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RK4_or_propagator_j2

Guidance, Navigation and Controls Subsystem

rk_4()

Code author: Shivnash chakrawarti
Created on: 08/06/2021
Last modified: 07/07/2021
Reviwed by: KT Prajwal Pratiksh
Description:
Uses Rk4 numerical integration method to calculate the value at the next instant of the initial state vector containing radial components and velocity components whose degree 1 differential equations are known with respect to time.It uses a particular step size h and the starting and ending time instants between which we need the values of the state vectors using the preceding state vector and It also takes another function as an argument which contains the degree 1 differential equations.It first calculates the total instants for which we want the state vector using the starting,ending time values and the step size which is given by us and arranges it in a form of an array.Then it creates an zero array whose number of columns are total instants and number of rows are the number of components of initial state vector that is radial and velocity components.And now the first column of this array is equated to the initial state vector and then using a for loop to apply RK4 method formula and storing the value at the next instant in that zero array and after the for loop is iterated the final array is formed which contains the values of positions and velocity vector at all the instants.
Formula & References:
Formula used-$y_{i+1} = y_i + \frac{h}{6}(\tilde{f}_1 + 2\tilde{f}_2 + 2\tilde{f}_3 + \tilde{f}_4)$ where $\tilde{f}_1 = f(t_i, y_i), \tilde{f}_2 = f(t_i + \frac{h}{2}, y_i + \frac{h\tilde{f}_1}{2}), \tilde{f}_3 = f(t_i + \frac{h}{2}, y_i + \frac{h\tilde{f}_2}{2}), \tilde{f}_4 = f(t_i + h, y_i + h\tilde{f}_3)$ This is RK4 method which is given in the book Orbital mechanics for Engineering students by Horward D.Curtis(4th edition).
Input parameters:

1. **function** : (Float) - function which returns the derivatives of radial and velocity components . *m/s,m/s^2*
2. **v_pos_vel** : (Float) - A column array containing initial position and velocity components of the state vector. *m,m/s*
3. **t_step** : (Float) - time interval between the instants that we choose . *seconds*
4. **t_end** : (Float) - time until which we are calculating the state vectors. *seconds*
5. **t_start** : (Float) - initial time at which state vector is known which is 0.0 . *seconds*

Output:
function returns an array containing the values of all the position and velocity components at every instant between t_start and t_end .The x,y,z components of position are in first 3 rows respectively and x,y,z components of velocity are in next 3 rows respectively and also returns the time instants in an array form.

circular_orbit_j_2()

Code author: Shivansh chakrawarti
Created on: 08/06/2021
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Description: Calculates the derivatives of position and velocity vector at a particular instant ,the derivative of position vector is velocity and j_2 perturbation is included so the function calculates the effect on velocity derivative which is acceleration due to earth's oblateness (ignoring terms after j_2 term) in the x,y,z directions and then adding the effect on the gravitational acceleration(without considering j_2) in the x,y,z direction and then the function returns the array containing the position and velocity derivatives.
Formula & References:
Formula used - $\frac{dr}{dt} = v$ and now effect due to j_2 -
j_x=-$\frac{3}{2} j_2 \frac{\mu}{|r|^2}(\frac{R}{|r|})^2 \frac{r_x}{|r|}(5(\frac{r_z}{|r|})^2 - 1)$
j_y=-$\frac{3}{2} j_2 \frac{\mu}{|r|^2}(\frac{R}{|r|})^2 \frac{r_y}{|r|}(5(\frac{r_z}{|r|})^2 - 1)$
j_z=-$\frac{3}{2} j_2 \frac{\mu}{|r|^2}(\frac{R}{|r|})^2 \frac{r_z}{|r|}(5(\frac{r_z}{|r|})^2 - 3)$
So,
$\frac{dv_x}{dt} = -\frac{\mu}{|r|^3} r_x + j_x$
$\frac{dv_y}{dt} = -\frac{\mu}{|r|^3} r_y + j_y$
$\frac{dv_z}{dt} = -\frac{\mu}{|r|^3} r_z + j_z$
This is given in the book Orbital Mechanics for Engineering students by Horward D. Cutris
Input parameters:

1. **v_pos_vel** : (Float) - a vector containing initial position and velocity which will be used to calculate the value of derivatives. *m,m/s*

2. **t** : (Float) - time at which the derivatives are being calculated. *seconds*

Output:
Returns an array containing the derivatives in x,y,z direction of position and velocity ,first 3 rows contains the value of position vector derivative and next 3 contains velocity vector derivative