Q1. Translate the sentences to FOL:

1. $\forall x\ (\text{Tomato}(x) \rightarrow (\text{Fruit}(x) \vee \text{Vegetable}(x)))$
2. $\exists x\ (\text{Mushroom}(x) \wedge \text{Poisonous}(x))$
3. $\forall x\ (\text{Triangle}(x) \leftrightarrow \exists e1,e2,e3\ (\text{Partof}(e1,x) \wedge \text{Partof}(e2,x) \wedge \text{Partof}(e3,x) \wedge \text{Closed}(e1,e2,e3) \wedge \neg\text{Equal}(e1,e2,e3)))$

Alternatively, $\forall x\ \text{closedsides}(x,3) \wedge \text{Sumofangles}(x,180) \rightarrow \text{Triangle}(x)$

4. $\forall x\ ((\text{Plant}(x) \wedge \text{Polinated}(x)) \rightarrow \text{Produce}(x,\text{seeds}))$
5. $\forall m\ ((\text{movie}(m) \wedge \text{made}(\text{StephenKing},m) \wedge \neg\text{Equal}(m, \text{Cujo})) \rightarrow \text{Favourite}(m, \text{John}))$
6. $\forall t1,t2,g\ (\text{game}(g,\text{football}) \wedge \text{greateratend}(\text{points}(t1), \text{points}(t2)) \wedge \text{team}(t1) \wedge \text{team}(t2) \rightarrow \text{winner}(t1)$
7. $\forall x,g\ \text{Fordexporer}(x) \wedge \text{gastank}(g,x) \wedge \text{less}(\text{percentfuelleft}(g),0.1) \rightarrow \text{warninglight}(x)$
8. $\forall x,y\ (\text{Computer}(x, \text{AI}) \wedge \text{Computer}(y, \text{Bob}) \rightarrow \exists m\ (\text{Manufacturer}(m) \wedge \text{Manuf\_of}(x, m) \wedge \text{Manuf\_of}(y, m)))$
9. $\forall x,m\ (\text{Laptop}(x) \wedge \text{Seller}(x, \text{DELL}) \wedge \text{Year}(2012) \rightarrow (\text{Memory}(m) \wedge \text{Gigabytes}(x, \text{greaterthanorequalto}(m,4))))$

Q2. Convert the following sentence to CNF:

$$\forall x\ P(x) \rightarrow [\ \forall y\ P(y) \rightarrow P(f(x,y))\ ] \wedge [\neg \forall y\ Q(x,y) \rightarrow P(y)\ ]$$

Step-1: Eliminate Implication:

$$\forall x\ \neg P(x) \vee [\forall y\ \neg P(y) \vee P(f(x,y))] \wedge [\neg \forall y\ \neg Q(x,y) \vee P(y)]$$

Step-2: Push $\neg$ Inwards and apply Demorgan's rule:

$$\forall x\ \neg P(x) \vee [\forall y\ \neg P(y) \vee P(f(x,y))] \wedge [\exists y\ \neg(\neg Q(x,y) \vee P(y))]$$

$$\forall x\ \neg P(x) \vee [\forall y\ \neg P(y) \vee P(f(x,y))] \wedge [\exists y\ \ Q(x,y) \wedge \neg P(y)]$$

Step-3: Standardize Variables apart**:**

$$\forall x\ \neg P(x) \vee [\forall y\ \neg P(y) \vee P(f(x,y))] \wedge [\exists z\ \ Q(x,z) \wedge \neg P(z)]$$

Step-4: Skolemization**:**

$$\forall x\ \neg P(x) \vee [\forall y\ \neg P(y) \vee P(f(x,y))] \wedge [Q(x,g(x)) \wedge \neg P(g(x))]$$

Step-5: Drop Universal Quantifiers**:**

$$(\ \neg P(x) \vee (\ (\neg P(y) \vee P(f(x,y))) \wedge (Q(x,g(x)) \wedge \neg P(g(x)))\ )\ )$$

Step-6: Distribute $\vee$ over $\wedge$**:**

$$(\neg P(x) \vee \neg P(y) \vee P(f(x,y)))\ \ \wedge\ \ (\neg P(x) \vee Q(x,g(x))) \wedge (\neg P(x) \vee \neg P\ (g(x)))$$

The sentence is now in CNF. Hence it is our solution.

Q3. a) Knowledge Base in First Order Logic:

Constants: Marcus, Caesar

Predicates: People(x), Pompeian(x), Roman(x), Ruler(x), Loyal(x,y), Hate(x,y), Assassinate(x,y)

1. Pompeian(Marcus)
2. $\forall$x Pompeian(x) $\rightarrow$ Roman(x)
3. Ruler(Caesar)
4. $\forall$x Roman(x) $\rightarrow$ ([Loyal(x, Caesar) $\vee$ Hate(x, Caesar)] $\wedge$ $\neg$[Loyal(x, Caesar) $\wedge$ Hate(x, Caesar)])
5. $\forall$x $\exists$y Loyal(x, y)
6. $\forall$x $\forall$y (People(x) $\wedge$ Ruler(y) $\wedge$ Assassinate(x, y)) $\rightarrow$ $\neg$Loyal(x, y)
7. Assassinate(Marcus, Caesar)
8. People(Marcus)

b) Prove Marcus hates Caesar using Natural Deduction:

