Assignment 1 :

1. Given,

$$\vec{A} = \hat{a}_x + 2\hat{a}_y + 5\hat{a}_z$$

$$\vec{B} = 5\hat{a}_\rho + (-\hat{a}_\phi) + 3\hat{a}_z$$

a. We have

$$B_\rho = 5 \; ; \; B_\phi = -1 \; ; \; B_z = 3$$

We know,

$$B_x = B_\rho \cos\phi - B_\phi \sin\phi = 5\cos\phi + \sin\phi$$

$$B_y = B_\rho \sin\phi + B_\phi \cos\phi = 5\sin\phi - \cos\phi$$

$$B_z = B_z = 3$$

at $P(0, 2, -3)$,

$$x = 0 \; ; \; y = 2 \; ; \; z = -3$$

$$\therefore \phi = \tan^{-1}(y/x) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\therefore B_x = \cancel{5}\,1 \; ; \; B_y = 5 \; ; \; B_z = 3$$

$$\therefore \vec{B} = \hat{a}_x + 5\hat{a}_y + 3\hat{a}_z$$

$$\therefore \vec{C} = \vec{A} + \vec{B} = 2\hat{a}_x + 7\hat{a}_y + 8\hat{a}_z$$

b. Component of $\vec{A}$ along $\vec{B}$ $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$$= \frac{1\times1 + 2\times5 + 3\times5}{\sqrt{1^2 + 5^2 + 3^2}}$$

$$= \frac{26}{\sqrt{35}} \text{ units}$$

2. Given,

$$\vec{E} = yz\,\hat{a}_x + xz\,\hat{a}_y + xy\,\hat{a}_z$$

Now,

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy)$$

$$= 0$$

$\therefore$ $\vec{E}$ is solenoidal in nature

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(zx)\right)\hat{a}_x + \left(\frac{\partial}{\partial x}xy - \frac{\partial}{\partial z}yz\right)\hat{a}_y$$

$$+ \left(\frac{\partial}{\partial x}zx - \frac{\partial}{\partial y}yz\right)\hat{a}_z$$

$$= (x-x)\,\hat{a}_x + (y-y)\,\hat{a}_y + (z-z)\,\hat{a}_z$$

$$= \vec{0}$$

$\therefore$ $\vec{E}$ is irrotational in nature.

3. $$\vec{E} = \frac{14}{r^2}\,\hat{a}_r$$

$$\therefore\ E_r = \frac{14}{r^2}\ ;\ E_\theta = 0\ ;\ E_\phi = 0$$

We know,

$$E_y = \sin\theta \sin\phi \, E_r + \cos\theta \sin\phi \, E_\theta + \cos\phi \, E_\phi$$

$$= \sin\theta \sin\phi \, E_r$$

at $(-2,3,1)$,

$$\theta = \cos^{-1}\left(1/\sqrt{2^2+3^2+1^2} \right)$$

$$= \cos^{-1}\left(1/\sqrt{14} \right)$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{14}}$$

$$\Rightarrow \sin\theta = \sqrt{1-\frac{1}{14}} = \sqrt{13/14}$$

and $\phi = \tan^{-1}\left(3/-2 \right)$

$$\Rightarrow \tan\phi = \frac{-3}{2}$$

$$\Rightarrow \sin\phi = \sqrt{\frac{9}{13}}$$

$$\therefore \quad E_y = \sqrt{\frac{13}{14}} \times \sqrt{\frac{9}{13}} \times \frac{14}{(-2)^2+3^2+1^2}$$

