Name : Shivansh Pandey
Roll no: 102103602
Group: 3COE24

Assignment:
Parameter Estimation

Que. 1
Sol^n

Given that $X_1, X_2, \ldots X_n$ is a random sample from a normal form distribution with mean θ_1 and variance $= \theta_2$ the likelihood func^n is $\Rightarrow$

$$L(\theta_1, \theta_2 | x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x-\theta_1)^2}{2\theta_2}}$$

By taking log on both sides, we get

$$\ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

To find MLE, will differentiate the log-likelihood with respect to θ_1 & θ_2 set derivative equal to zero

(i) For θ_1 :

$$\frac{\delta}{\delta\theta_1} \ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

Setting this's equal to zero

$$\frac{1}{\theta_2} \cdot \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0$$

$\therefore \hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$. So the MLE for θ_1 is the sample mean.

(ii) For θ_2 :

$$\frac{\delta}{\delta\theta_2} \ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

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Setting this equal to zero:

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2 = 0$$

$$\Rightarrow \frac{n}{2\hat{\theta}_2} = \frac{-1}{2\theta_2^2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

$$\Rightarrow \hat{\theta}_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

So MLE for θ_2 is the sample variance.

Ans:

To find the MLE of θ for a random sample $X_1, X_2 --- X_n$ from a Bernoulli distribution with parameter θ and a known m. The likelihood for this scenario is:

$$L(\theta | x_1, x_2 --- x_n) = \prod_{i=1}^{n} P(X_i = x_i | \theta)$$

Since X_i follows a bernoulli distribution,

$$P(X_i = x_i|\theta) = \theta^{x_i}(1-\theta)^{m-x_i} \text{ for each } i$$

Taking the log on both sides:

$$\ln L(\theta | x_1, x_2, ----, x_n) = \sum_{i=1}^{n}\ln(\theta^{x_i}(1-\theta)^{m-x_i})$$

$$= \sum_{i=1}^{n}(x_i \ln\theta + (m-x_i)\ln(1-\theta))$$

Now differentiate with respect to θ and set to zero.

Date: __.__.____

$$\frac{d}{d\theta}\left(\ln L(\theta \mid x_1, x_2 \ldots x_n)\right) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n}\frac{x_i}{\theta} = \frac{nm - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n}\frac{x_i}{nm}$$

So, $\therefore$ maximum likelihood estimate for θ is

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n}\frac{x_i}{n.m}$$