So we have to derive Hate(Marcus, Caesar) from Knowledge Base

9. Roman(Marcus) [By Modus Ponens on 1,2 with θ={x|Marcus}]
10. [Loyal(Marcus, Caesar) $\vee$ Hate(Marcus, Caesar)] $\wedge$ $\neg$[Loyal(Marcus, Caesar) $\wedge$ Hate(Marcus, Caesar)]
    [By Modus Ponens on 9,4 with θ={x|Marcus}]
11. Loyal(Marcus, Caesar) $\vee$ Hate(Marcus, Caesar) [By AND elimination on 10]
12. $\neg$Loyal(Marcus, Caesar) [By Modus Ponens on 3,6,7,8 with θ={x|Marcus, y|Caesar}]
13. Hate(Marcus, Caesar) [By Resolution on 11,12]

Thus, we derived Marcus Hates Caesar.

c) Label derived sentences with the prior sentences and unifier used.

a. By Modus Ponens on sentences 1,2 with substitution θ={x|Marcus} derived sentence 9
b. By Modus Ponens on sentences 9,4 with substitution θ={x|Marcus} derived sentence 10
c. By AND elimination on sentence 10 derived sentence 11
d. By Modus Ponens on sentences 3,6,7,8 with substitution θ={x|Marcus, y|Caesar} derived sentence 12
e. By Resolution on sentences 11,12 derived sentence 13

Q-4) Sammy's Sport Shop Problem:

Predicates: obs(x,z), lab(x,z), cont(x,z), Box(x), Color(z), Equal(x1,x2,x3)

a) **FOL Knowledge Base:**

[Initial Facts]

1. obs(1,Y)
2. obs(2,W)
3. obs(3,Y)
4. lab(1,W)
5. lab(2,Y)
6. lab(3,B)
7. $\neg$Equal(1,2,3)
8. $\neg$Equal(1,3,2)
9. $\neg$Equal(2,1,3)
10. $\neg$Equal(2,3,1)
11. $\neg$Equal(3,1,2)
12. $\neg$Equal(3,2,1)
13. $\neg$Equal(W,Y,B)
14. $\neg$Equal(W,B,Y)
15. $\neg$Equal(Y,W,B)
16. $\neg$Equal(Y,B,W)
17. $\neg$Equal(B,Y,W)
18. $\neg$Equal(B,W,Y)

[Inferences from the given information]

19. $\forall$x,z obs(x,z) $\rightarrow$ cont(x,z) $\vee$ cont(x, B)

[Labels displayed wrong]

20. $\forall$x,z lab(x,z) $\rightarrow$ $\neg$cont(x, z)

[No Two boxes should have the same contents]

21. $\forall$x1,x2,x3,z cont(x1,z) $\wedge$ $\neg$Equal(x1,x2,x3) $\rightarrow$ $\neg$cont(x2, z) $\wedge$ $\neg$cont(x3, z)

[There should be at least One box of each color]

22. $\forall$x,z1,z2,z3 $\neg$Equal(z1,z2,z3) $\rightarrow$ cont(x,z1) $\vee$ cont(x,z2) $\vee$ cont(x,z3)

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b) Prove cont(2,W) using Natural deduction:

23. cont(3,Y) ∨ cont(3, B) [By Modus Ponens on 3,7,19 with θ={x/3, z/Y}]
24. ¬cont(3, B) [By Modus Ponens on 6,8,20 with θ={x/2, z/W}]
25. cont(3,Y) [By Resolution on 23,24]
26. ¬cont(1,Y) ∧ ¬cont(2,Y) [By Modus Ponens on 11,21,25 with θ={x1/3, x2/1, x3/2, z/Y}]
27. cont(1,Y) ∨ cont(1, B) [By Modus Ponens on 1,19 with θ={x/1, z/Y}]
28. ¬cont(1, Y) [By AND Elimination on 26]
29. cont(1,B) [By Resolution on 27,28]
30. cont(2,W) ∨ cont(2,B) [By Modus Ponens on 2,19 with θ={x/2, z/W}]
31. ¬cont(2,B) ∧ ¬cont(3,B) [By Modus Ponens on 7,21,29 with θ={ x1/1, x2/2, x3/3, z/B}]
32. ¬cont(2, B) [By AND Elimination on 31]
33. cont(2,W) [By Resolution on 30,32]

Hence box 2 must contain while balls.

Q 5) Tic- Tac- Toe Game Puzzle:

Predicates: p(X,i,j) , p(O,i,j), b(i,j), move(X,i,j), canWin(X,i,j), canWin(X), forcedMove(X,i,j), TwoInaRow(X,i), TwoInaCol(X,i),
TwoInaDiag(X,i,j)

// As there are only 2 diagonals present in the puzzle so TwoInaDiag predicate tells us the diagonal in which i,j is present. so that means (i,j) in TwoInaDiag predicate has to be either (1,1) / (2,2) / (3,3) / (1,3)/ (3,1)

FOL KnowledgeBase:

[When X can win]

1. ∀i, j TwoInaRow(X,i) ∧ b(i,j) → canWin(X,i,j)
2. ∀i, j TwoInaCol(X,j) ∧ b(i,j) → canWin(X,i,j)
3. ∀i, j TwoInaDiag(X,i,j) ∧ b(i,j) → canWin(X,i,j)

[When O can win then Block that potential win by O with forcedMove of X]

4. ∀i, j TwoInaRow(O,i) ∧ b(i,j) → forcedMove(X,i,j)
5. ∀i, j TwoInaCol(O,j) ∧ b(i,j) → forcedMove (X,i,j)
6. ∀i, j TwoInaDiag(O,i,j) ∧ b(i,j) → forcedMove (X,i,j)

[Decide optimal move for X and move X]

7. ∀i, j canWin(X,i,j) → move(X,i,j)
8. ∃i, j canWin(X,i,j) → canWin(X)
9. ∀i, j ForcedMove(X,i,j) ∧ ¬CanWin(X) → move(X,i,j)

Thus, all these rules will infer all desirable moves for **X** based on the given strategies.

<---------------------------------------**Thank you so much, Patrick** -->