

# 432 Class 20 Slides

[github.com/THOMASELOVE/2020-432](https://github.com/THOMASELOVE/2020-432)

2020-04-07

# Preliminaries

```
library(here); library(janitor); library(magrittr)
library(broom); library(knitr); library(rms)
library(survival); library(survminer)
library(tidyverse)
```

```
theme_set(theme_bw())
```

```
brca <- read_csv(here("data", "brca.csv")) %>%
  type.convert()
```

# Working with Time-to-Event Data

- The brca trial of three treatments for Breast Cancer
- Cox Proportional Hazards Regression with `coxph`
- Diagnostics for Cox Proportional Hazards Regression
- Using `cph` from the `rms` package to fit a Cox model

# The brca trial

# The brca trial

The brca data describes a parallel randomized trial of three treatments, adjuvant to surgery in the treatment of patients with stage-2 carcinoma of the breast. The three treatment groups are:

- S\_CT = Surgery plus one year of chemotherapy
- S\_IT = Surgery plus one year of immunotherapy
- S\_Both = Surgery plus one year of chemotherapy and immunotherapy

The measure of efficacy were “time to death” in weeks. In addition to treat, our variables are:

- trial\_weeks: time in the study, in weeks, to death or censoring
- last\_alive: 1 if alive at last follow-up (and thus censored), 0 if dead
- age: age in years at the start of the trial

## brca tibble (note big problem: n = 31!)

Source: Chen and Peace (2011) *Clinical Trial Data Analysis Using R*, CRC Press, section 5.1

```
# A tibble: 31 x 5
```

	subject <fct>	treat <fct>	trial_weeks <int>	last_alive <int>	age <int>
1	A01	S_CT	102	0	55
2	A02	S_IT	192	0	62
3	A03	S_Both	73	0	72
4	A04	S_CT	58	1	48
5	A05	S_CT	48	1	26
6	A06	S_IT	182	1	52
7	A07	S_IT	196	1	50
8	A08	S_CT	177	1	49
9	A09	S_IT	191	1	62
10	A10	S_Both	36	0	60

```
# ... with 21 more rows
```

# Analytic Objectives

This is a typical right-censored survival data set with interest in the comparative analysis of the three treatments.

- 1 Does immunotherapy added to surgery plus chemotherapy improve survival? (Comparing  $S_{\text{Both}}$  to  $S_{\text{CT}}$ )
- 2 Does chemotherapy add efficacy to surgery plus immunotherapy? ( $S_{\text{Both}}$  vs.  $S_{\text{IT}}$ )
- 3 What is the effect of age on survival?

# Create survival object

- `trial_weeks`: time in the study, in weeks, to death or censoring
- `last_alive`: 1 if alive at last follow-up (and thus censored), 0 if dead

So `last_alive = 0` if the event (death) occurs.

*What's next?*



# Create survival object

- `trial_weeks`: time in the study, in weeks, to death or censoring
- `last_alive`: 1 if alive at last follow-up (and thus censored), 0 if dead

So `last_alive = 0` if the event (death) occurs.

```
brca$S <- with(brca, Surv(trial_weeks, last_alive == 0))
```

```
head(brca$S)
```

```
[1] 102  192   73  58+  48+ 182+
```

# Build Kaplan-Meier Estimator

```
kmfit <- survfit(S ~ treat, dat = brca)
```

```
print(kmfit, print.rmean = TRUE)
```

```
Call: survfit(formula = S ~ treat, data = brca)
```

	n	events	*rmean	*se(rmean)	median	0.95LCL
treat=S_Both	10	4	188	23.7	NA	139
treat=S_CT	11	6	153	21.1	144	102
treat=S_IT	10	5	188	17.9	192	144

0.95UCL

treat=S_Both	NA
treat=S_CT	NA
treat=S_IT	NA

\* restricted mean with upper limit = 242

```
> summary(kmfit)
```

```
Call: survfit(formula = S ~ treat, data = brca)
```

treat=S\_Both

time	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
36	10	1	0.900	0.0949	0.732	1
73	9	1	0.800	0.1265	0.587	1
139	8	1	0.700	0.1449	0.467	1
185	6	1	0.583	0.1610	0.340	1

