt.plot(S)
plt.show()
```

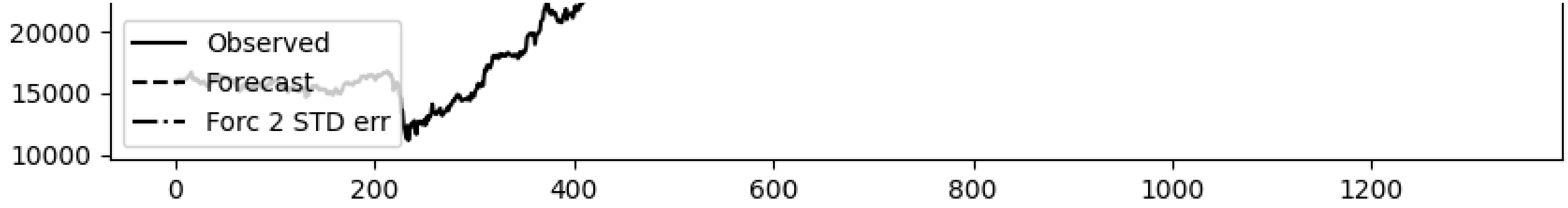

8000
6000
4000
2000
0
−2000
−4000
0
200
400
600
800
1000
1200

In [ ]:
```
vecm_model.plot_forecast(steps=100, alpha=0.05)
plt.show()
```


NSE
25000
22500
20000
17500
15000
12500
10000
7500
0
200
400
600
800
1000
1200
Observed
Forecast
Forc 2 STD err


INFY
1800
1600
1400
1200
1000
800
600
0
200
400
600
800
1000
1200
Observed
Forecast
Forc 2 STD err


IT
40000
35000
30000
25000



we can see the deviation from long-term equilibrium is moving within a channel. Let's take a look at the ADF test for the deviation.

In [ ]:
```
# ADF Test Result for Deviation from Long-Term Equilibrium
S_adf = ADF(S, trend="n", method="bic")
print("Augmented Dickey-Fuller Unit Root Test\n", S_adf.regression.summary())
print("\nTest statistics and critical values: \n", S_adf)
```

```
Augmented Dickey-Fuller Unit Root Test
                                  OLS Regression Results
=======================================================================================
Dep. Variable:                      y   R-squared (uncentered):                   0.004
Model:                            OLS   Adj. R-squared (uncentered):              0.003
Method:                 Least Squares   F-statistic:                              4.879
Date:                Mon, 08 Apr 2024   Prob (F-statistic):                      0.0274
Time:                        07:22:03   Log-Likelihood:                         -8788.3
No. Observations:                1226   AIC:                                  1.758e+04
Df Residuals:                    1225   BIC:                                  1.758e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Level.L1      -0.0078      0.004     -2.209      0.027      -0.015      -0.001
==============================================================================
Omnibus:                      152.715   Durbin-Watson:                   2.043
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1122.535
Skew:                          -0.305   Prob(JB):                    1.76e-244
Kurtosis:                       7.648   Cond. No.                         1.00
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Test statistics and critical values:
    Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -2.209
P-value                         0.026
Lags                                0
-------------------------------------

Trend: No Trend
Critical Values: -2.57 (1%), -1.94 (5%), -1.62 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

we can see that we easily reject that the deviation has a unit root because the test statistic (-2.209) is below 5% critical value (-1.94). P-value is < 0.05

Team Member 3

STOCKS CHOSEN:

Apple Incorp (AAPL), Intel Corporation (INTC), and The Walt Disney Company (DIS).

The AAPL and INTC are NasdaqGS real time price datas, DIS is NYSE delayed price data. All the three categories are different. Apple is in Consumer Electronics, Intel is in semiconductor and Disney is in the entertainment industry.

