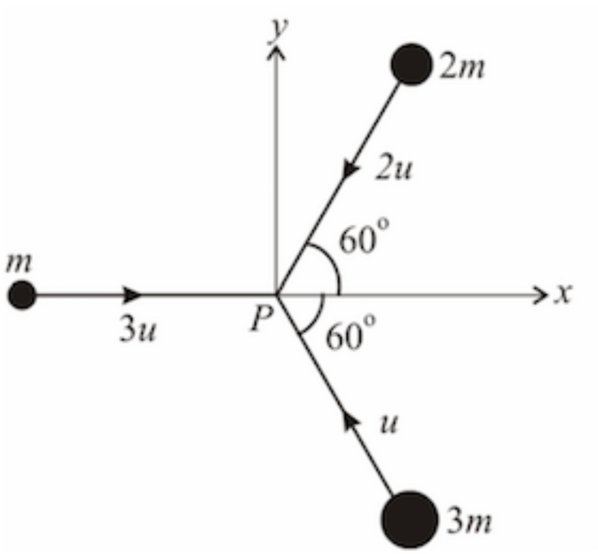
Solved Problems - IV

JAM 2012

1. *[Q.3]* Three masses m, 2m and 3m are moving in x-y plane with speeds 3u, 2u and u, respectively, as shown in the figure. The three masses collide at the same time at P and stick together. The velocity of the resulting mass would be?



Basic Physics: ***Law of conservation of momentum***

Mass	**Velocity**	**Momentum**
m	$3u\,\widehat{x}$	$3mu\,\widehat{x}$
$2m$	$-2u\cos 60^\circ\ \widehat{x} - 2u\sin 60^\circ\,\hat{y} \Rightarrow$	$-2mu\,\widehat{x} - 2\sqrt{3}mu\,\hat{y}$
$3m$	$-u\cos 60^\circ\ \widehat{x} + u\sin 60^\circ\,\hat{y} \Rightarrow$	$-\dfrac{3}{2}mu\,\widehat{x} + \dfrac{3\sqrt{3}}{2}mu\,\hat{y}$
After collison: $6m$	Let $\overrightarrow{V} = V_x\,\widehat{x} + V_y\,\hat{y} \Rightarrow$	$6mV_x\,\widehat{x} + 6mV_y\,\hat{y}$

Total momentum before collision = Total momentum after collision

Therefore equating component wise,

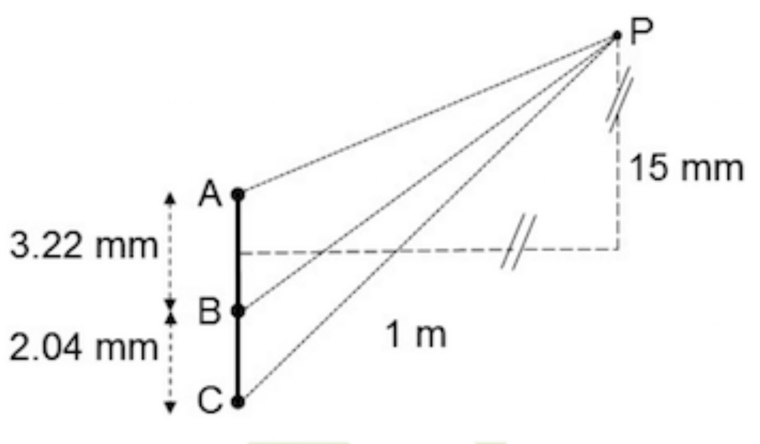
$$\left(3mu - 2mu - \frac{3}{2}mu\right)\widehat{x} = 6mV_x\widehat{x} \Rightarrow -\frac{1}{2}u = 6V_x$$

$$\left(0 - 2\sqrt{3}mu + \frac{3\sqrt{3}}{2}mu\right)\hat{y} = 6mV_y\hat{y} \Rightarrow \ -\frac{\sqrt{3}}{2}u = 6V_y$$

Solving, $\vec{V} = \boxed{\frac{u}{12}\left(-\hat{x} - \sqrt{3}\,\hat{y}\right)}$

JAM 2015

1. *[Q.C14]* For the arrangement given in the following figure, the coherent light sources A, B and C have individual intensities of 2 mW/m², 2 mW/m² and 5 mW/m², respectively at point P. The wavelength of each of the sources is 600 nm. The resultant intensity at point P (in mW/m²) is _____________.