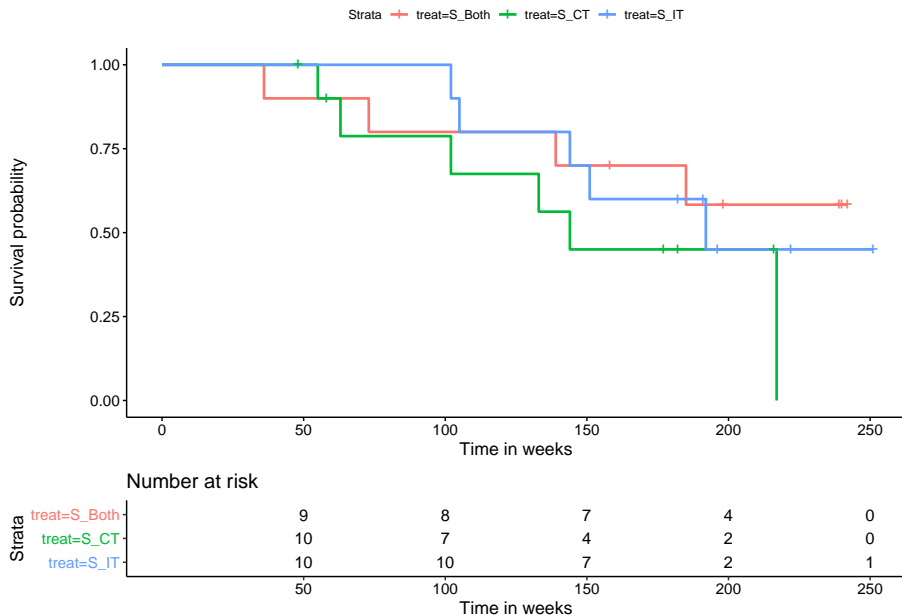
treat=S\_CT

time	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
55	10	1	0.900	0.0949	0.732	1.000
63	8	1	0.787	0.1340	0.564	1.000
102	7	1	0.675	0.1551	0.430	1.000
133	6	1	0.562	0.1651	0.316	1.000
144	5	1	0.450	0.1660	0.218	0.927
217	1	1	0.000	NaN	NA	NA

treat=S\_IT

time	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
102	10	1	0.90	0.0949	0.732	1.000
105	9	1	0.80	0.1265	0.587	1.000
144	8	1	0.70	0.1449	0.467	1.000
151	7	1	0.60	0.1549	0.362	0.995
192	4	1	0.45	0.1743	0.211	0.961

# K-M Plot via survminer



## K-M Plot via survminer (code)

```
ggsurvplot(kmfit, data = brca,  
            risk.table = TRUE,  
            risk.table.height = 0.25,  
            xlab = "Time in weeks")
```

# Testing the difference between curves

```
survdif(S ~ treat, dat = brca)
```

Call:

```
survdif(formula = S ~ treat, data = brca)
```

	N	Observed	Expected	$(O-E)^2/E$	$(O-E)^2/V$
treat=S_Both	10	4	5.62	0.4676	0.7725
treat=S_CT	11	6	3.80	1.2772	1.7647
treat=S_IT	10	5	5.58	0.0605	0.0981

Chisq= 1.9 on 2 degrees of freedom, p= 0.4

What do we conclude?

# A Cox Model for Treatment

## Fit Cox Model `mod_T`: Treatment alone

```
mod_T <- coxph(S ~ treat, data = brca)
mod_T
```

Call:

```
coxph(formula = S ~ treat, data = brca)
```

	coef	exp(coef)	se(coef)	z	p
treatS_CT	0.8313	2.2963	0.6547	1.270	0.204
treatS_IT	0.2481	1.2816	0.6740	0.368	0.713

Likelihood ratio test=1.75 on 2 df, p=0.4164  
n= 31, number of events= 15



```
> summary(mod_T)
```

```
Call:
```

```
coxph(formula = S ~ treat, data = brca)
```

```
n= 31, number of events= 15
```

	coef	exp(coef)	se(coef)	z	Pr(> z )
treatS_CT	0.8313	2.2963	0.6547	1.270	0.204
treatS_IT	0.2481	1.2816	0.6740	0.368	0.713

	exp(coef)	exp(-coef)	lower .95	upper .95
treatS_CT	2.296	0.4355	0.6364	8.286
treatS_IT	1.282	0.7803	0.3420	4.803

```
Concordance= 0.577 (se = 0.083 )
```

```
Likelihood ratio test= 1.75 on 2 df, p=0.4
```

```
Wald test = 1.82 on 2 df, p=0.4
```

```
Score (logrank) test = 1.89 on 2 df, p=0.4
```

# Interpreting the Summaries

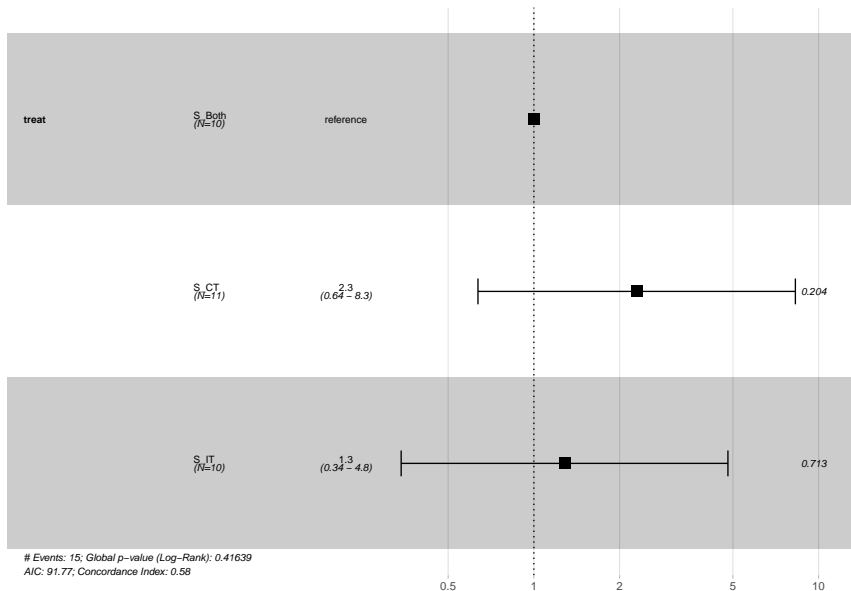
```
tidy(mod_T, exponentiate = TRUE) %>%  
  select(term, estimate, std.error, conf.low, conf.high) %>%  
  kable(digits = 3)
```

term	estimate	std.error	conf.low	conf.high
treatS_CT	2.296	0.655	0.636	8.286
treatS_IT	1.282	0.674	0.342	4.803

