

$$V(x, y) = \begin{cases} 0, & 0 \leq x \leq L_x \text{ \& } 0 \leq y \leq L_y \\ \infty & \text{otherwise} \end{cases}$$

PARTICLE IN A

TWO-DIMENSIONAL

INFINITE POTENTIAL

SATURDAY

APRIL

06

096-269 • Week 14

MAY 2013

M T W T F S S M T W T F S S
1 2 3 4 5 6 7 8 9 10 11 12
13 14 15 16 17 18 19 20 21 22 23 24 25 26
27 28 29 30 31

Symmetric
2D

$$\psi(x, y) = \sqrt{\frac{2}{L}} \sin \frac{n_x \pi x}{L} \sin \frac{n_y \pi y}{L}$$

$$\therefore \frac{\hbar^2}{2m} \left(\frac{1}{L} \sin \frac{n_x \pi x}{L} \sin \frac{n_y \pi y}{L} \cdot \frac{n_x^2 \pi^2}{L^2} + \sin \frac{n_x \pi x}{L} \sin \frac{n_y \pi y}{L} \cdot \frac{n_y^2 \pi^2}{L^2} \right) = E \psi$$

$$\therefore E_{m,n} = \frac{\hbar^2}{2m} \cdot \frac{(n_x^2 + n_y^2) \pi^2}{L^2} = \boxed{\frac{\hbar^2 \pi^2}{2m L^2} (n_x^2 + n_y^2)}$$

Asymmetric
2D

$$\psi(x, y) = A \sin \frac{n_x \pi x}{L_x} \sin \frac{n_y \pi y}{L_y}$$

$$\therefore \frac{\hbar^2}{2m} A \left\{ \sin \frac{n_x \pi x}{L_x} \times \sin \frac{n_y \pi y}{L_y} \times \frac{n_x^2 \pi^2}{L_x^2} + \sin \frac{n_x \pi x}{L_x} \sin \frac{n_y \pi y}{L_y} \times \frac{n_y^2 \pi^2}{L_y^2} \right\} = E \psi$$

$$\therefore E_{(m,n)} = \frac{\hbar^2 \pi^2}{2m} \left\{ \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right\} \text{ KNOW!}$$

Energy Eigen values.

$$\text{Normalisation} \quad A^2 \int_0^{L_x} \int_0^{L_y} \sin^2 \frac{n_x \pi x}{L_x} \sin^2 \frac{n_y \pi y}{L_y} dx dy = 1$$

$$\Rightarrow A^2 \int_0^{L_x} \sin^2 \frac{n_x \pi x}{L_x} dx \int_0^{L_y} \sin^2 \frac{n_y \pi y}{L_y} dy = 1$$

$$\Rightarrow \frac{A^2}{4} \left[\frac{x}{2} \right]_0^{L_x} \left[1 - \cos \frac{2n_x \pi x}{L_x} \right] \left[\frac{y}{2} \right]_0^{L_y} \left[1 - \cos \frac{2n_y \pi y}{L_y} \right] = 1$$

$$\Rightarrow A^2 \left[x - \frac{\sin \frac{2n_x \pi x}{L_x}}{\frac{2n_x \pi}{L_x}} \right]_0^{L_x} \left[y - \frac{\sin \frac{2n_y \pi y}{L_y}}{\frac{2n_y \pi}{L_y}} \right]_0^{L_y} = \frac{4}{A^2}$$

$$\Rightarrow [L_x] \cdot [L_y] = \frac{4}{A^2} \quad \therefore A = \frac{2}{\sqrt{L_x L_y}} = \sqrt{\frac{2}{L_x}} \sqrt{\frac{2}{L_y}}$$

\therefore Normalised wave function,

$$\psi(x, y) = \sqrt{\frac{2}{L_x}} \sqrt{\frac{2}{L_y}} \sin \frac{n_x \pi x}{L_x} \sin \frac{n_y \pi y}{L_y} \text{ KNOW!}$$

Similarly extend to 3D.

- Jacobian - matrix/det of all 1st order partial derivatives of a vector-valued function. $J_{ij} = \frac{\partial f_i}{\partial x_j}$ or for $f_1, f_2(x, y)$ - $J = \begin{vmatrix} f_{1x} & f_{1y} \\ f_{2x} & f_{2y} \end{vmatrix}$

- Trace - sum of elements of main diagonal

- Zener diode - Choose R_s such that $I_Z \gg I_L$

- C_v & degree of freedom (n):

$$C_v = \frac{dU}{dT} = \frac{d}{dT} \left[\frac{n}{2} RT \right] = \frac{n}{2} R ; \quad C_p = R + C_v$$

- Vibrations of diatomic molecules - 2-body oscillator

$$T = 2\pi \sqrt{\frac{\mu}{c}}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2} = \text{reduced mass}, \quad c = \text{force const.}$$

$$\text{Vibrational states} - E_n = \left(n + \frac{1}{2} \right) h \nu_0, \quad n = 0, 1, 2, 3, \dots$$

- Beats (waves) - $\nu_{\text{beat}} = \nu_1 - \nu_2$

- Doppler effect - $\nu = \nu_0 \left[\frac{v + v_{\text{obs}}}{v + v_{\text{source}}} \right]$ $\{ v = \text{speed of wave} \}$
obs $\xrightarrow{+ve}$ source

- Poynting vector - See 9-Jan.