In [ ]:
```
tickers = ['AAPL', 'INTC', 'DIS']
```

In [ ]:
```
T3_df = assets(tickers)
```

```
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
```

Figure 3.1: Daily Price Data of Apple, Intel and Walt Disney Stock

In [ ]:
```
T3_df.head()
```

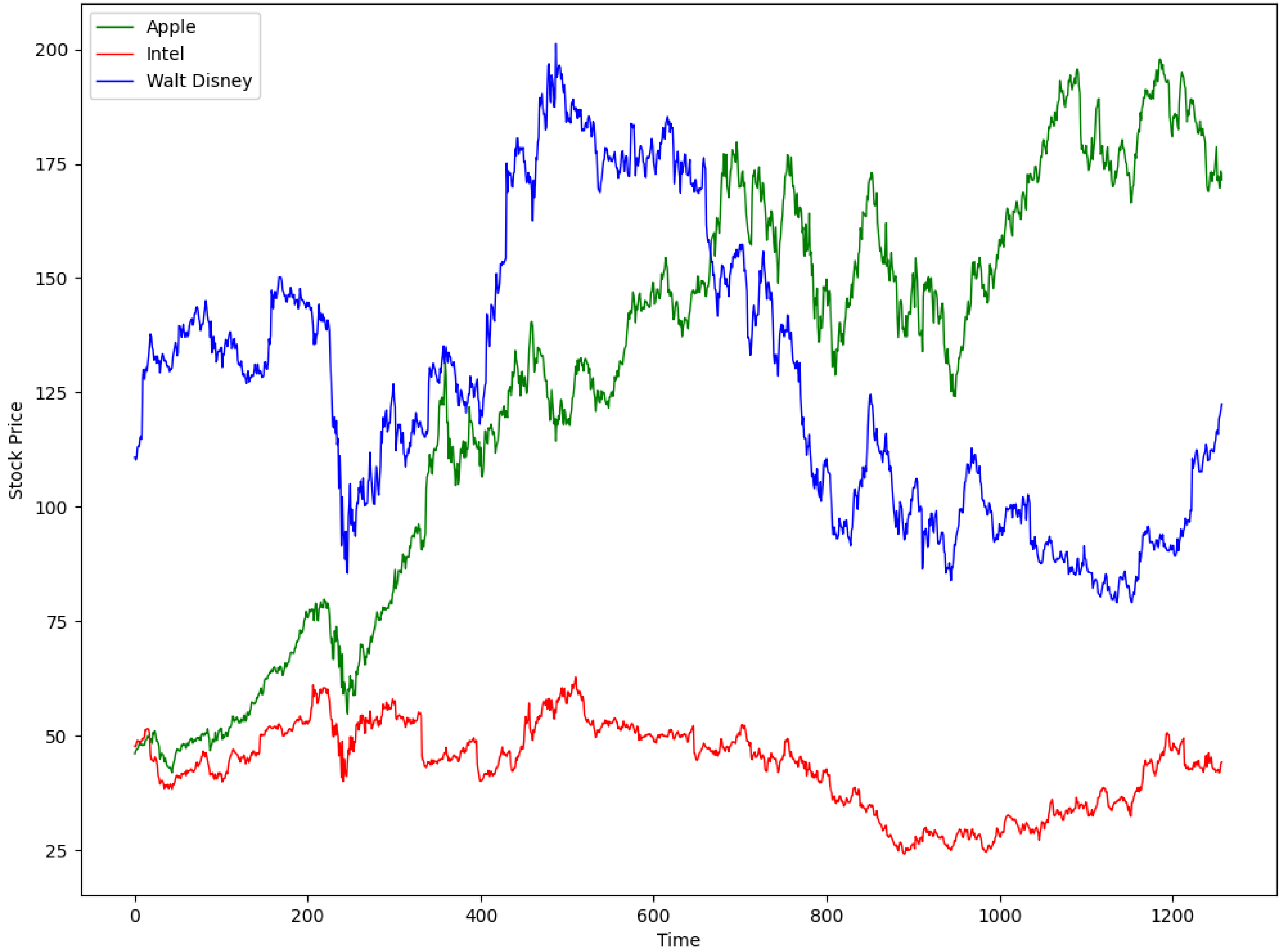
Out[ ]:

	Date	AAPL	INTC	DIS
0	2019-04-01	46.026638	47.702271	110.790527
1	2019-04-02	46.695717	47.571014	110.248924
2	2019-04-03	47.015808	48.551136	110.800369
3	2019-04-04	47.097637	48.936180	112.996284
4	2019-04-05	47.412914	48.656147	113.242462

Figure 3.2: Time series Line graph of Price Data of Apple, Intel and Walt Disney Stock