- A subject treated with S\_CT is estimated to have 2.296 times the hazard (95% CI: 0.636, 8.286) of a subject treated with S\_Both (the baseline).
- A subject treated with S\_IT is estimated to have 1.282 times the hazard (95% CI 0.342, 4.803) of a subject treated with S\_Both.

```
ggforest(mod_T, data = brca)
```

Hazard ratio



# Summarizing `mod_T`

All of this comes from `glance(mod_T)`

- $n = 31$  cases, with `nevent` = 15 events (so 16 censored)
- log rank test statistic = 1.752,  $p = 0.416$
- Score test statistic = 1.895,  $p = 0.388$
- Wald test statistic = 1.820,  $p = 0.403$ 
  - Each tests  $H_0$ : Treatment adds no value
- (Cox-Snell) R-Squared = 0.055, Maximum Pseudo R-Square = 0.944
  - Cox and Snell's pseudo- $R^2$  reflects the improvement of this model over the model with the intercept alone, with higher values indicating more substantial improvement over an intercept-only model.
  - Not really a percentage of anything: often the maximum value here is less than 1.

## Summarizing `mod_T`

Again, all of this comes from `glance(mod_T)` - see next slide

- Concordance = 0.577 (standard error = 0.083)
  - Really only appropriate when we have at least one quantitative predictor in the Cox model
  - Assesses probability of agreement between survival time and the risk score generated by the predictors
  - 1 indicates perfect agreement, 0.5 indicates no better than chance
- log Likelihood = -43.886, AIC = 91.773, BIC = 93.189
  - Usual summaries, used to compare models, mostly

## glance(mod\_T) Fit Quality Summaries (edited)

---

n	31.000
nevent	15.000
statistic.log	1.752
p.value.log	0.416
statistic.sc	1.895
p.value.sc	0.388
statistic.wald	1.820
p.value.wald	0.403
r.squared	0.055
r.squared.max	0.944
concordance	0.577
std.error.concordance	0.083
logLik	-43.886
AIC	91.773
BIC	93.189

---

# Checking the Proportional Hazards Assumption

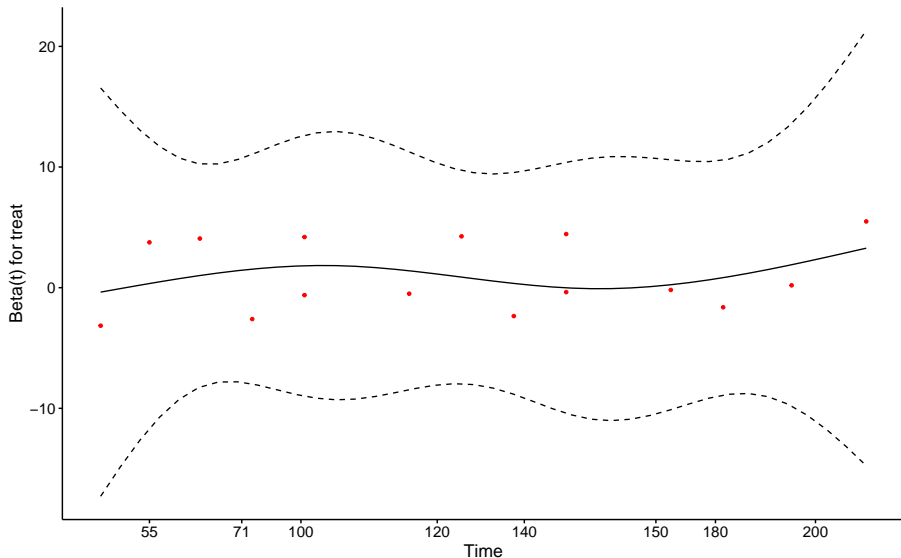
- If the proportional hazards assumption is appropriate, we should see a slope of essentially zero in the residuals that are plotted against time on the next slide.
- If we see a slope that seriously different from zero, that will suggest a violation of the proportional hazards assumption.
- A hypothesis test is also performed, where a significant result also indicates a potential problem with the assumption.

If we did see a violation of assumptions, we could either add a non-linear predictor term or use a different kind of survival model.