- Hydrogen spectra -

$$\text{ser. Name } \begin{matrix} L & B & P & B & P \\ m & 1 & 2 & 3 & 4 & 5 \end{matrix} \quad \frac{1}{\lambda} = R \left[\frac{1}{m^2} - \frac{1}{n^2} \right] ; \quad n = m+1, m+2, \dots$$

- Displacement current inside a capacitor :-

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = \epsilon_0 \oint \vec{ds} \cdot \frac{\partial \vec{E}}{\partial t} \approx \epsilon_0 \frac{dE}{dt}$$

- De Broglie wavelength of electron :-

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{1.227 \text{ nm}}{\sqrt{V}} \rightarrow V = \text{accelerating potential}$$

- Compton effect :- inelastic scattering of photon by a charged particle -
(electron)

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$$

Compton wavelength.

scattering angle.

8 Diode equation :
(Shockley)

$$I_D = I_0 \left[e^{\frac{V_D}{nV_T}} - 1 \right]$$

($n = 1$ for Ge, 2 for Si)

$$\left\{ V_T = \frac{kT}{q} \right\}$$

$\approx 26 \text{ mV @ } 300 \text{ K}$

9 • RI and ϵ_r :-

$$\mu = \sqrt{\epsilon_r \mu_r} \approx \sqrt{\epsilon_r} \text{ for most natural materials.}$$

10 • Basic eqn. of entropy :-

(Macroscopic)

$$\Delta S = \int \frac{dQ_{rev}}{T}$$

11

• Calorie \rightarrow specific heat capacity of water in $\text{J K}^{-1} \text{g}^{-1}$

$$\therefore 1 \text{ Calorie} = 4.184 \text{ J} \approx 4.2 \text{ J}$$

12

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

$$\frac{\mu_0}{4\pi} = 10^{-7}$$

1

• Magnetic field due to finite piece of conductor :-

2

$$B = \frac{\mu_0 I}{4\pi d} [\cos \theta]_f^i ; d = \perp^r \text{ dist, } \theta = \text{angle b/w } d\vec{l} \text{ \& } \vec{r}$$

3. Taylor series of log and ln :-

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

4

• Capacitance of a single shell :-

5

$$C = 4\pi\epsilon_0 R$$

6 • Perfectly inelastic collision -

* Particles stick together after colliding

* Momentum conserved

* KE loss ✓

• Disk with mass per unit area \propto distance from centre:

$$I_G = \frac{3}{5} MR^2, \quad I_D = \frac{3}{10} MR^2$$

• Magnetic field at the centre of a current-loop:

$$B = \frac{\mu_0 I}{2R}$$

- LORRENT'S TRANSFORMATIONS :

$t' = \gamma \left(t - \frac{vx}{c^2} \right)$	$t = \gamma \left(t' + \frac{vx'}{c^2} \right)$
$x' = \gamma (x - vt)$	$x = \gamma (x' + vt)$

- $C_p - C_v$ relation for real gases :-

$$C_p = C_v + \frac{T v \alpha^2}{k_T}$$

where T = temp., v = specific volume at T
 α = coeff. of volume expansion
 k_T = isothermal compressibility

- Sum of Trigonometric series :-

$$\left[\sum_{n=0}^{\infty} r^n \sin n\theta = \frac{r \sin \theta}{r^2 - 2r \cos \theta + 1} \quad \left| \quad \sum_{n=0}^{\infty} r^n \cos n\theta = \frac{1 - r \cos \theta}{r^2 - 2r \cos \theta + 1} \right. \right]$$

- Entropy change: (C be the heat capacity of substance)

$$\left[\begin{array}{l} * \text{ Substance :- } \Delta S = C \ln \frac{T_2}{T_1} \\ \{T \text{ varying}\} \end{array} \quad \left| \quad \begin{array}{l} * \text{ Reservoir :- } \Delta S = \frac{C \Delta T}{T} \\ (T \text{ const}) \quad \{ \Delta T \text{ of substance} \} \end{array} \right. \right]$$

- Force on a whole body (rod) pivoted at an end :-
Effective Torque $T_{\text{ext}} = \frac{\vec{L}}{2} \times \vec{F}$ { $\because F$ effectively acts on CM }

transient
response of L

$$\Rightarrow I = e^{Rt/L} \times \frac{\varepsilon}{L} \cdot e^{+\frac{Rt}{L}} \times \frac{L}{+R} \Big|_0^t$$

$$\Rightarrow I = e^{-Rt/L} \times \frac{\varepsilon}{R} (e^{+\frac{Rt}{L}} - 1) =$$

$$\boxed{\frac{\varepsilon}{R} (1 - e^{-Rt/L})}$$

substitute $I, \frac{R}{L}$ for $Q, \frac{1}{RC}$ in capacitor.

