Simulating quantum black holes with matrices

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Introduction

Black holes are thermodynamic objects with entropy. The information paradox in quantum gravity is the problem of identifying what pure quantum state formed a black hole in the first place by analyzing thermal blackbody radiation which is essentially featureless and appears mixed. Recent developments have attacked the problem using quantum information theory. One of the new insights is that black holes exhibit a characteristic chaotic behavior. We can model these effects using matrix quantum mechanics. We simulate and study the time evolution of various characteristic observables of black hole dynamics.

Motivation & Background

The BFSS action is given by

$$S[X] = \int dt Tr(\frac{1}{2}\dot{X}^i\dot{X}^i - \frac{1}{4}[X^i, X^j][X^i, X^j]) \tag{1}$$

From the BFSS action, we derive the equation of motion. The classical dynamics is described by the equation of motion is

$$\frac{\partial^2 X_M}{\partial t^2} - \sum_N [X_N, [X_M, X_N]] = 0 \tag{2}$$

where X_M are N $\times$ N matrices. We impose the Gauge constraint given by

$$\sum_M [X_M, \frac{\partial X_M}{\partial t}] = 0 \tag{3}$$

The system in the N $\to \infty$ limit behaves like a theory of membranes in K dimensions of spherical topology and thus the matrices which are now functions on a sphere may be expanded in matrix spherical harmonics. The spherical harmonics $Y_{jm}(j = 0, 1, ...$ and $m = -j, -j+1, ..., j)$ give a convenient orthonormal basis of $L^2(S^2)$. Expanding in terms of matrix spherical harmonics,

$$X_M(t) = \sum_{j=0}^{N-1} \sum_{m=-j}^{j} a_{jm}^M(t)\hat{Y}_j^m \tag{4}$$

with initial conditions $X_M(0)$ defined by $a_{jm}^M(0)$. Making use of the properties of the matrix spherical harmonics relations

$$Y_m^{j\dagger} = Y_{-m}^j \tag{5}$$

We get that the coefficients must satisfy

$$a_{jm}^k = a_{j-m}^{k*} \text{for} X^k = X^{k\dagger} \tag{6}$$

We truncate at $j = j_{max}$: i.e., we only consider terms upto

$$j \in 0, 1, ..., j_{max}. \tag{7}$$

The other observables can be easily derived in terms of the coefficients a_{jm}^k with the help of matrix spherical harmonics properties,

$$[Y_{m1}^{j1}, Y_{m2}^{j2}] = \sum_{j_3 m_3} f_{j_1 m_1 j_2 m_2}^{j_3 m_3} Y_{m3}^{j_3} \tag{8}$$

$$\text{where } f_{j_1 m_1 j_2 m_2}^{j_3 m_3} = \frac{1}{N} Tr(Y_{-m_3}^{j_3}[Y_{m_1}^{j_1}, Y_{m_2}^{j_2}]) \tag{9}$$

- A solution to a BFSS matrix model can be interpreted as the dynamical embedding of a M2 brane (the fundamental membrane of M-theory) in 8D non-commutative space.
- The aim is to study a typical solution in both the classical and quantum BFSS theory and understand how a typical black hole microstate behaves in real time.
- This would open new understanding on the physical mechanisms through which information paradoxes can be resolved. At the classical level one would like to understand how much of the dynamics is captured by chaos and random matrices.

Method

In this project we expanded the observables of interest in terms of the coefficients a_{jm}^N. A function was created to initialise the observables of interest at time t=0 and the values were seeded for the rest of the simulation. The coefficients a_{jm}^N were allowed to evolve through time. During this process we learned the variety of features of the observables and how they scale with time.