# Graphical PH Check `ggcoxzph(cox.zph(mod_T))`

Global Schoenfeld Test p: 0.4691

Schoenfeld Individual Test p: 0.4691





## A Two-Predictor Cox Model (Age + Treatment)

## Fit Cox Model mod\_AT: Age + Treatment

```
mod_AT <- coxph(S ~ age + treat, data = brca)
mod_AT
```

Call:

```
coxph(formula = S ~ age + treat, data = brca)
```

	coef	exp(coef)	se(coef)	z	p
age	0.07807	1.08119	0.03672	2.126	0.0335
treatS_CT	0.59960	1.82139	0.65741	0.912	0.3617
treatS_IT	0.28799	1.33375	0.68566	0.420	0.6745

Likelihood ratio test=6.99 on 3 df, p=0.07224  
n= 31, number of events= 15

```
> summary(mod_AT)
```

```
Call:
```

```
coxph(formula = S ~ age + treat, data = brca)
```

```
n= 31, number of events= 15
```

	coef	exp(coef)	se(coef)	z	Pr(> z )
age	0.07807	1.08119	0.03672	2.126	0.0335 *
treatS_CT	0.59960	1.82139	0.65741	0.912	0.3617
treatS_IT	0.28799	1.33375	0.68566	0.420	0.6745

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

	exp(coef)	exp(-coef)	lower .95	upper .95
age	1.081	0.9249	1.0061	1.162
treatS_CT	1.821	0.5490	0.5021	6.607
treatS_IT	1.334	0.7498	0.3479	5.113

```
Concordance= 0.701 (se = 0.065 )
```

```
Likelihood ratio test= 6.99 on 3 df, p=0.07
```

```
Wald test = 5.85 on 3 df, p=0.1
```

```
Score (logrank) test = 6.15 on 3 df, p=0.1
```

# Interpreting the Coefficients of mod\_AT

```
tidy(mod_AT, exponentiate = TRUE) %>%  
  select(term, estimate, std.error, conf.low, conf.high) %>%  
  kable(digits = 2)
```

term	estimate	std.error	conf.low	conf.high
age	1.08	0.04	1.01	1.16
treatS_CT	1.82	0.66	0.50	6.61
treatS_IT	1.33	0.69	0.35	5.11

- If Harry and Sally receive the same treat but Harry is one year older, the model estimates Harry will have 1.08 times the hazard of Sally (95% CI 1.01, 1.16).

## Interpreting the Coefficients of mod\_AT

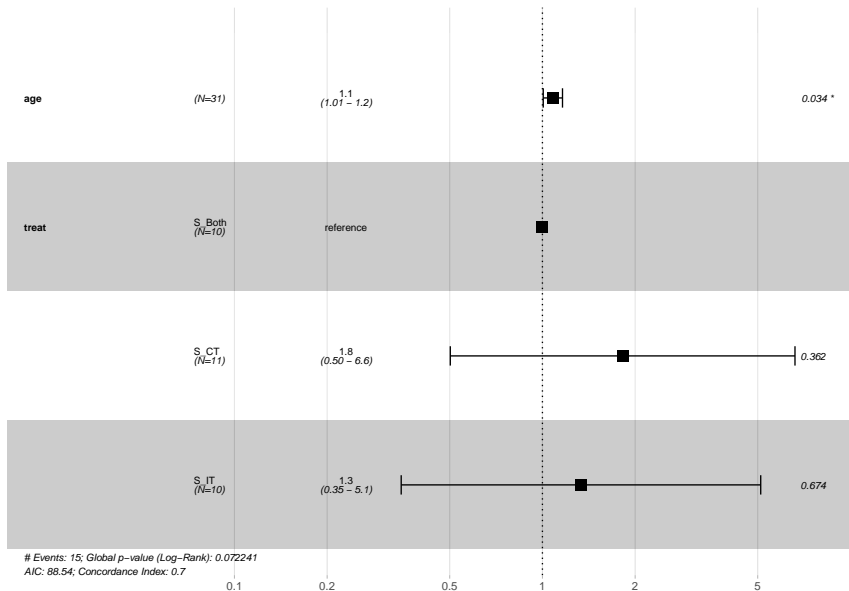
```
tidy(mod_AT, exponentiate = TRUE) %>%  
  select(term, estimate, std.error, conf.low, conf.high) %>%  
  kable(digits = 2)
```

term	estimate	std.error	conf.low	conf.high
age	1.08	0.04	1.01	1.16
treatS_CT	1.82	0.66	0.50	6.61
treatS_IT	1.33	0.69	0.35	5.11

- If Harry receives S\_CT and Sally receives S\_Both, and they are the same age, the model estimates Harry will have 1.82 times the hazard of Sally (95% CI 0.50, 6.61).
- If Cyrus receives S\_IT and Sally receives S\_Both, and they are the same age, the model estimates Cyrus will have 1.33 times the hazard of Sally (95% CI 0.33, 5.11).

```
ggforest(mod_AT, data = brca)
```

Hazard ratio



# Comparing the Two Models

n = 31, nevent = 15 for each model.

```
bind_rows(glance(mod_T), glance(mod_AT)) %>%  
  mutate(model = c("mod_T", "mod_AT")) %>%  
  select(model, p.value.log, concordance, r.squared,  
         max_r2 = r.squared.max, AIC, BIC) %>%  
  kable(digits = c(0,3,3,3,3,1,1))
```

model	p.value.log	concordance	r.squared	max_r2	AIC	BIC
mod_T	0.416	0.577	0.055	0.944	91.8	93.2
mod_AT	0.072	0.701	0.202	0.944	88.5	90.7

What do the glance results indicate?