I_0

KNOW!

for small oscillations, $k = V''(x_0)$

Where V is the potential energy function and x_0 is the equilibrium position

Resolving power of grating =
No. of Lines

Resolving power a system of gratings =
 $R_1 \cdot R_2 \cdot R_3 \dots$

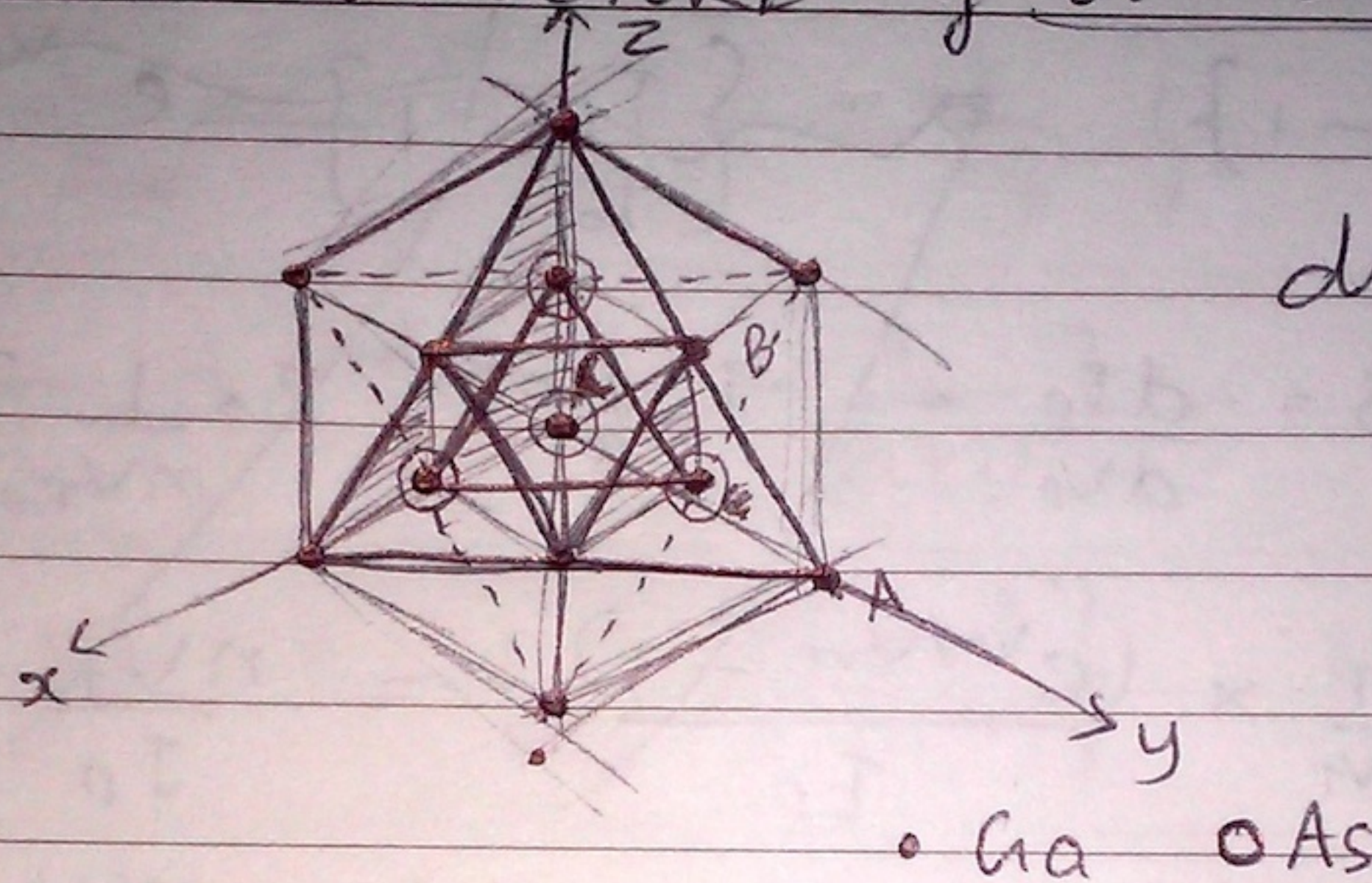
$$\lambda^2 - (\text{trace}) \lambda + \text{determinant} = 0$$

Where trace = sum of elements of main diagonal

λ = Eigen value.

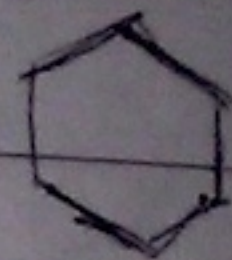
DIAMOND STRUCTURE of GaAs.

1 2 3 4 5 6 7 8 9 10 11 12 13 14
15 16 17 18 19 20 21 22 23 24 25 26 27 28
29 30



double fcc

→ hcp II
(ABCA)



TAM 15 ahd.

Q10: Each Ga atom in the xy plane is bonded
to 2 As atoms ABOVE the plane & 2 As atoms
BELOW the plane. So, when fracturing the
crystal in xy (001) plane, either of these two
pairs of bond per atom has to be broken.

∴ Ans :- 2

