UNITS & MEASUREMENTS

MEASUREMENT OF MASS & TIME

MASS

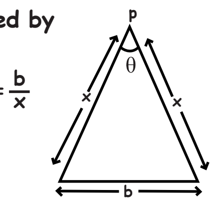
- Unified atomic mass unit(amu) is used to measure mass of atoms & molecules
- 1amu =(1/12)th mass of one C^{12} atom
- 1amu = 1.66×10^{-27} kg
- Electron mass- 10^{-30} kg
- Earth mass : 10^{25} kg
- Observable Universe 10^{55} kg

TIME

- SI unit is second (based on caesium clock with an uncertainity less than 1 part in 10^{-13} ie,3µs loss every year)
- Timespan of unstable particle: 10^{-24} s
- Age of universe: 10^{17} s

MEASUREMENT OF LENGTH

- Large distance is measured by parallax method
- Parallax angle= $\frac{\text{BASIS}}{\text{DISTANCE}} = \frac{b}{x}$

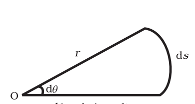


- $1^\circ = 1.745 \times 10^{-2}$ rad
- $1' = 2.91\times10^{4}$ rad.
- $1'' = 4.85\times10^{6}$ rad.
- For very small sizes, optical microscope, tunneling microscope, electron microscope are used.
- 1 AU = 1.496×10^{11} m
- 1 ly = 9.46×10^{15} m
- 1parsec= 3.08×10^{16} m
- Size of proton: 10^{-15} m
- Radius Of Earth: 10^{7}m
- Distance to Boundary Of Observable Universe : 10^{26} m

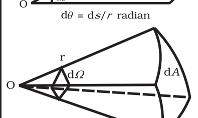
SI SYSTEM

7 Base units and 2 supplementary units

NO.	Base Units		
	Quantity	Unit	Symbol
1	Length	meter	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol



$d\theta = ds/r$ radian



$d\Omega = dA/r^2$ steradian

NO.	Supplementary Units		
	Quantity	Unit	Symbol
1	Plane angle	radian	rad
2	Solid angle	steradian	sr

What is the unit of permittivity of free space ε_0?
(a) coloumb/newton-metre
(b) newton-metre^2 /coloumb^2
(c) coloumb^2/newton-metre^2
(d) coloumb^2/(newton-metre)^2

SIGNIFICANT FIGURES

The digits in a measured quantity which are reliable and confidence in our measurement + the digit which is uncertain.

RULES FOR SIGNIFICANT FIGURES

1. All non-zero digits are significant. For example, 42.3 has three significant figures; 243.4 has four significant figures; and 24.123 has five significant figures.
2. A zero becomes significant figure if it appears between two non-zero digits. For example, 5.03 has three significant figures; 5.604 has four significant figures; and 4.004 has four significant figures.
3. Leading zeros or the zeros placed to the left of the number are never significant. For example,0.543 has three significant figures; 0.045 has two significant figures; and 0.006 has one significant figure.
4. Trailing zeros or the zeros placed to the right of the number are significant. For example, 4.330 has four significant figures; 433.00 has five significant figures; and 343.000 has six significant figures.
5. In exponential notation, the numerical portion gives the number of significant figures. For example,1.32 x 10^{-2} has three significant figures and 1.32 x 10^{4} has three significant figures.

RULES FOR ROUNDING OF A MEASUREMENT

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged. For example,x = 7.82 is rounded off to 7.8 and x = 3.94 is rounded off to 3.9.
2. If the digit to be dropped is more than 5, then the preceding digit is raised by one. For example, x = 6.87 is rounded off to 6.9 and x = 12.78 is rounded off to 12.8.
3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one. For example, x = 16.351 is rounded off to 16.4 and x = 6.758 is rounded off to 6.8.
4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is even, is left unchanged. For example, x = 3.250 becomes 3.2 on rounding off and x = 12.650 becomes 12.6 on rounding off.
5. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd. For example, x = 3.750 is rounded off to 3.8, again x = 16.150 is rounded off to 16.2.

RULES FOR ROUNDING OF A MEASUREMENT

ADDITION & SUBTRACTION

In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places

3.1421
0.241
+0.09 ⟵ (has two decimal places)
3.4731 ⟵ (Answer should be reported to two decimal places after rounding off)

Answer = 3.47

MULTIPLICATION & DIVISION

When numbers are multiplied or divided, the number of significant figures in the answer equals the smallest number of significant figures in any of the original numbers

51.028
x 1.31 ⟵ (Three significant figures)
66.84668 ⟵ (Answer should have three significant figures after rounding off)

Answer = 66.8

If L=2.331cm, B= 2.1cm,then L+B = ?
(a) 4.431 cm (b) 4.43 cm
(c) 4.4 cm (d) 4 cm

Dimensional Analysis

Dimensions of a physical quantity are the powers to which units of base quantity are raised. Eg: $[M]^a [L]^b [T]^c [A]^d [K]^e$

APPLICATIONS

- checking the correctness of various formulae
  Eg: If Z=A+B,[Z]=[A]=[B]
- conversion of one system of unit into another
  $n_1u_1 = n_2u_2$
  Eg: $n_1[M_1^A L_1^B T_1^C] = n_2[M_2^A L_2^B T_2^C]$
  $n_1 = n_2\left[\frac{M_2}{M_1}\right]^A \left[\frac{L_2}{L_1}\right]^B \left[\frac{T_2}{T_1}\right]^C$
- Deducing relation among physical quantity

DIMENSIONAL FORMULA

1) Pressure=stress=Young's modulus=$ML^{-1}T^{-2}$
2) Work=Energy=Torque=ML^2T^{-2}
3) Power P=ML^2T^{-3}
4) Gravitational constant G=$M^{-1}L^3T^{-2}$
5) Force constant=Spring constant=MT^{-2}
6) Coefficient of viscosity=$ML^{-1}T^{-1}$
7) Latent heat = L^2T^{-2}
8) Electric potential=$\frac{P}{I}$= $ML^2T^{-3}A^{-1}$
9) Resistance=$\sqrt{\frac{\mu_0}{\varepsilon_0}}$ =$ML^2T^{-3}A^{-2}$
10) Capacitance=$M^{-1}L^{-2}T^4A^2$
11) Permittivity ε_0=$M^{-1}L^{-3}T^4A^2$
12) Angular momentum = planck's constant =$M^1L^2T^{-1}$
13) M= $k\sqrt{\frac{hc}{G}}$ L=$k\sqrt{\frac{hG}{c^2}}$ T=$k\sqrt{\frac{hG}{c^5}}$

Time period $T \propto \sqrt{\frac{l}{g}} \propto \sqrt{\frac{m}{k}} \propto \sqrt{\frac{R}{g}}$
$\frac{L}{R} = RC = \sqrt{LC}$

DIMENSIONLESS QUANTITIES

1) Strain
2) Refractive index
3) Relative density
4) Plane angle
5) Solid angle

In SI Units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is:
a)$A^{-1}TML^3$ b)$AT^2M^{-1}L^{-1}$
c)$AT^{-3}ML^{3/2}$ d)$A^2T^3M^{-1}L^{-2}$

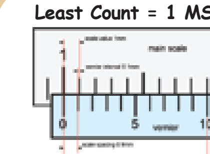
INSTRUMENTS

Least Count:
Smallest quantity an instrument can measure

- mm scale → 1mm
- vernier scale → 0.1mm
- screw gauge → 0.01mm

VERNIER CALIPERS

Least Count = 1 MSD - 1VSD



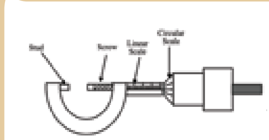
If n VSD Coincides with (n-1) MSD,
then (n-1) MSD= n VSD
∴ 1VSD = $\frac{n-1}{n}$ MSD

Least Count = 1MSD - $\frac{n-1}{n}$MSD = $\frac{1MSD}{n}$

Total Reading = Main Scale Reading + (coinciding Vernier Scale division x least count)

In a vernier calipers, one main scale division is x cm & n division of vernier scale coincide with n-1 divisions of the main scle. The least count (in cm) of the calipers is;
a) $\left(\frac{n-1}{n}\right)x$ b) $\frac{nx}{(n-1)}$ c) $\frac{x}{(n-1)}$ d) $\frac{x}{n}$

SCREW GAUGE



Pitch = $\frac{\text{Main Scale Reading}}{\text{No.of rotations}}$

Least Count = $\frac{\text{pitch}}{\text{Total no.of divisions on circular scale}}$

Total Reading = Linear Scale Reading + circular scale reading x least count

The least count of the main scale of a screw gauge is 1mm. The minimum no.of divisions on its circular scale required to measure 5µm diameter of wire is;
a) 200 b) 50 c) 400 d) 100

ERRORS IN MEASUREMENT

Difference between true value & measured value of a quantity

- Systematic Errors: Errors which tend to occur only in one direction, either positive or negative
  - Instrumental: Due to inbuilt defect of measuring instrument
  - Experimental: Limitations in experimental technique
  - Personal: Due to individual bias,Lack of proper setting of apparatus
- Random Errors: Irregular and random in magnitude & direction

- Least count error is the smallest value that can be measured by instrument (occurs with random & systematic errors)
- Absolute Error :- $\Delta a = a_i - a_{mean}$, $a_{mean} = \frac{a_1+a_2+a_3+\ ....+a_n}{n}$
- Relative Error:- $\frac{\Delta a_{mean}}{a_{mean}}$ $\Delta a_{mean} = \frac{|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+\ ....+|\Delta a_n|}{n}$
- Percentage Error:- $\frac{\Delta a_{mean}}{a_{mean}} \times 100$

COMBINATION OF ERRORS

Operations	Formula Z	Absolute error ΔZ	Relative error ΔZ/Z	Percentage error 100 x ΔZ/Z
Sum	$A+B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A+B}$	$\frac{\Delta A+\Delta B}{A+B} \times 100$
Difference	$A-B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A-B}$	$\frac{\Delta A+\Delta B}{A-B} \times 100$
Multiplication	$A\times B$	$A\Delta B+B\Delta A$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A+A\Delta B}{B^2}$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right) \times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A} \times 100$
Root	$A^{\frac{1}{n}}$	$\frac{1}{n}A^{\frac{1}{n}-1}\ \Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A} \times 100$

General rule:

If $Z = \frac{A^pB^q}{C^r}$,Then the maximum fractional relative error in Z will be:

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

In an expirement four quantities a,b,c and d are measured with percentage error1%, 2%, 3% and 4% respectively. Quantity P is calculated as shown below. What is the percentage error in P?

$$P = \frac{a^2b^2}{cd}$$

(a) 14% (b) 10%
(c) 7% (d) 4%