# Significance Test via Likelihood Ratio ANOVA

```
anova(mod_AT, mod_T)
```

Analysis of Deviance Table

Cox model: response is S

Model 1: ~ age + treat

Model 2: ~ treat

	loglik	Chisq	Df	P(> Chi )
1	-41.268			
2	-43.886	5.237	1	0.02211 *

---

Signif. codes:

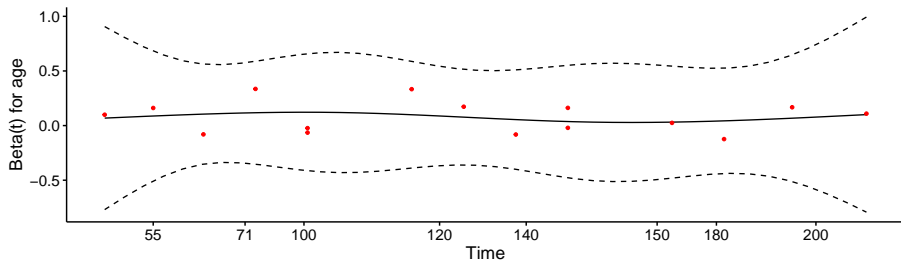
0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



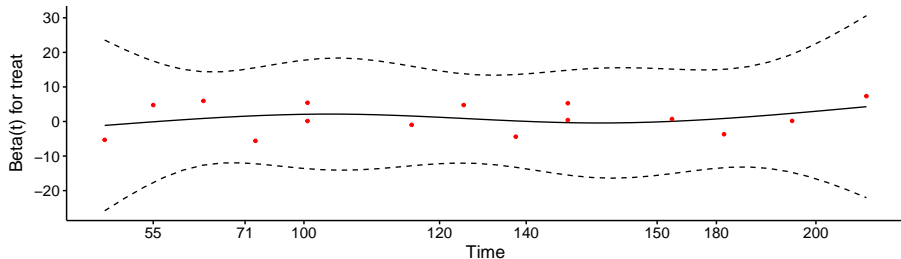
# Graphical PH Check `ggcoxzph(cox.zph(mod_AT))`

Global Schoenfeld Test p: 0.4817

Schoenfeld Individual Test p: 0.7172



Schoenfeld Individual Test p: 0.2967



# Back to the Analytic Objectives

- 1 Does immunotherapy added to surgery plus chemotherapy improve survival? (Comparing  $S_{\text{Both}}$  to  $S_{\text{CT}}$ )
- 2 Does chemotherapy add efficacy to surgery plus immunotherapy? ( $S_{\text{Both}}$  vs.  $S_{\text{IT}}$ )
- 3 What is the effect of age on survival?

So ... what do you think?

## Building a Cox Model with `cph` from the `rms` package

## Using `rms::cph` to fit a fancier AxT

```
brca <- read_csv(here("data/brca.csv")) %>% type.convert()

d <- datadist(brca)
options(datadist="d")

brca$S <- with(brca, Surv(trial_weeks, last_alive == 0))

cph_AxT <- cph(S ~ rcs(age, 4) + treat + age %ia% treat,
               data = brca,
               x = TRUE, y = TRUE, surv = TRUE)
```

## cph\_AxT (and see next slide)

### Cox Proportional Hazards Model

```
cph(formula = S ~ rcs(age, 4) + treat + age %ia% treat, data =  
x = TRUE, y = TRUE, surv = TRUE)
```

Model Tests				Discrimination Indexes	
Obs	31	LR chi2	11.66	R2	0.332
Events	15	d.f.	7	Dxy	0.488
Center	14.2906	Pr(> chi2)	0.1123	g	1.980
		Score chi2	11.89	gr	7.245
		Pr(> chi2)	0.1042		

	Coef	S.E.	Wald Z	Pr(> Z )
age	0.3011	0.2330	1.29	0.1963
age'	-1.2521	0.7528	-1.66	0.0963
age''	2.7316	1.5490	1.76	0.0778
treat=S_CT	-4.9327	6.6650	-0.74	0.4592

```
> cph_AxT
```

```
Cox Proportional Hazards Model
```

```
cph(formula = S ~ rcs(age, 4) + treat + age %ia% treat, data = brca,  
     x = TRUE, y = TRUE, surv = TRUE)
```

		Model Tests		Discrimination Indexes	
Obs	31	LR chi2	11.66	R2	0.332
Events	15	d.f.	7	Dxy	0.488
Center	14.2906	Pr(> chi2)	0.1123	g	1.980
		Score chi2	11.89	gr	7.245
		Pr(> chi2)	0.1042		

