

# CSCE-629 Analysis of Algorithms

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**Instructor:** Dr. Jianer Chen

**Office:** HRBB 315C

**Phone:** 845-4259

**Email:** chen@cse.tamu.edu

**Office Hours:** T,Th 2:00pm-3:30pm

**Teaching Assistant:** Qin Huang

**Office:** HRBB 315A

**Phone:** 402-6216

**Email:** huangqin@email.tamu.edu

**Office Hours:** T,Th 9:00am-12:00noon

## Assignment # 5 (Due November 21, 2017)

1. Let  $G$  be a directed graph. The *component graph*  $G_c$  of  $G$  is a directed simple graph, defined as follows: (1) each vertex in  $G_c$  corresponds to a strongly connected component (scc) in  $G$ , and (2) there is an edge  $[v, w]$  in  $G_c$  if and only if there are edges from the scc in  $G$  that corresponds to  $v$  to the scc in  $G$  that corresponds to  $w$ . Given an  $O(n+m)$ -time algorithm that on a directed graph  $G$ , constructs the component graph  $G_c$  of  $G$ .
2. Consider a hash table of size  $m = 1009$  and the hash functions  $h_1(k) = k \bmod m$  and  $h_2(k) = \lfloor m \cdot ((k \cdot A) \bmod 1) \rfloor$ , where  $A = (\sqrt{5} - 1)/2$ . Compute the locations to which the keys 61, 62, 63, 64, and 65 are mapped by each hash function.
3. Suppose that we are storing a set of  $n$  keys into a hash table of size  $m$ . Show that if the keys are drawn from a universe  $U$  with  $|U| > nm$ , then no matter what hash function  $h$  we use,  $U$  has a subset of size  $n$  consisting of keys that are all hashed by  $h$  to the same slot. Note that this shows that in the worst case, searching in a set of  $n$  keys by hashing can take time as bad as  $\Theta(n)$ .