Results

- **BFSS Radius**
  The BFSS Radius is given by $\frac{1}{N}\sum_k [Tr[X^k.X^k]$

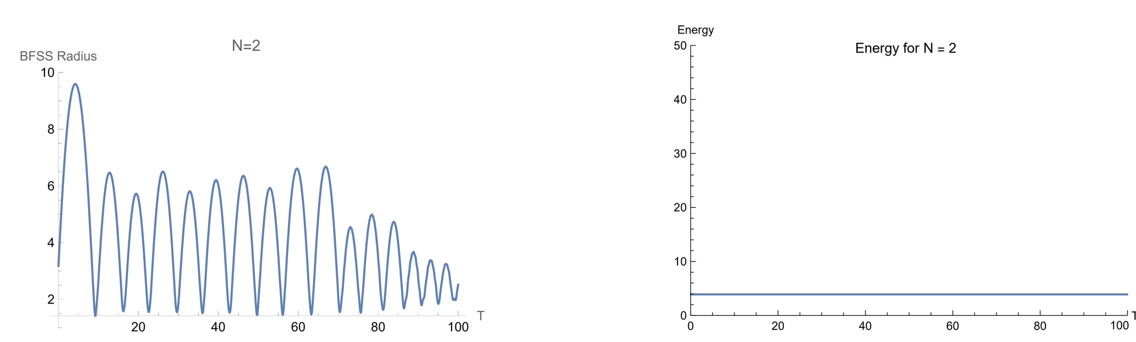

Figure 1. BFSS Radius calculated for the case N=2 and allowed to evolve for until 100 seconds keeping the energy conserved.

We also observed chaos in some observables and explored the same by adding a slight perturbation to it. These perturbations measurements were very sensitive as the perturbation quantities needed to be of working precision of 15, precision goal of 10 and accuracy goal of 10 in Mathematica.