	Coef	S.E.	Wald Z	Pr(> Z )
age	0.3011	0.2330	1.29	0.1963
age'	-1.2521	0.7528	-1.66	0.0963
age''	2.7316	1.5490	1.76	0.0778
treat=S_CT	-4.9327	6.6650	-0.74	0.4592
treat=S_IT	0.1210	4.8180	0.03	0.9800
age * treat=S_CT	0.1006	0.1157	0.87	0.3846
age * treat=S_IT	-0.0005	0.0835	-0.01	0.9949

## summary(cph\_AxT)

Effects

Response : S

Factor	Low	High	Diff.	Effect	S.E.
age	49.5	61.5	12	0.48200	0.99998
Hazard Ratio	49.5	61.5	12	1.61930	NA
treat - S_Both:S_CT	2.0	1.0	NA	-0.49745	0.80805
Hazard Ratio	2.0	1.0	NA	0.60808	NA
treat - S_IT:S_CT	2.0	3.0	NA	-0.40504	0.78888
Hazard Ratio	2.0	3.0	NA	0.66695	NA

Lower 0.95 Upper 0.95

-1.47790 2.4419

0.22811 11.4950

-2.08120 1.0863

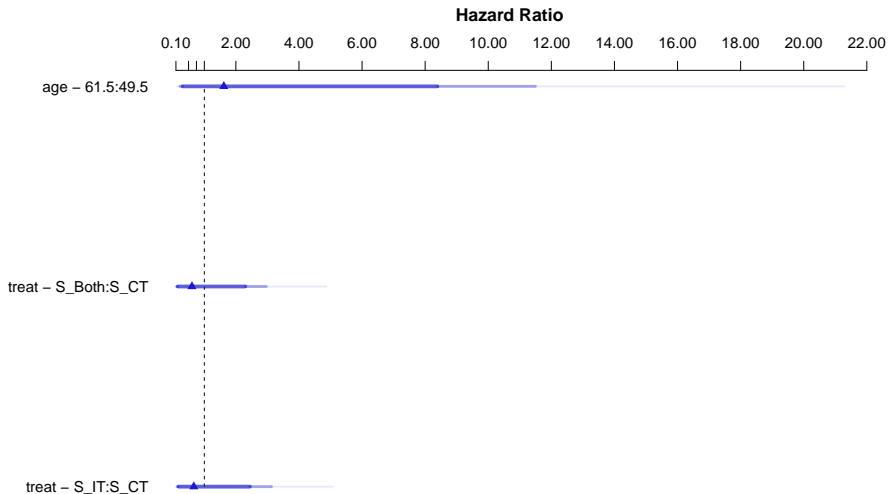
0.12478 2.9633

-1.95120 1.1411

0.14210 3.1303

Adjusted to: age=54 treat=S\_CT

```
plot(summary(cph_AxT))
```



Adjusted to:age=54 treat=S\_CT



```
set.seed(432)
validate(cph_AxT)
```

```
Warning in fitter(..., strata = strata, rownames = rownames,
offset = offset, : Ran out of iterations and did not
converge
```

Divergence or singularity in 1 samples

	index.orig	training	test	optimism	index.corrected
Dxy	0.4883	0.5965	0.3693	0.2273	0.2610
R2	0.3320	0.4741	0.2061	0.2680	0.0640
Slope	1.0000	1.0000	0.3819	0.6181	0.3819
D	0.1191	0.2078	0.0650	0.1428	-0.0237
U	-0.0223	-0.0226	1.0971	-1.1196	1.0973
Q	0.1414	0.2303	-1.0321	1.2624	-1.1210
g	1.9803	4.2315	1.2169	3.0145	-1.0342

n

Dxy 39

R2 39

# ANOVA for cph\_AxT model

```
> anova(cph_AxT)
```

Wald Statistics

Response: S

Factor	Chi-Square	d.f.	P
age (Factor+Higher Order Factors)	7.71	5	0.1727
All Interactions	0.96	2	0.6175
Nonlinear	3.73	2	0.1548
treat (Factor+Higher Order Factors)	2.58	4	0.6297
All Interactions	0.96	2	0.6175
age * treat (Factor+Higher Order Factors)	0.96	2	0.6175
TOTAL NONLINEAR + INTERACTION	3.74	4	0.4423
TOTAL	8.55	7	0.2868

## survplot in rms (code)

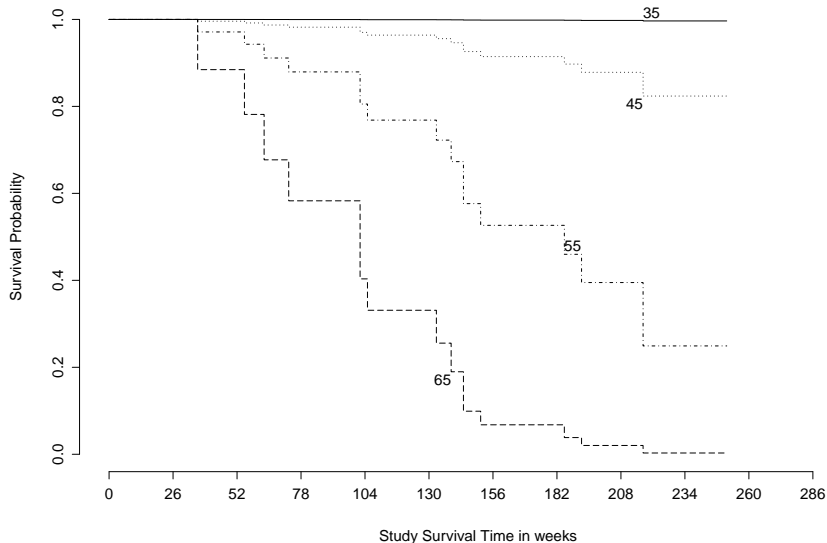
For age comparison:

```
survplot(cph_AxT,  
         age = c(35, 45, 55, 65),  
         time.inc = 26,  
         type = "kaplan-meier",  
         xlab = "Study Survival Time in weeks")
```