In [ ]:
```
plt.plot(T3_df["AAPL"], linewidth=1, c="g", label="Apple")
plt.plot(T3_df["INTC"], linewidth=1, c="r", label="Intel")
plt.plot(T3_df["DIS"], linewidth=1, c="b", label="Walt Disney")
plt.xlabel("Time")
```

```
plt.ylabel("Stock Price")
plt.legend()
plt.show()
```


Apple
Intel
Walt Disney
Stock Price
200
175
150
125
100
75
50
25
0
200
400
600
800
1000
1200
Time

Two statistical tests would be used to check the stationarity of a time series – Augmented Dickey Fuller ("ADF") test and Kwiatkowski-Phillips-Schmidt-Shin ("KPSS") test.

Figure 3.3: ADF Test for AAPL, INTC and DIS

In [ ]:
```
# ADF Test Results with 5% Significance Level for AAPL, INTC, DIS
aapl_adf = ADF(T3_df['AAPL'], trend="n", method="bic")
intc_adf = ADF(T3_df['INTC'], trend="n", method="bic")
dis_adf = ADF(T3_df['DIS'], trend="n", method="bic")


pd.DataFrame(
    {
        "Apple Inc.": (aapl_adf.stat,  aapl_adf.critical_values["5%"]),
        "| Intel Corp.": (intc_adf.stat,  intc_adf.critical_values["5%"]),
        "| Walt Disney Co.": (dis_adf.stat, dis_adf.critical_values["5%"],),
    },
    index=["ADF Test Statistic", "5% Critical Value"],
)
```

Out[ ]:

	Apple Inc.	| Intel Corp.	| Walt Disney Co.
ADF Test Statistic	0.857132	-0.482154	-0.238878
5% Critical Value	-1.941216	-1.941216	-1.941216

From figure 3.3, we can see that none of the financial assets has an ADF test statistic lower than 5% critical value. Hence, we cannot reject $H0$ hypothesis and there are unit roots in three time series. ADF test shows non stationary data. Now let us use KPSS test for the same issue of non stationarity.

Figure 3.4: KPSS Stationarity Test

In [ ]:
```
print(KPSS(T3_df['AAPL'], trend="ct", lags=-1))
print(KPSS(T3_df['INTC'], trend="ct", lags=-1))
print(KPSS(T3_df['DIS'], trend="ct", lags=-1))
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.776
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.478
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.776
P-value                         0.000
Lags                               23
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: 0.22 (1%), 0.15 (5%), 0.12 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

Based upon the significance level of 0.05 and the p-value of KPSS test, there is evidence for rejecting the null hypothesis in favor of the alternative. Hence, the series is non-stationary as per the KPSS test. Both tests conclude that the series is not stationary - The series is not stationary

Figure 3.5: VAR Order Selection

In [ ]:
```
# Selection of Number of Lags for VAR Model
vecm_data = pd.concat(
```

```
    [T3_df['AAPL'], T3_df['INTC'], T3_df['DIS']], axis=1
)

# Fit VAR model and run lag selection tool
model = VAR(vecm_data)
x = model.select_order(maxlags=7, trend="c")
x.summary()
```

Out[ ]: VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	18.10	18.12	7.290e+07	18.11
1	3.433	3.482*	30.97	3.451
2	3.409	3.495	30.24	3.442*
3	3.408*	3.531	30.21*	3.454
4	3.416	3.576	30.46	3.477
5	3.426	3.623	30.77	3.500
6	3.432	3.666	30.94	3.520
7	3.436	3.706	31.05	3.538

From figure 3.5, we can see all the information criteria select lag 3 for the level of the VAR model.

Now let's check Johansen trace test results.

Figure 3.6: VECM Coefficient Matrix (Johansen Trace Test)

In [ ]:
```
# Johansen Trace Test Result for AAPL, INTC and DIS
jtest = coint_johansen(vecm_data, det_order=0, k_ar_diff=1)
# Print the results
print(f"Eigenvalues of VECM coefficient matrix : {jtest.eig}\n")

pd.DataFrame(
    {
        "Test statistic": jtest.trace_stat,
        "Critical values (90%)": jtest.trace_stat_crit_vals[:, 0],
        "Critical values (95%)": jtest.trace_stat_crit_vals[:, 1],
```