Basic Physics: ***Double slit experiment***

At P, the path difference for sources A and B,

$$\Delta_{AB} = \frac{xd}{D} = \frac{15mm \times 3.22mm}{1}m = 48.3 \times 10^{-6}m.$$

$$\therefore \frac{\Delta_{AB}}{\lambda} = \frac{48.3 \times 10^{-6}}{600 \times 10^{-9}} = 80.5 \Rightarrow \Delta_{AB} = \left(80 + \frac{1}{2}\right)\lambda$$

i.e. Path difference is half-integral multiple of wavelength: *condition for dark fringe.*

Thus A and B cancels at P; resultant intensity $= I_c = \boxed{5\ mW/m^2}$

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2. *[Q.C15]* One gram of ice at 0°C is melted and heated to water at 39°C. Assume that the specific heat remains constant over the entire process. The latent heat of fusion of ice is 80 Calories/gm. The entropy change in the process (in Calories per degree) is _____________.

Basic Physics: ***Latent heat, Specific heat, Entropy change***

→ Melting at constant T (ice at 0°C to water at 0°C): *Latent heat*

$$\Delta S_1 = \int_R \frac{dQ}{T} = \frac{Q_R}{T} = \frac{80 \text{ cal/g} \times 1\text{g}}{273\text{K}} = 0.293 \text{ cal/K}$$

→ Heating up of water (water at 0°C to water at 39°C): *Specific heat*

Specific heat capacity of water $= 4186 \text{ Jkg}^{-1}\text{K}^{-1} = 1000 \text{ cal kg}^{-1}\text{K}^{-1}$
$\therefore$ Heat capacity of 1g of water, $C = 4.186 \text{ J/K} = 1 \text{ cal/K}$

$$\therefore \Delta S_2 = \int_R \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{CdT}{T} = C \ln\left[\frac{T_2}{T_1}\right] = 1 \times \ln\left[\frac{39+273}{273}\right] = 0.134 \text{ cal/K}$$

$\therefore$ Total entropy change, $\Delta S = \Delta S_1 + \Delta S_2 = \boxed{0.43\ (cal/K)}$

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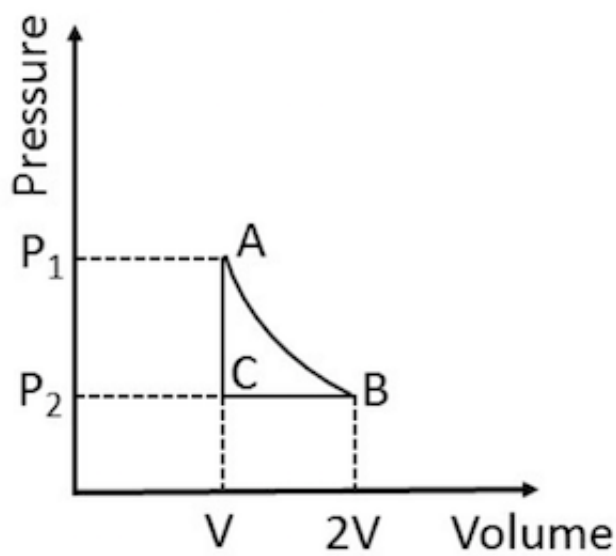
3. *[Q.C11]* In the thermodynamic cycle shown in the figure, one mole of a monatomic ideal gas is taken through a cycle. AB is a reversible isothermal expansion at a temperature of 800 K in which the volume of the gas is doubled. BC is an isobaric contraction to the original volume in which the temperature is reduced to 300 K. CA is a constant volume process in which the pressure and temperature return to their initial values. The net amount of heat (in Joules) absorbed by the gas in one complete cycle is __________.