For treat comparison:

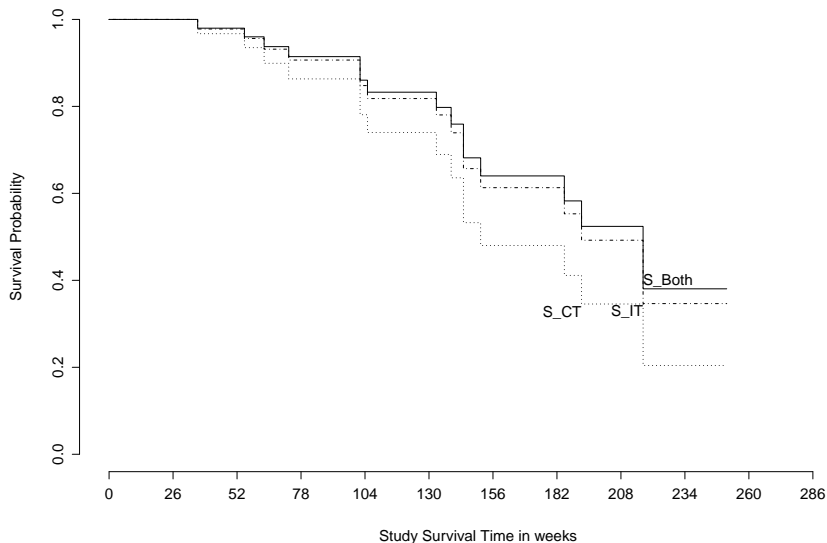
```
survplot(cph_AxT,  
         treat,  
         time.inc = 26,  
         type = "kaplan-meier",  
         xlab = "Study Survival Time in weeks")
```

# survplot in rms (Result)



Adjusted to: treat=S\_CT

# survplot for treat in rms (Result)



Adjusted to: age=54

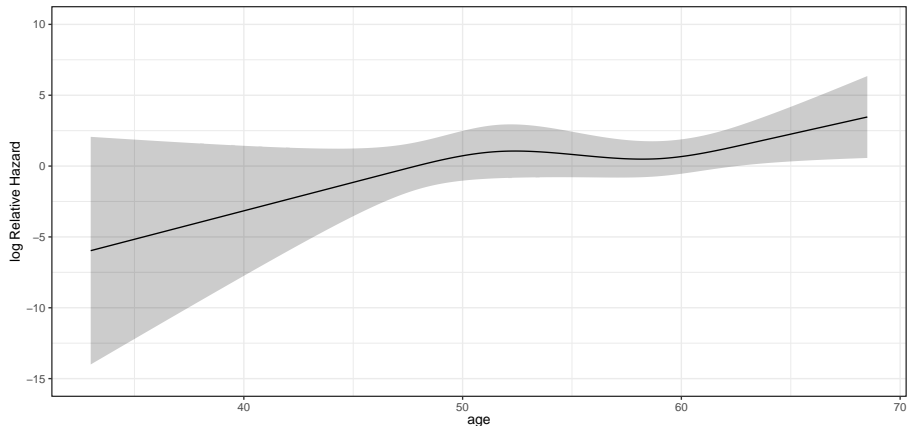
## cph\_AxT nomogram (code)

```
sv <- Survival(cph_AxT)
surv4 <- function(x) sv(208, lp = x)

plot(nomogram(cph_AxT,
              fun = surv3,
              funlabel = c("4 year survival")))
```

