Tutorial → 4

Q1) $T(n) = 3T(n/2) + n^2$

Ans Here $a = 3, b = 2$ | $K = 2$ and $p = 0$

$\log_b a = \log_2 3$

$= 1.92 \leftarrow K = 2$

$\log_b a < K$ Hence 3rd Case Applied.

i.e $\log_b a < K$ if $p \geq 0$ $O(n^K \log^p n)$

here $p = 0$

$\boxed{T.C = O(n^2)}$

Q2) $T(n) = 4T(n/2) + n^2$

Ans here $a = 4, b = 2$ | $K = 2, p = 0$

$\log_a b = 2$ | $K = 2$

Hence Case IInd applied

if $p > -1$ $O(n^K \log^{p+1} n)$

$TC = O(n^2 \log n)$.

Q3) $T(n) = T(n/2) + 2^n$.

Ans $C = \log_2^2$ $f(n) = 2^n$ $n^c = n^0 = 1$

$= 0$ $f(n) > 1$

$2^n > 1$

$\boxed{T(n) = O(2^n)}$

Q4) $T(n) = 2^n T(n/2) + n^n$

Can't be solved using Master method because 'a' is not constant but depends on 'n'

Q5) $T(n) = 16T(n/4) + n$

Ans $\log_b a = \log_4 16$ | $k=1$, $p=0$

$= 2$ | $k=1$

$2 > 1$

$\therefore \boxed{TC = O(n^2)}$

Q6) $T(n) = 2T(n/2) + n\log n$.

Ans $\log_b a = \log_2 2 = 1$ | $k=1$, $p=1$

$\log_b a = 1$

$\boxed{TC = O(n \log^2 n)}$

Q7) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Ans $a = 2$, $b = 2$ | $k=1$, $p=-1$

$\log_b a = \log_2 2$

$= 1$ | $k=1$

$\log_b a = k$

$\boxed{TC = O(n \log \log n)}$

Q8) $T(n) = 2T(n/4) + n^{0.51}$

Ans $a = 2, b = 4$

$\log_b a = \log_4 2 = 0.5$ | $k = 0.51, p = 0$

$\log_b a = 0.5$

$0.5 < k$

T.C = $O(n^{0.51})$

Q9) $T(n) = 0.5T(n/2) + 1/n$

Ans $a = 0.5, b = 2$ | $k = -1, p = 0$

$\log_b a = \log_2 0.5 = -1$ | $k = -1$

$\log_b a = -1 = k$

T.C = $O(n^{-1} \log^1 n)$

TC = $O\left(\frac{\log n}{n}\right)$

Q10) $T(n) = 16T(n/4) + n!$

Ans $a = 16, b = 4$ | $k = 1, p = 0$

$\log_b a = 2$

$\log_b a > k$

T.C = $O(n^2)$.

Q11) $T(n) = 4T(n/2) + \log n$

Ans $a = 4, b = 2$ | $k = 0, p = 1$

$\log_b a = 2$

$\log_b a > k$

$\boxed{T.C = O(n^2)}$

Q12) $T(n) = \text{Sqrt}(n)\,T(n/2) + \log n$.

Ans Can't be Solve by Master's method because here 'a' is not constant.

Q13) $T(n) = 3T(n/2) + n$

Ans Here, $a = 3,\ b = 2$ | $K = 1,\ P = 0$

$\log_2 3 = 1.58$

$\log_b a > k$

$\boxed{T.C = O(n^{1.58})}$

Q14) $T(n) = 3T(n/3) + \text{Sqrt}(n)$

Ans $a = 3,\ b = 3$ | $K = \frac{1}{2},\ P = 0$

$\log_b a = 1$

$\log_b a > k$

$\boxed{T.C = O(n^1)}$

Q15) $T(n) = 4T(n/2) + cn$

Ans $\log_b a = \log_2 4$ | $K = 1,\ P = 0$

$\log_b a = 2$

$\log_b a > k$

$\boxed{T.C = O(n^2)}$

Q16) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

Ans $a = 3, b = 4$ | $k = 1, P = 1$

$\log_b a = 0.79$

$\log_b a < k$

T.C = O(nlogn)

Q17) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Ans $a = 3, b = 3$ | $k = 1, P = 0$

$\log_b a = 1$

$\log_b a = k$

T.C = O(nlogn)

Q18) $T(n) = 6T\left(\frac{n}{3}\right) + n^2\log n$

Ans $a = 6, b = 3$ | $k = 2, P = 1$

$\log_b a = 1.63$

$\log_b a < k$

T.C = $O(n^2\log n)$

Q19) $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Ans $\log_b a = \log_2 4$ | $k = 1, P = -1$

$= 2$

$\log_b a > k$

$\boxed{T.C = O(n^2)}$

Q20) $T(n) = 64T(n/8) - n^2 \log n$

Ans $\log_b a = \log_8 64$ | $k=2, p=1$

$= 2$

$\log_b a = k$

$\boxed{T.C = O(n^2 \log^2 n)}$

Q21) $T(n) = 7T(n/3) + n^2$

Ans $a=7, b=3$ | $k=2, p=0$

$\log_b a = \log_3 7 = 1.73$

$\log_b a < k$

$T.C = O(n^2)$

Q22) $T(n) = T(n/2) + n(2 - \cos n)$

Ans $a=1, b=2$ | $k=1, p=0$

$\log_b a = \log_2 1$

$= 0$

$\log_b a < k$

$\boxed{T.C = O(n)}$