```
        "Critical values (99%)": jtest.trace_stat_crit_vals[:, 2],
    },
    index=["rank=0", "rank<=1", "rank<=2"],
)
```

```
Eigenvalues of VECM coefficient matrix : [0.00550217 0.00345251 0.00098268]
```

Out[ ]:

	Test statistic	Critical values (90%)	Critical values (95%)	Critical values (99%)
rank=0	12.508523	27.0669	29.7961	35.4628
rank<=1	5.578712	13.4294	15.4943	19.9349
rank<=2	1.234858	2.7055	3.8415	6.6349

Figure 3.6: VECM Model

In [ ]:

```
# VECM model
vecm_model = VECM(endog=vecm_data, k_ar_diff=1, deterministic="ci").fit()
print(vecm_model.summary())
```

```
Det. terms outside the coint. relation & lagged endog. parameters for equation AAPL
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.AAPL        0.0289      0.033      0.889      0.374      -0.035       0.093
L1.INTC       -0.1993      0.072     -2.778      0.005      -0.340      -0.059
L1.DIS        -0.0446      0.029     -1.543      0.123      -0.101       0.012
Det. terms outside the coint. relation & lagged endog. parameters for equation INTC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.AAPL       -0.0235      0.015     -1.611      0.107      -0.052       0.005
L1.INTC       -0.1174      0.032     -3.648      0.000      -0.180      -0.054
L1.DIS        -0.0138      0.013     -1.066      0.286      -0.039       0.012
Det. terms outside the coint. relation & lagged endog. parameters for equation DIS
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.AAPL        0.0652      0.035      1.862      0.063      -0.003       0.134
L1.INTC       -0.0561      0.077     -0.728      0.467      -0.207       0.095
L1.DIS        -0.0729      0.031     -2.344      0.019      -0.134      -0.012
                Loading coefficients (alpha) for equation AAPL
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1        -2.512e-06   1.22e-05     -0.205      0.837   -2.65e-05    2.15e-05
                Loading coefficients (alpha) for equation INTC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1         9.899e-06   5.49e-06      1.803      0.071    -8.6e-07    2.07e-05
                Loading coefficients (alpha) for equation DIS
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1        -1.345e-05   1.32e-05     -1.022      0.307   -3.92e-05    1.23e-05
          Cointegration relations for loading-coefficients-column 1
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
beta.1         1.0000          0          0      0.000       1.000       1.000
beta.2      -813.5871    317.003     -2.567      0.010   -1434.901    -192.274
beta.3       188.8296     91.045      2.074      0.038      10.384     367.275
```

```
const       1.011e+04   1.05e+04      0.966      0.334   -1.04e+04    3.06e+04
==============================================================================
```

In [ ]:
```
# VECM model
vecm_model = VECM(endog=vecm_data, k_ar_diff=1, deterministic="ci").fit()
print(vecm_model.summary())
```

Det. terms outside the coint. relation & lagged endog. parameters for equation AAPL

	coef	std err	z	P>|z|	[0.025	0.975]
L1.AAPL	0.0289	0.033	0.889	0.374	-0.035	0.093
L1.INTC	-0.1993	0.072	-2.778	0.005	-0.340	-0.059
L1.DIS	-0.0446	0.029	-1.543	0.123	-0.101	0.012

Det. terms outside the coint. relation & lagged endog. parameters for equation INTC

	coef	std err	z	P>|z|	[0.025	0.975]
L1.AAPL	-0.0235	0.015	-1.611	0.107	-0.052	0.005
L1.INTC	-0.1174	0.032	-3.648	0.000	-0.180	-0.054
L1.DIS	-0.0138	0.013	-1.066	0.286	-0.039	0.012

Det. terms outside the coint. relation & lagged endog. parameters for equation DIS

	coef	std err	z	P>|z|	[0.025	0.975]
L1.AAPL	0.0652	0.035	1.862	0.063	-0.003	0.134
L1.INTC	-0.0561	0.077	-0.728	0.467	-0.207	0.095
L1.DIS	-0.0729	0.031	-2.344	0.019	-0.134	-0.012

Loading coefficients (alpha) for equation AAPL

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-2.512e-06	1.22e-05	-0.205	0.837	-2.65e-05	2.15e-05

Loading coefficients (alpha) for equation INTC

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	9.899e-06	5.49e-06	1.803	0.071	-8.6e-07	2.07e-05

Loading coefficients (alpha) for equation DIS

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-1.345e-05	1.32e-05	-1.022	0.307	-3.92e-05	1.23e-05

Cointegration relations for loading-coefficients-column 1

	coef	std err	z	P>|z|	[0.025	0.975]
beta.1	1.0000	0	0	0.000	1.000	1.000
beta.2	-813.5871	317.003	-2.567	0.010	-1434.901	-192.274
beta.3	188.8296	91.045	2.074	0.038	10.384	367.275