Basic Physics: ***Thermodynamics: Cyclic process***

In a cyclic process, initial state = final state. $\therefore \Delta U = 0 \Rightarrow \Delta Q = \Delta W$

Work done in the isothermal process,

$$\Delta W_{AB} = -nRT\ln\left[\frac{V_f}{V_i}\right] = -1 \times R \times 800 \times \ln\left[\frac{2V}{V}\right] = -800R\ln 2.$$

Work done in the isobaric process,

$$\Delta W_{BC} = -P\left(V_f - V_i\right) = -nR\left(T_f - T_i\right)$$

$$= -1 \times R \times (300 - 800) = +500R$$

Work done in the isochoric process = 0.

$\therefore$ Total work, $\Delta W = -800R\ln 2 + 500R = -453.26\text{ J}$

$\therefore \Delta Q = -453.26\text{ J}$ (The negative sign shows that heat is *absorbed*).

$\Rightarrow$ Final answer: Heat absorbed $= \boxed{453.26\text{ J}}$

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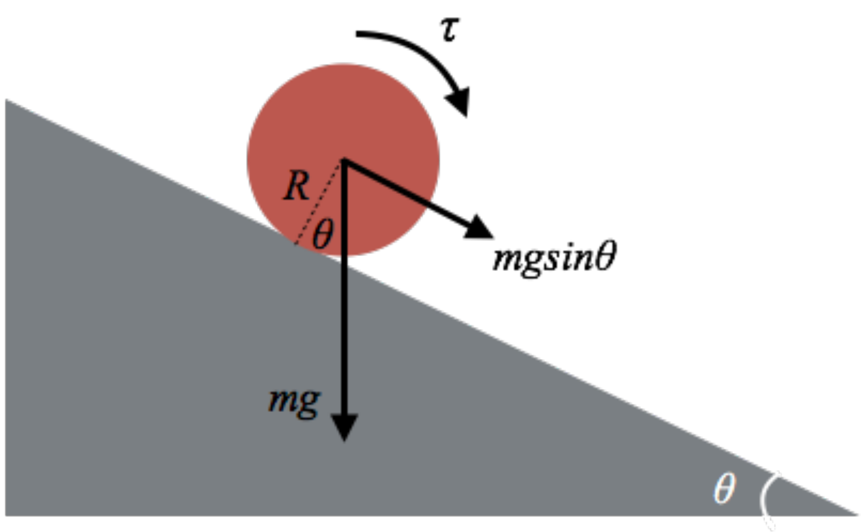
4. *[Q.C16]* A uniform disk of mass m and radius R rolls, without slipping, down a fixed plane inclined at an angle 30° to the horizontal. The linear acceleration of the disk (in m/sec²) is ____________.

Basic Physics: ***Dynamics of Rolling motion***



Point to remember: **Rolling = Translation + Rotation**

$\therefore$ Here, Applied force $\xrightarrow{\text{produces}}$ Linear acceleration + Torque

$$\Rightarrow F_{\text{applied}} = F_{\text{linear}} + F_{\text{torque}} \Rightarrow mg\sin\theta = ma + \frac{\tau}{R}$$

$$\text{We have } \tau = I\alpha = I\frac{a}{R} \ \{ \text{as } a = R\alpha \ (\text{like } v = R\omega)$$

$$\therefore mg\sin\theta = ma + \frac{Ia}{R^2} = ma\left(1 + \frac{I}{mR^2}\right)$$

$$\therefore \text{ Linear acceleration, } \boxed{a = \frac{g\sin\theta}{[1 + I/mR^2]}} \quad \ldots (1)$$

$$\text{For a disk, } I = \frac{1}{2}mR^2 \Rightarrow \frac{I}{mR^2} = \frac{1}{2}.$$

$$\therefore a = \frac{g\sin(30^\circ)}{1 + 1/2} = \frac{g/2}{3/2} = \frac{g}{3} = \boxed{3.27\ ms^{-2}}$$