$$= \sqrt{\frac{9}{14}} = \frac{3}{\sqrt{14}} \text{ units}$$

4. Given, $\vec{F} = y\,\hat{a}_x + x\,\hat{a}_y$

(a) $\int \vec{F}\cdot\vec{dl} = \int_1^2 y\,dx + \int_1^4 x\,dy$

$$= \int_1^2 x^2\,dx + \cancel{x} \int_1^4 y^{1/2} dy$$

$$= \frac{x^3}{3}\Big/_1^2 + \frac{2}{3}\, y^{3/2}\Big/_1^4$$

$$= \frac{8}{3} - \frac{1}{3} + \frac{2}{3}\left(8 - 1\right)$$

$$= \frac{7}{3} + \frac{2 \times 7}{3}$$

$$= 7$$

(b) The line joining $(1, 1, -1)$ to $(2, 4, -1)$ is

$$\frac{x-1}{y-1} = \frac{2-1}{4-1}$$

$$\Rightarrow 3(x-1) = y - 1$$

$$\Rightarrow 3x - y - 2 = 0$$

$$\therefore \int \vec{F}\cdot\vec{dl} = \int x\,dy + y\,dx$$

$$= \int_1^4 \frac{y+2}{3}\,dy + \int_1^2 (3x-2)\,dx$$

$$= \frac{1}{3}\left(\frac{y^2}{2} + 2y\right)\Big/_1^4 + \left(\frac{3x^2}{2} - 2x\right)\Big/_1^2$$

$$= \cancel{(8-1) + \left(2 + \frac{1}{2}\right)}$$

$$= \frac{1}{3}\left(16 - \frac{5}{2}\right) + \left(2 + \frac{1}{2}\right)$$

$$= \frac{9}{2} + \frac{5}{2} = 7$$

$\therefore \vec{F}$ is a conservative field.

5. Given

$$\vec{A} = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$$

$$\vec{B} = \hat{a}_x + (-2\hat{a}_y) + 2\hat{a}_z$$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|\times|\vec{B}|} = \frac{4-6+10}{\sqrt{4^2+3^2+5^2}\sqrt{1+2^2+2^2}} = \frac{8}{3\sqrt{50}} = 0.3772$$

$$\Rightarrow \theta = 67.84^\circ$$

6. Given $\vec{E} = y\hat{a}_x + x\hat{a}_y$

We have

$$dW = \vec{F}.\vec{dl} = (ydx + xdy)q$$

$$\Rightarrow dW = q\,d(xy)$$

$$\Rightarrow W = q\int_1^8 d(xy)$$

$$= 7q$$

$$= -7\mu J$$

7. Given,

$$A = 1\ cm^2$$

$$d = 2mm$$

We know,

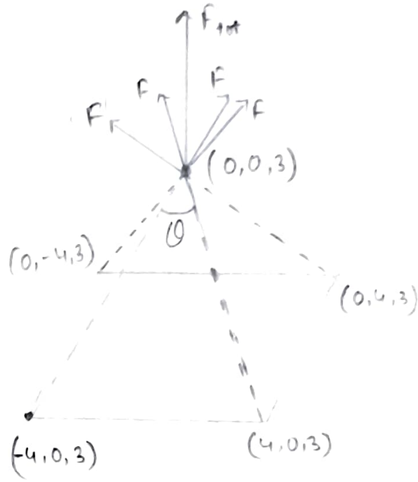
$$C = \frac{\epsilon_0 A}{d} \Rightarrow C = \frac{\epsilon_0 A}{d}\int_2^4 \epsilon_r\, d\epsilon_r$$

$$\Rightarrow \cancel{C = \frac{8.85 \times 10^{-12}}{\quad}}$$

$$\Rightarrow C = \frac{8.85 \times 10^{-12} \times 10^{-4}}{2 \times 10^{-3}} \times \left. \frac{\epsilon_r^2}{2} \right|_2^4$$

$$= 2.655 \text{ pF}$$

8. Here,



$$\vec{F}_{tot} = 4F\cos\theta \, \hat{a}_z$$

& $$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$$= 9 \times 10^9 \times \frac{(100 \times 10^{-6})^2}{5^2}$$

$$= \frac{18}{5} \text{ N}$$

$$\therefore F_{tot} = \left(4 \times \frac{18}{5} \times \frac{3}{5}\right) \hat{a}_z \text{ N}$$

$$= 8.64 \, \hat{a}_z \text{ N}$$

9. Let force due to charge Q_1 be $\vec{F}_1$ and due to charge Q_2 be $\vec{F}_2$.

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1 q_i}{|r_i - r_1|^3} (\vec{r}_i - \vec{r}_1)$$

$$= \cancel{9 \times 10^9} \frac{1}{4\pi\epsilon_0} \frac{Q_1 q_i}{(\sqrt{2^2+1^2})^3} (-2\hat{a}_x - \hat{a}_y)$$

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0} \frac{Q_2 q_i}{|\vec{r_i} - \vec{r_2}|^3} (\vec{r_i} - \vec{r_2})$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q_2 q_i}{(\sqrt{3^2})^3} (-3\hat{a}_y)$$

$$\therefore \vec{F}_{tot} = \vec{F_1} + \vec{F_2}$$

$$= \frac{Q_1 q_i}{4\pi\epsilon_0} \frac{(-2\hat{a}_x - \hat{a}_y)}{5\sqrt{5}} + \frac{Q_2 q_i}{4\pi\epsilon_0} \frac{(-3\hat{a}_y)}{27}$$

$$= \frac{q_i}{4\pi\epsilon_0} \left[\frac{-2Q_1 \hat{a}_x}{5\sqrt{5}} - \cancel{\frac{Q_1 + 3Q_2}{}} \left(\frac{Q_1}{5\sqrt{5}} + \frac{Q_2}{9} \right) \hat{a}_y \right]$$

Given,

$$\vec{F_x} = \vec{F_y}$$

$$\Rightarrow \frac{-2Q_1}{5\sqrt{5}} = -\frac{Q_1}{5\sqrt{5}} - \frac{Q_2}{9}$$

$$\Rightarrow \frac{Q_1}{5\sqrt{5}} = \frac{Q_2}{9}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{5\sqrt{5}}{9}$$

10. We know,

$$V = \frac{Q}{4\pi\epsilon_0 r} + C$$

Given,

at $V(0,6,-8) = 2V$

then

$$2 = \frac{9\times10^{9}\times5\times10^{-9}}{\sqrt{6^2+8^2}} + C$$

$$\Rightarrow 2 = 4.5 + C$$

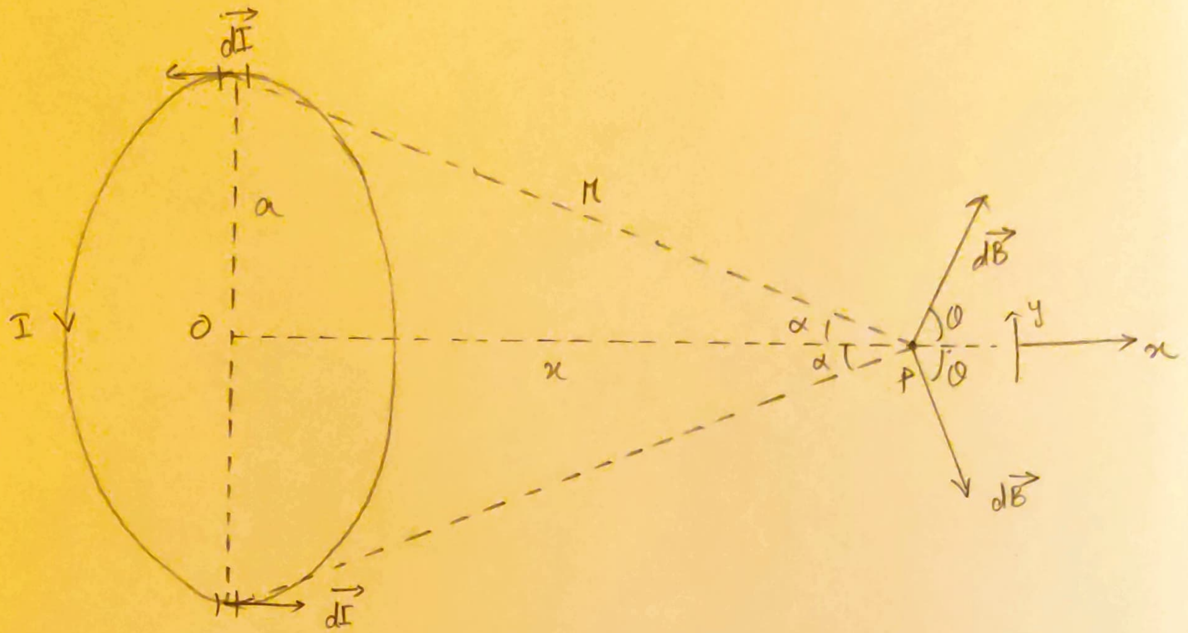
$$\Rightarrow C = -2.5$$

$\therefore$ Potential at $(-3,2,6)$ is $= \dfrac{9\times10^{9}\times5\times10^{-9}}{|(-3,2,6)-(0,0,0)|} - 2.5$

$$= \frac{45}{\sqrt{49}} - 2.5$$

$$= 3.929\ V$$

11.



From Biot Savart's Law,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{\cancel{\vec{I}\times\vec{dl}}\ \ I\vec{dl}\times\hat{r}}{r^2}$$

The y-components of the magnetic flux at P will cancel out and total magnetic flux density at P will be due to the ~~d~~ x-components only

$$\therefore \int_{loop} dB\cos\theta\,\hat{x} = \vec{B}$$

Now, $\cos\theta = \cos(90-\alpha) = \sin\alpha = \frac{a}{r}$

$\sin\theta = \cos\alpha = \frac{x}{r}$

$$\therefore \vec{B} = \int_{loop} \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \frac{a}{r}\ \hat{a}_x$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{Ia}{r^3} \int_{loop} dl \, \hat{a}_x$$

$$= \frac{\mu_0}{4\pi} \times \frac{Ia}{r^3} \times 2\pi a \, \hat{a}_x$$

$$= \frac{\mu_0}{2} \frac{Ia^2}{r^3} \hat{a}_x$$

(i) When P is at center of loop, $x = 0$

$$\therefore r = (a^2+x^2)^{3/2} = a^3$$

$$\therefore \vec{B} = \frac{\mu_0}{2} \frac{I}{a} \hat{a}_x$$

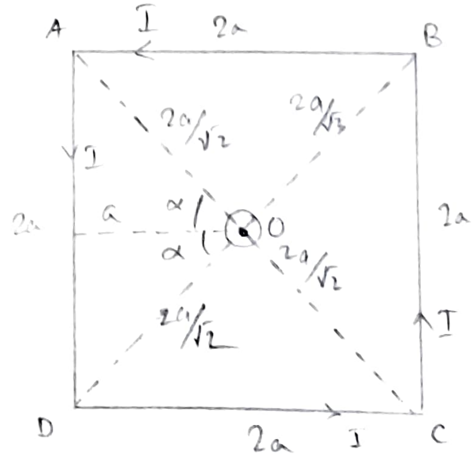
(ii) When P is very far from center of loop,

$$x >> a$$

$$\Rightarrow r = (a^2+x^2)^{3/2} = x^3$$

$$\therefore \vec{B} = \frac{\mu_0}{2} \frac{Ia^2}{x^3} \hat{a}_x$$

12. We know, magnetic field due to a straight current carrying conductor is



$$\vec{B} = \frac{\mu_0 I}{4\pi a} (2\sin\alpha) \, \hat{a}_z$$

$$= \frac{\mu_0 I}{4\pi a} \frac{a}{\sqrt{2} a} \hat{a}_z$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{I}{\sqrt{2}a} \hat{a}_z$$

$\therefore$ Magnetic field due to the current carrying conductors are

$$\vec{B} = 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\sqrt{2}a} \hat{a}_z$$

$$= \frac{\mu_0 I}{\pi\sqrt{2}a} \hat{a}_z$$

$$\therefore \vec{H} = \frac{I}{\pi\sqrt{2}a} \hat{a}_z$$

13. Given,

$$\vec{A} = \frac{-\rho^2}{2} \hat{a}_z$$

Total magnetic field is given by

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{-\rho^2}{2} \end{vmatrix}$$

$$= \frac{\partial}{\partial \rho}\left(-\frac{\rho^2}{2}\right) \hat{a}_\phi$$

$$= -\rho \, \hat{a}_\phi$$

$\therefore$ Magnetic flux density is given by

$$\phi_B = \int \vec{B} \cdot \vec{dS}$$

$$= -\int \rho \, d\rho \, dz = \frac{-\rho^2}{2}\Big|_0^1 * z\Big|_0^1 = \frac{-1}{2} \, \cancel{Wb/} \, Wb$$

14.

(i) Given.

$$\vec{B} = \frac{10}{\rho} e^{-j3z} \hat{a}_\rho$$

Instantaneous form.

$$\vec{B}' = Re\left[\vec{B} e^{j\omega t}\right]$$

$$= Re\left[\frac{10}{\rho} e^{j(\omega t - 3z)} \hat{a}_\rho\right]$$

$$= \frac{10}{\rho} \cos(\omega t - 3z)\, \hat{a}_\rho$$

(ii) Given,

$$\vec{A} = (1 - j\sqrt{3})\, e^{-jkx}\, \hat{a}_z$$

$$= 2 e^{-j\pi/3} e^{-jkx} \hat{a}_z$$

$$= 2 e^{-j(kx + \pi/3)} \hat{a}_z$$

Instantaneous form,

$$\vec{A}' = Re\left[\vec{A} e^{j\omega t}\right]$$

$$= 2\cos(-kx + \omega t - \pi/3)\, \hat{a}_z$$

15.

Given,

$$\vec{H} = 6\cos\left(2\times10^{8}t - 6x\right)\hat{a}_y \quad A/m$$

Given, $\omega = 2\times10^{8}$ rad/s

$k = 6$ rad/m

$$\therefore \quad V_p = \frac{\omega}{k} = \frac{1}{3}\times10^{8} \ m/s$$

We know,

$$V_p = \frac{1}{\sqrt{\mu\epsilon}} \Rightarrow \frac{V_p}{c} = \frac{1}{\sqrt{\mu_r\epsilon_r}}$$

$$\Rightarrow \frac{1/3\times10^{8}}{3\times10^{8}} = \frac{1}{\sqrt{\epsilon_r}}$$

$$\Rightarrow \epsilon_r = 3$$

Again

$$\vec{E} = v_p\vec{B}$$

$$= \frac{1}{3}\times10^{8}\times6\mu_0\cos\left(2\times10^{8}t - 6x\right)\hat{a}_z \quad V/m$$

$$= 2\times10^{8}\mu_0\cos\left(2\times10^{8}t - 6x\right)\hat{a}_z \quad V/m$$

16. Given,

$$\vec{H} = 20\cos\left(\pi\times10^{8}t - 5x\right)\hat{a}_z$$

(a) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{5}$ m

Wave velocity,

$$V_p = \frac{\omega}{k}$$

$$= \frac{\pi \times 10^8}{5} \ m/s$$

$$= 6.28 \times 10^7 \ m/s$$

(b) Electric field compponent,

$$\vec{E} = 6.28 \times 10^7 \times 20 \cos(\pi \times 10^8 t - 5x)\ \hat{a}_y$$

$$= 3.14 \times 10^8 \cos(\pi \times 10^8 t - 5x)\ \hat{a}_y$$

Again,

$$\frac{V_p}{c} = \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \frac{6.28 \times 10^7}{3 \times 10^8} = \frac{1}{\sqrt{\epsilon_r}}$$

$$\Rightarrow \epsilon_r \approx 23$$

$$\therefore \ \chi_e = 1 + \epsilon_r \approx 24$$

$$\therefore \ \vec{P} = \chi_e \epsilon_0 \vec{E}$$

$$= 24 \times 8.85 \times 10^{-12} \times 3.14 \times 10^8 \cos(\pi \times 10^8 t - 5x)\ \hat{a}_y$$

$$= 6.67 \times 10^{-2} \cos(\pi \times 10^8 t - 5x)\ \hat{a}_y$$

17. Giuen,

$$\vec{H} = 10\cos\left(10^8 t - \frac{x}{\sqrt{3}}\right)\hat{a}_y \quad mA/m$$

Here,

$$H_0 = 10 \quad mA/m$$

$$\omega = 10^8 \quad s^{-1}$$

$$\beta = \frac{1}{\sqrt{3}} \quad m^{-1}$$

$\therefore$ $$\eta = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} \quad \text{and} \quad \beta = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\Rightarrow \frac{\beta}{\omega} = \frac{\sqrt{\epsilon_r}}{c}$$

$$\Rightarrow \sqrt{\epsilon_r} = \frac{3\times 10^8}{\sqrt{3}\times 10^8}$$

$$\Rightarrow \sqrt{\epsilon_r} = \sqrt{3}$$

$$\Rightarrow \epsilon_r = 3$$

$\therefore$ $$\eta = \sqrt{\frac{\mu_0}{3\epsilon_0}}$$

$$= \sqrt{\frac{4\pi\times 10^{-7}}{8.85\times 10^{-12}\times 3}}$$

$$= 217.55 \ \Omega$$

$\therefore$ $$\vec{P}_{av} = \frac{1}{2}\eta H_0^2 \hat{a}_x = \frac{1}{2}\times 217.55\times 10^2 \hat{a}_x = 108.27\times 10^2 \hat{a}_x$$

$\therefore$ $$P_{surface} = \int \vec{P}_{av}\cdot(dy\,dz\,\hat{a}_x) = 108.27(2-0)(2-1)\times 10^2$$

$$= 217.55\times 10^2 \ mW/m$$

$$= 21.755 \ W/m$$

18.

Given,

$\sigma = 5.8 \times 10^{7}$ ℧/m

$\epsilon_r = 1$

$\mu_r = 1$

$f = 60$ Hz

We ~~kee~~ have,

$$\frac{\sigma}{\omega\epsilon} = \frac{5.8 \times 10^{7}}{2\times\pi\times 60 \times 8.85\times10^{-12}}$$

$$= 1.74 \times 10^{16} \gg 1$$

$\therefore$ Copper is a good conductor

$\therefore$ Attenuation factor, $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$

$$= \frac{5.8\times10^{7}}{2}\sqrt{\frac{4\pi\times10^{-7}}{8.85\times10^{-12}}}$$

$$= 1.092 \times 10^{10}$$

Propagation factor, $\beta = \omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right)$

$$= 2\times\pi\times60\sqrt{1/c^2}\left(1 + \frac{(5.8\times10^{7})^2}{8\times(2\times\pi\times60)^2\times(8.85\times10^{-12})}\right)$$

$$= 4.2011 \times 10^{14} \text{ rad/m}$$

Wave velocity, $v_p = \frac{\omega}{\beta} = 8.97\times10^{-13}$ m/s

Wavelength $= \dfrac{2\pi}{\beta} = 1.4956 \times 10^{-14}$ m

Magnitude of intrinsic impedance $= \sqrt{\dfrac{\omega \epsilon}{6}}$

$= 7.58 \times 10^{-9}\ \Omega$

Angle of intrinsic impedance $= \pi/4$ rad