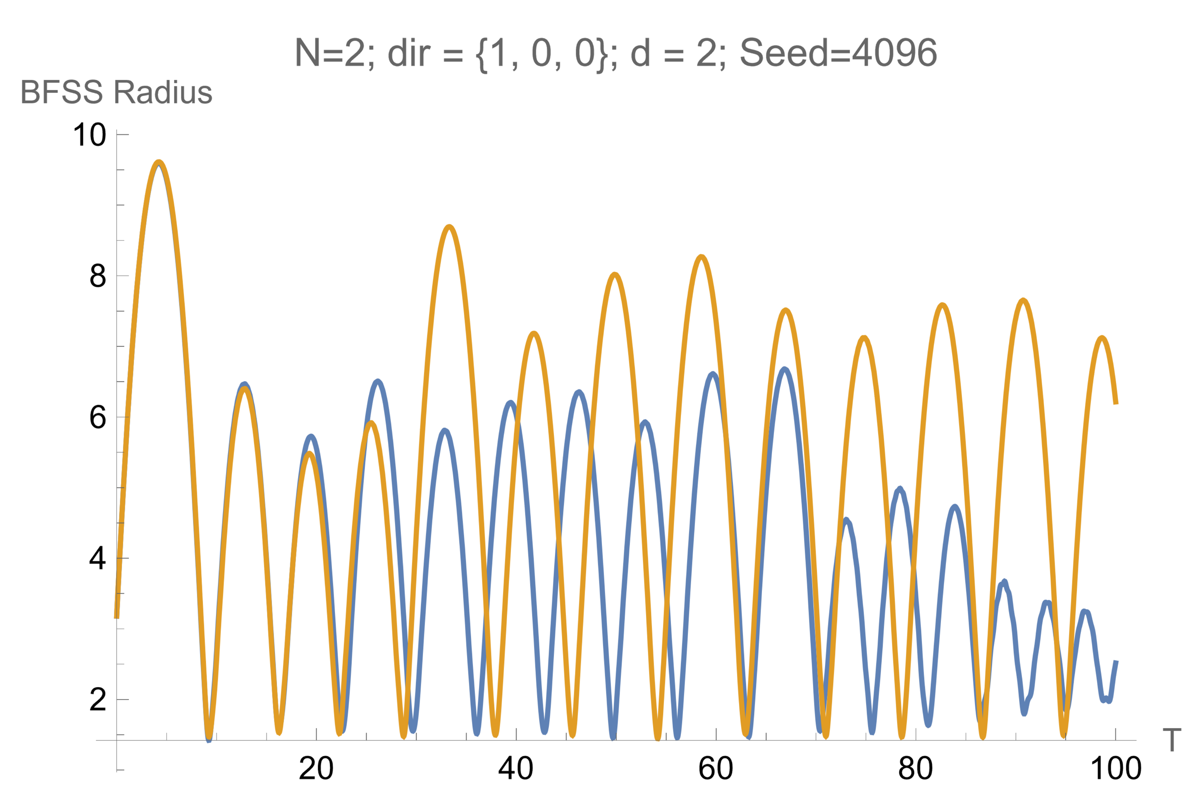

Figure 2. Chaos in BFSS Radius observed by through perturbation. Blue - Non Perturbed trajectory. Orange - Perturbation of order 10^{-2} is applied

- **BFSS Symmetric Traceless**

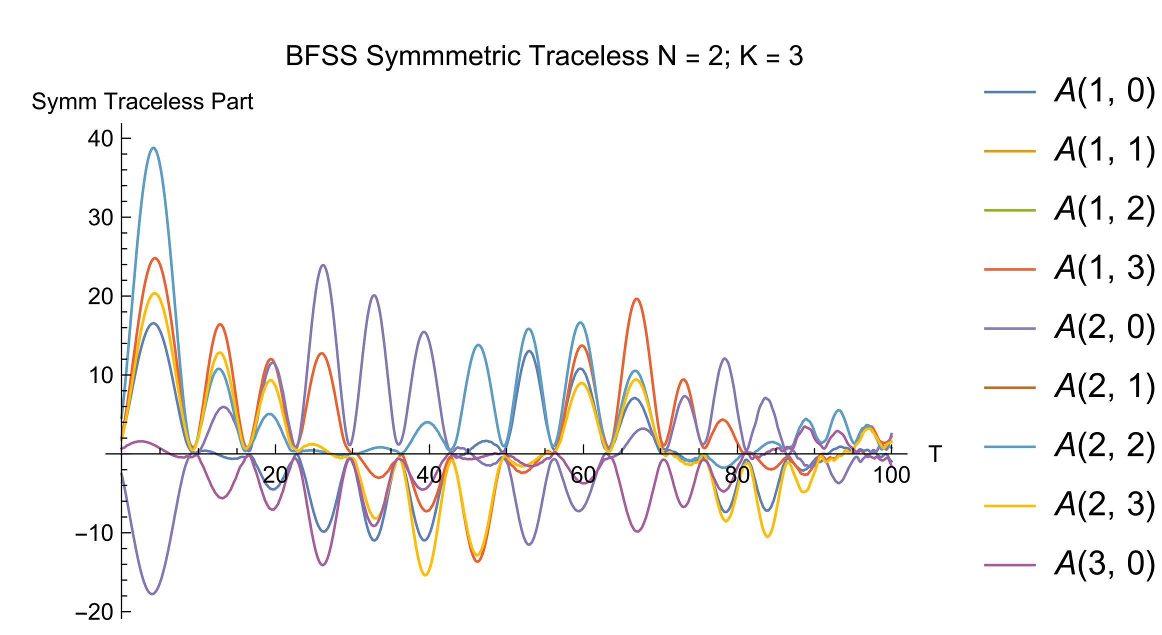

Figure 3. Different components of BFSS Symmetric Traceless

- The Traceless part of the BFSS symmetric observable is given by $\frac{1}{N}[Tr[X^i.X^j] - \frac{1}{K}\sum[X^k.X^k]\delta_{ij}$. This is a table of values with i, j, k each running from 1 to K. As observed from the Figure 3, the components oscillate with time and gradually decay. We seek to analyse the time scales at which they decay and the frequency of the oscillations of the different components.
- Some other observables of interest and other standard observables like angular momentum, energy were also plotted.
- For interconversion from matrices to matrix spherical harmonics basis, helper functions were created in *Mathematica*. All functions necessary for the simulation files were arranged and stored in the helper files. Most of the simulations were done in *Mathematica* although some harder problems needed *Python* and *Julia* for progress.
- The entire project is organised and uploaded on private GitHub repository for easier and coordinated group work.