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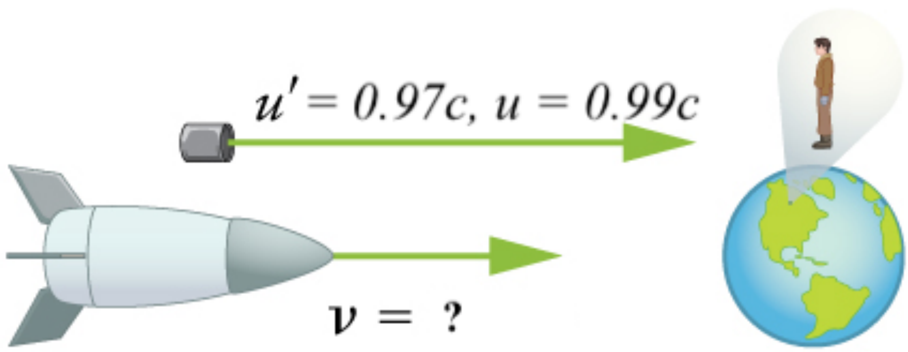
5. *[Q.A8]* A proton from outer space is moving towards earth with velocity 0.99 as measured in earth's frame. A spaceship, traveling parallel to the proton, measures proton's velocity to be 0.97 . The approximate velocity of the spaceship, in the earth's frame, is?

Basic Physics: ***Relativisitic velocity addition***

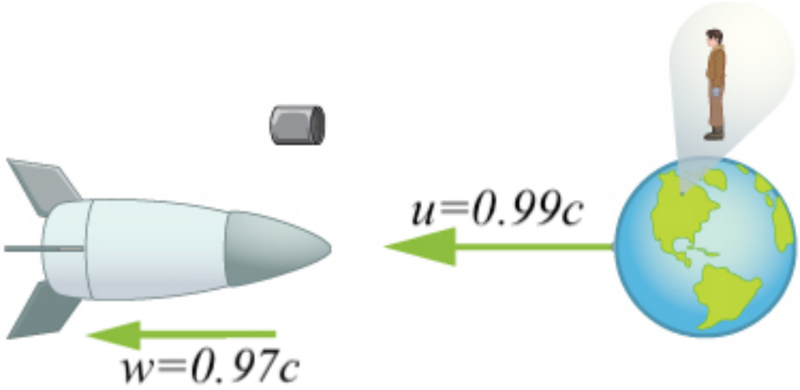
Direct method: (see Upadhyaya, P.108)



$$u' = \frac{u - v}{1 - uv/c^2} \Rightarrow 0.97c = \frac{0.99c - v}{1 - 0.99c \cdot v/c^2}$$

Rearranging and solving gives $\boxed{v = 0.5c}$, but lengthy.

Short cut: Find the *common* frame of reference for the given two velocities:
i.e. *the **proton's** frame:* $(u = \ - u, \ w = \ - u')$



$\therefore$ Relative velocity of spaceship w.r.t. Earth,

$$v_{\text{rel}} = \frac{u - w}{1 - uw/c^2} = \frac{0.99c - 0.97c}{1 - 0.99 \times 0.97} = \boxed{0.5c}$$

JAM 2016

1. *[Q.A20]* A train passes through a station with a constant speed. A stationary observer at the station platform measures the tone of the train whistle as 484 Hz when it approaches the station and 442 Hz when it leaves the station. If the sound velocity in air is 330 m/s, then the tone of the whistle and the speed of the train are?

Basic Physics: ***Doppler effect (non-relativistic)***

$$f_{\text{observed}} = \left[\frac{v_{\text{wave}} + v_{\text{observer}}}{v_{\text{wave}} + v_{\text{source}}}\right] f_{\text{original}}$$

where *the direction from the observer to the source is taken as positive*.

When train is approaching (source moving *towards* observer, i.e. velocity -ve):

$$484 = \left[\frac{330 + 0}{330 - v}\right] f_o \Rightarrow f_o = 484\left[1 - \frac{v}{330}\right] \ \ldots (1)$$

When train is leaving (source moving *away from* observer, i.e. velocity +ve):

$$442 = \left[\frac{330 + 0}{330 + v}\right] f_o \Rightarrow f_o = 442\left[1 + \frac{v}{330}\right] \ \ldots (2)$$

Solving $(1), (2) \Rightarrow v = \dfrac{42 \times 330}{926} = 14.97 ms^{-1} = 14.97 \times \dfrac{18}{5} kmh^{-1} \approx \boxed{54\ kmh^{-1}}$

$$\therefore (1) \Rightarrow f_0 = 484\left[1 - \frac{14.97}{330}\right] = \boxed{462.04\ Hz}$$