```
const        1.011e+04    1.05e+04       0.966       0.334    -1.04e+04     3.06e+04
==============================================================================
```

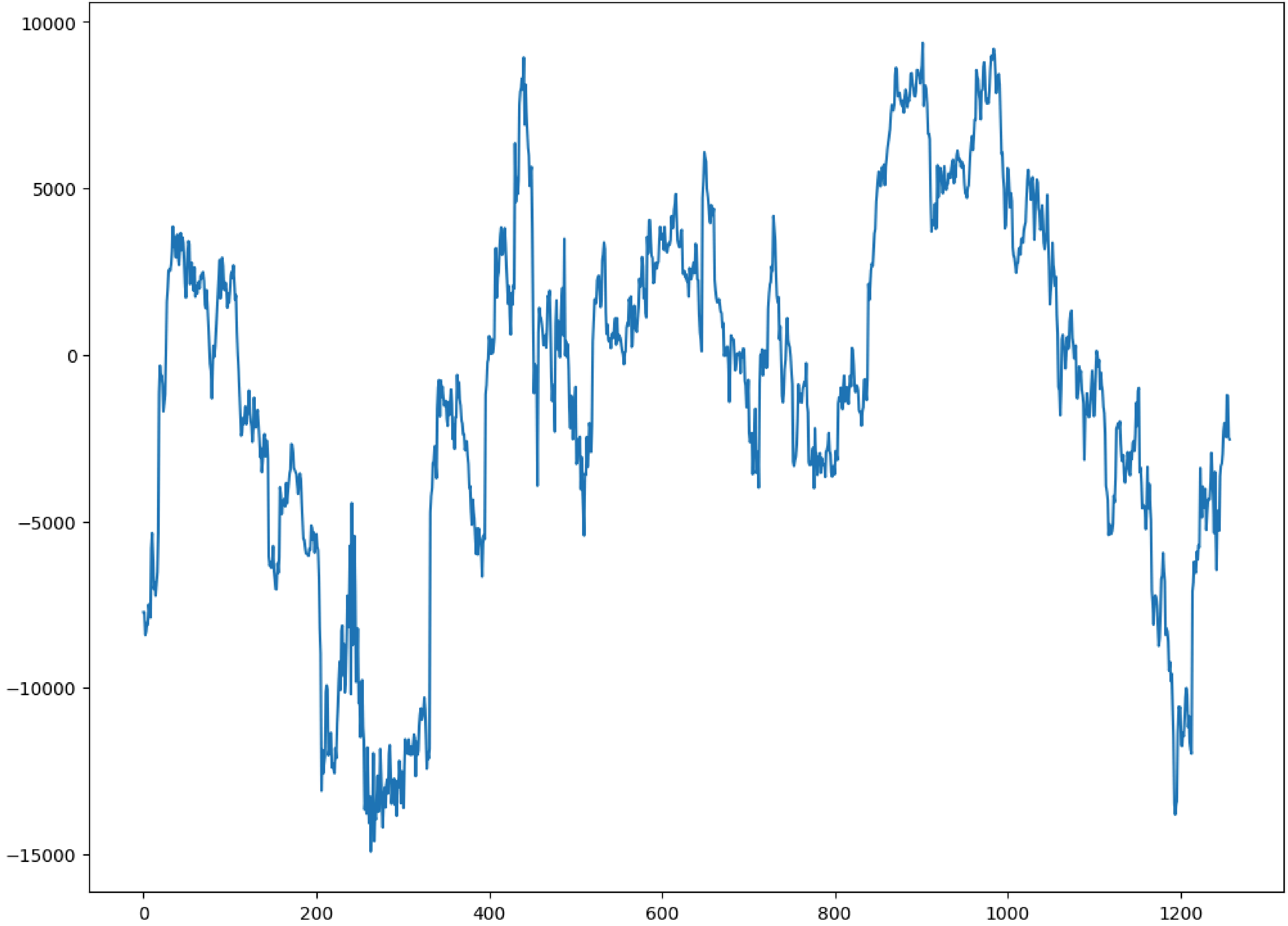
Figure 3.6 shows the result of the Johansen trace test. The top part in Figure 3.6 shows the result for three tests: $H0$: rank = 0 , $H0$: rank = 1 , and $H0$: rank = 2 . Let's use 5% as our decision point. For $H0$: rank = 0 we can see the test statistic is 12.508522 and the 5% critical value is 29.7961. We can not reject $H0$ and say the rank is 0 or a cointegration relationship would exist. There is one linear combination of three time series variables that is stationary. The coefficients of the linear combination are in the second part under 'Cointegration relations for loading-coefficients-column 1' heading. We can write the linear combination as follows:

S=1.011e+04+1·AAPL+188.8112·INTC-813.5078·DIS

The above equation is the deviation from the long-term equilibrium of the three time series. Let's check out the plot to see if this deviation is stationary.

Figure 3.7: Time Plot for Deviation from Long-Term Equilibrium

In [ ]:
```
# Time Plot for Deviation from Long-Term Equilibrium
"""
The precise coefficients of the linear combination are in:
vecm_model.const_coint and vecm_model.beta
S=1.011e+04+1*AAPL+188*8112·INTC-813*5078·DIS
"""
S = (
    vecm_model.const_coint[0][0]
    + vecm_model.beta[0][0] * vecm_data.AAPL
    + vecm_model.beta[1][0] * vecm_data.INTC
    + vecm_model.beta[2][0] * vecm_data.DIS
)
plt.plot(S)
plt.show()
```


10000
5000
0
−5000
−10000
−15000
0
200
400
600
800
1000
1200

From figure 3.7, we can see the deviation from long-term equilibrium is moving within a channel. Let's take a look at the ADF test for the deviation.

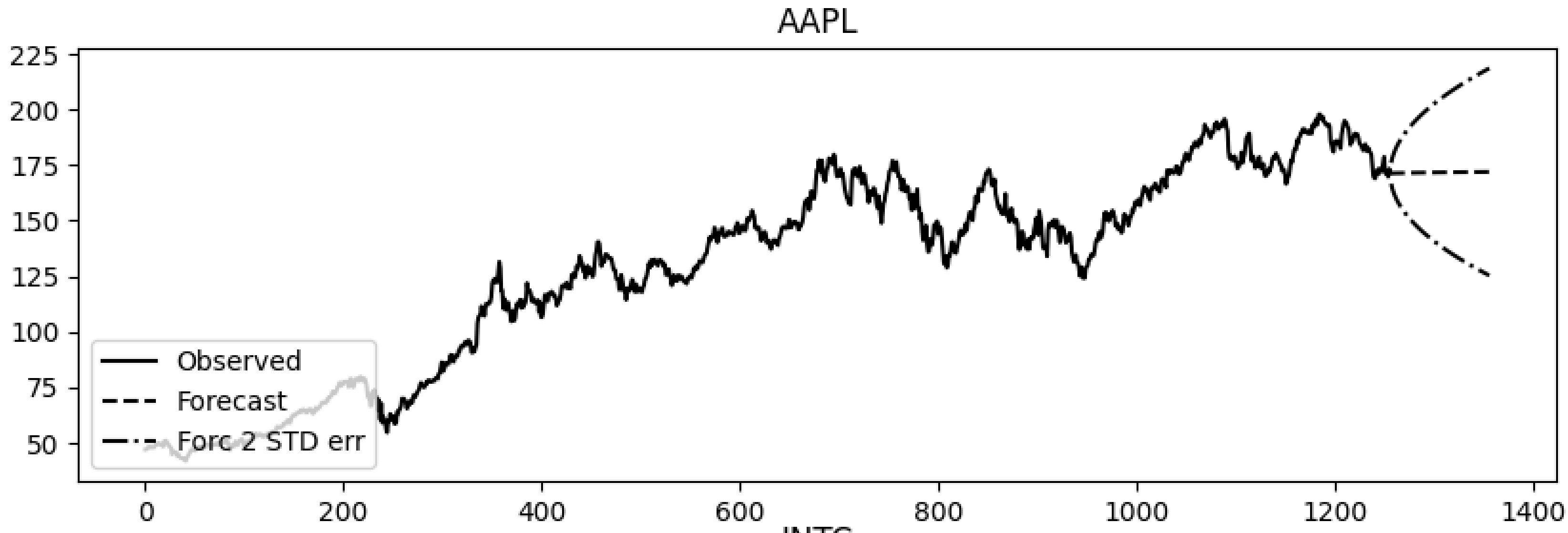
Figure 3.8: ADF Test Result for Deviation from Long-Term Equilibrium

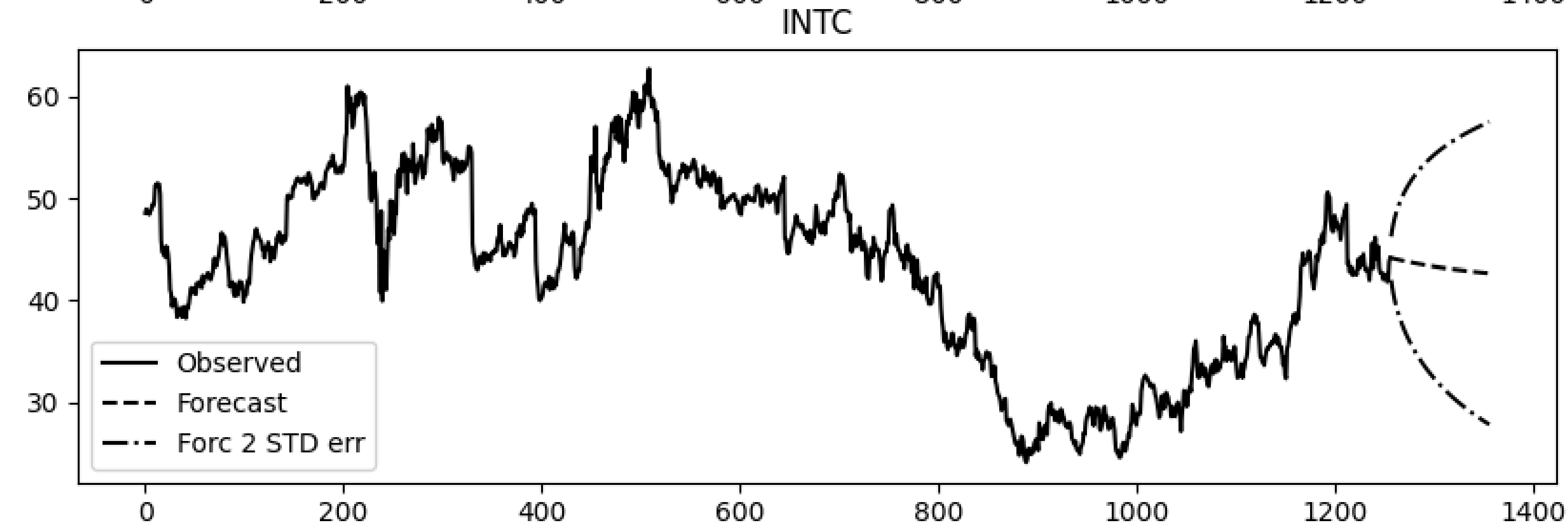
In [ ]:
```
# ADF Test Result for Deviation from Long-Term Equilibrium
S_adf = ADF(S, trend="n", method="bic")
print("Augmented Dickey-Fuller Unit Root Test\n", S_adf.regression.summary())
print("\nTest statistics and critical values: \n", S_adf)
```

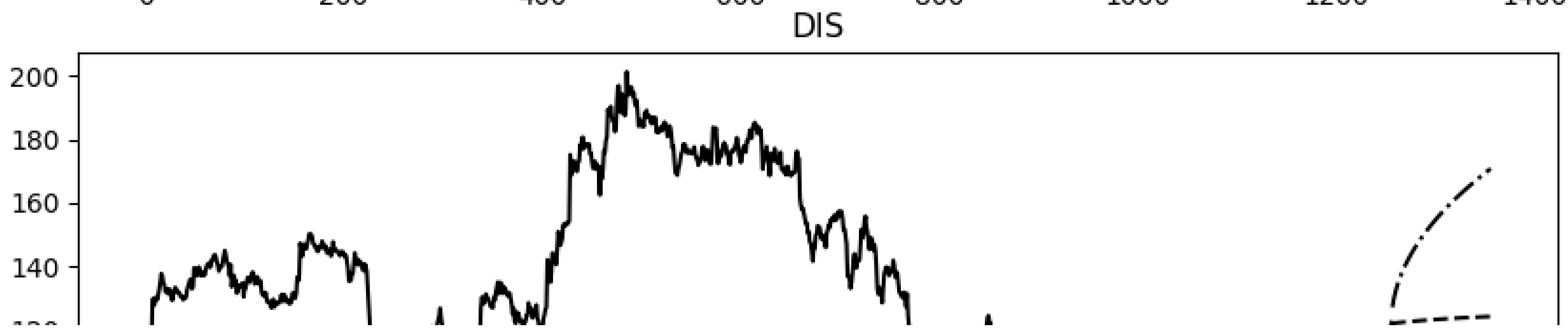
```
Augmented Dickey-Fuller Unit Root Test
                                  OLS Regression Results
=======================================================================================
Dep. Variable:                      y   R-squared (uncentered):                  0.020
Model:                            OLS   Adj. R-squared (uncentered):             0.019
Method:                 Least Squares   F-statistic:                             12.94
Date:                Mon, 08 Apr 2024   Prob (F-statistic):                   2.75e-06
Time:                        07:22:05   Log-Likelihood:                        -10237.
No. Observations:                1256   AIC:                                 2.048e+04
Df Residuals:                    1254   BIC:                                 2.049e+04
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Level.L1      -0.0110      0.004     -2.580      0.010      -0.019      -0.003
Diff.L1       -0.1174      0.028     -4.188      0.000      -0.172      -0.062
==============================================================================
Omnibus:                      366.336   Durbin-Watson:                   1.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5587.802
Skew:                           0.923   Prob(JB):                         0.00
Kurtosis:                      13.167   Cond. No.                         6.58
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Test statistics and critical values:
    Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -2.580
P-value                         0.010
Lags                                1
-------------------------------------