Perturbation analysis

A perturbative analysis was done by adding random perturbations to each of the coefficients a_{jm}^N that were of the order of 10^{-8}. The degree of the order was chosen in such a way that the perturbed quantity follow their non perturbed trajectories around t=0 and taking into consideration the time it takes for the simulation to complete.

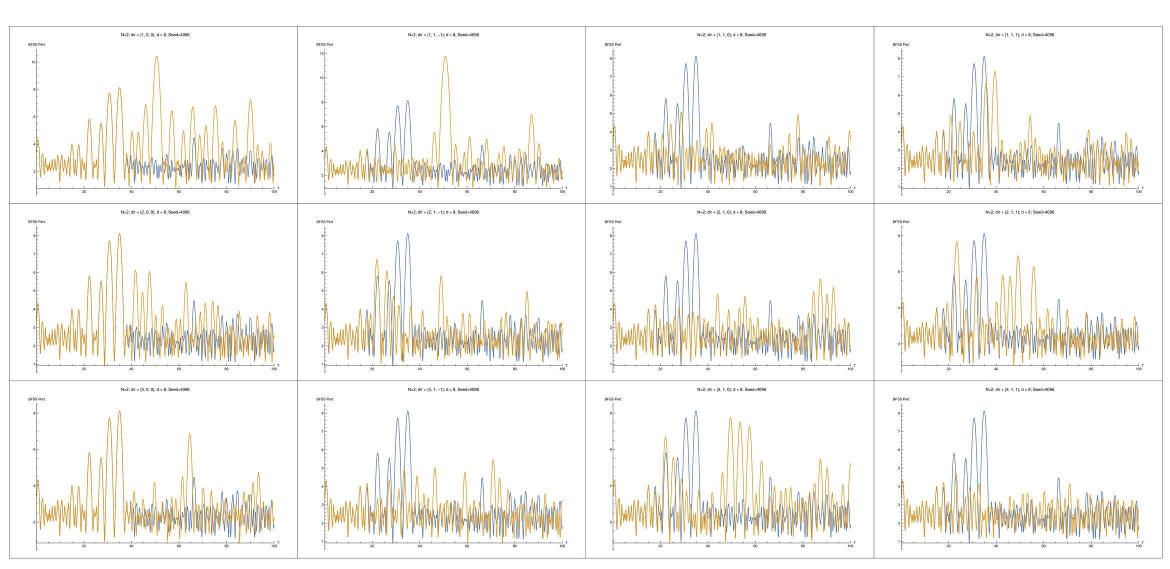

Figure 4. Perturbation analysis of the BFSS Radius by adding random perturbations of the order of 10^{-8} for N=2 and along different directions labelled by $\{k, j+1, m+j+1\}$ where k runs from 1 to K, j runs from 0 to $N-1$ & m runs from $-j$ to j. Blue - Non Perturbed trajectory. Orange - Perturbation of order 10^{-8} is applied

Future Prospects

The future goals of the project would be observe the time series analysis. There appears to be a correlation between the time chaos starts to be observed and the amount of perturbation applied. By understanding the above dynamics of the black hole through Matrix Harmonics and M-Theory we aim to apply it to quantum information theory by developing quantum circuits to simulate black hole scramblings.

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References

[1] Sinya Aoki, Masanori Hanada, and Norihiro Iizuka. Quantum black hole formation in the BFSS matrix model. *Journal of High Energy Physics*, 2015(7), 2015.

[2] Xizhi Han and Sean A. Hartnoll. Deep Quantum Geometry of Matrices. *Physical Review X*, 10(1):1–62, 2020.

[3] Daniel Harlow. Jerusalem lectures on black holes and quantum information. *Reviews of Modern Physics*, 88(1):015002, 2016.

[4] Yoshifumi Hyakutake. Black hole and fuzzy objects in the BFSS matrix model. *Physical Review D*, 98(4):46023, 2018.

[5] Badis Ydri. *Matrix Models of String Theory*. 2053-2563. IOP Publishing, 2018.

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