# Plotting age effect implied by cph\_AxT model

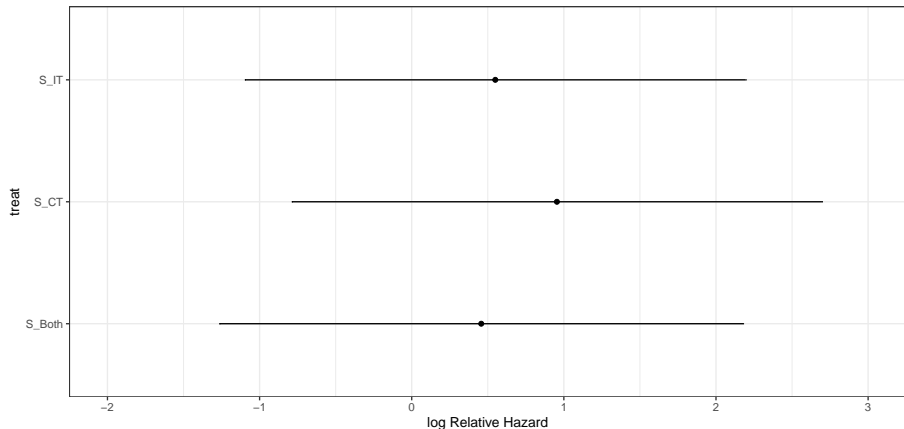
```
ggplot(Predict(cph_AxT, age))
```



Adjusted to: treat=S\_CT

# Plotting treat effect implied by cph\_AxT model

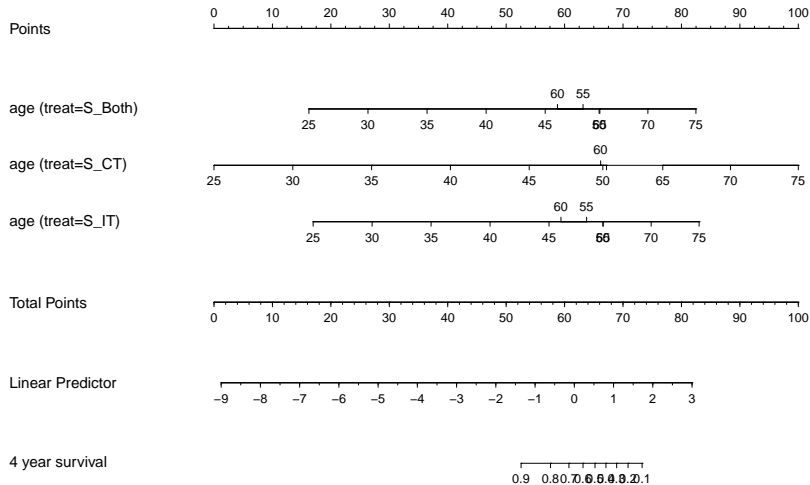
```
ggplot(Predict(cph_AxT, treat))
```



Adjusted to: age=54



# cph\_AxT nomogram (Results)

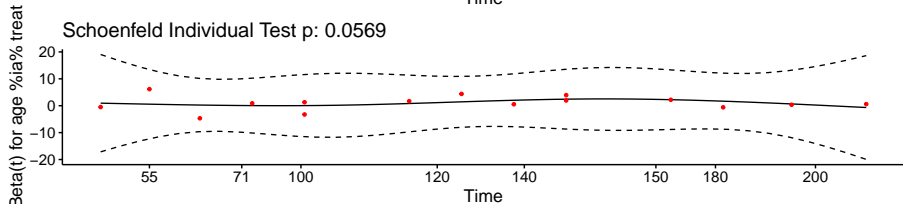
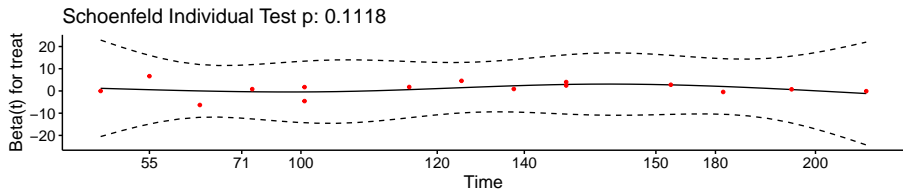
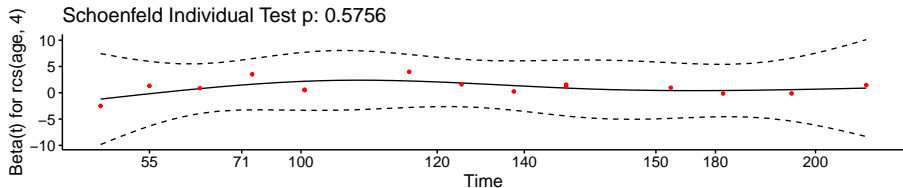


# Checking the Proportional Hazards Assumption

```
cox.zph(cph_AxT, transform = "km", global = TRUE)
```

	chisq	df	p
rccs(age, 4)	1.98	3	0.576
treat	4.38	2	0.112
age %ia% treat	5.73	2	0.057
GLOBAL	10.67	7	0.154

Global Schoenfeld Test p: 0.1537



# Additional Diagnostic Plots for your Cox model?

- `survminer` has a function called `ggcoxdiagnostics()` which plots different types of residuals as a function of time, linear predictor or observation id.
- The type of residual can be “martingale”, “deviance”, “score”, “schoenfeld”, “dfbeta”, “dfbetas”, or “scaledsch”.

# What to do if the PH assumption is violated

- If the PH assumption fails on a categorical predictor, fit a Cox model stratified by that predictor (use `strata(var)` rather than `var` in the specification of the `coxph` model.)
- If the PH assumption is violated, this means the hazard isn't constant over time, so we could fit separate Cox models for a series of time intervals.
- Use an extension of the Cox model that permits covariates to vary over time.

Visit

<https://cran.r-project.org/web/packages/survival/vignettes/timedep.pdf> for details on building the relevant data sets and models, with examples.