Trend: No Trend
Critical Values: -2.57 (1%), -1.94 (5%), -1.62 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

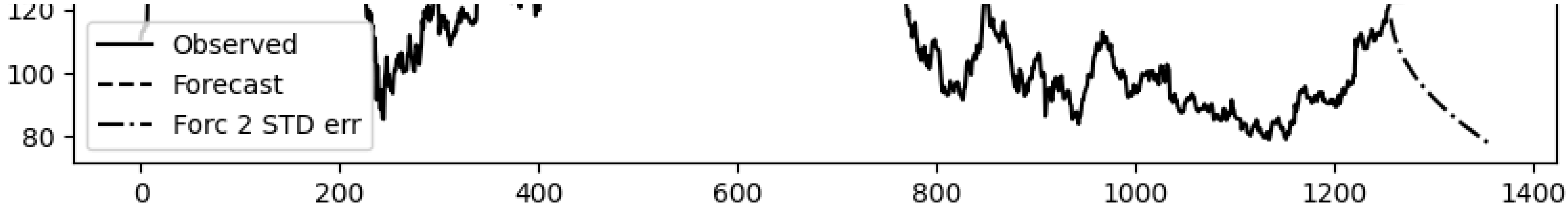
From figure 3.8, we can see that we easily reject that the deviation has a unit root because the test statistic (-2.580) is below 5% critical value (-1.94). P-value is < 0.05

In [47]:
```
vecm_model.plot_forecast(steps=100, alpha=0.05)
plt.show()
```


AAPL
Observed
Forecast
Forc 2 STD err


INTC
Observed
Forecast
Forc 2 STD err


DIS


120
100
80
Observed
Forecast
Forc 2 STD err
0
200
400
600
800
1000
1200
1400

In [ ]: