CSCE-629 Analysis of Algorithms

Fall 2017

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Assignment # 6
(Due December 5, 2017)

1. A *vertex cover* in an undirected graph G is a set C of vertices in G such that every edge in G has at least one end in C. Consider the following two versions of the VERTEX-COVER problem:

VC-D: Given a graph G and an integer k, decide whether G contains a vertex cover of at most k vertices.

VC-O: Given a graph G, construct a minimum vertex cover for G

Prove: VC-D is solvable in polynomial time if and only if VC-O is solvable in polynomial time.

2. Prove that the VC-D problem given in Question 1 is in $\mathcal{NP}$.

3. Using the fact that the INDEPENDENT SET problem is $\mathcal{NP}$-complete, prove that the following problem is $\mathcal{NP}$-complete:

CLIQUE: Given a graph G and an integer k, is there a set C of k vertices in G such that for every pair v and w of vertices in C, v and w are adjacent in G?

4. Give formal proofs for the following lemmas:
Lemma 4.1. If Q is $\mathcal{NP}$-hard and $Q \in \mathcal{P}$, then $\mathcal{P} = \mathcal{NP}$.
Lemma 4.2. If $Q_1 \leq_m^p Q_2$ and $Q_2 \leq_m^p Q_3$, then $Q_1 \leq_m^p Q_3$.