Solved Problems - V

Thermal properties of matter

52. A body cools from 67°C to 37°C. If this takes time t when the surrounding temperature is 27°C, what will be the time taken if the surrounding temperature is 7°C?

A. $2t$
B. $t/3$
C. $t/2$
D. $t/4$

Concept: ***Newton's Law of Cooling*** (see +1 Physics - Part 2 - p. 291)

$$-\frac{dQ}{dt} = k\,\Delta T$$

where ΔT is the *temperature difference* between body and surroundings (the derivative is with respect to time t).

Alternate form: $\boxed{\ln \Delta T = -Kt + c}$ (K is a constant for a given body)

Thus for initial and final temperature differences ΔT_0 and ΔT_t:

$$\ln\Delta T_0 - \ln\Delta T_t = -0 + c - (-Kt + c) \Rightarrow \ln\left[\frac{\Delta T_0}{\Delta T_t}\right] = Kt$$

Thus for first case, $\ln\left[\frac{67-27}{37-27}\right] = Kt \Rightarrow K = \frac{1}{t}\ln\left[\frac{40}{10}\right] = \frac{\ln 4}{t}$

For second case, $\ln\left[\frac{67-7}{37-7}\right] = Kt' \Rightarrow t' = \frac{1}{K}\ln\left[\frac{60}{30}\right]$

$$= \frac{t}{\ln 4}\ln 2 = \frac{t\ln 2}{\ln 2^2} = \frac{t\ln 2}{2\ln 2} = \boxed{t/2}$$

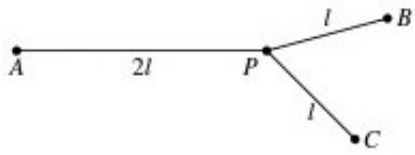
53. Three rods (lengths $2l$, l, l) made of the same material and having the same area of cross-section are joined as shown in figure. The end points A, B and C are maintained at constant temperatures 100°C, 50°C and 0°C, respectively. Assuming that there is no loss of heat from the surface of the rods, find the temperature that the junction P ultimately reaches.

A. 50°C

B. 40°C

C. 30°C

D. 20°C



Concept: ***Thermal conductivity*** (see Q.8 in Solved Problems 3, very similar).

At equilibrium, *net* heat flow at P = 0 (i.e. heat flowing in = heat flowing out)

We have $H = KA\dfrac{T_1 - T_2}{l}$. Given K and A are same for the three rods.

Let T be the final temperature at P.

$$\therefore KA\frac{100 - T}{2l} + KA\frac{50 - T}{l} + KA\frac{0 - T}{l} = 0$$

$$\Rightarrow \frac{KA}{l}\left\{\frac{100 - T}{2} + 50 - T + \ - T\right\} = 0$$

$$\Rightarrow 50 - \frac{T}{2} + 50 - 2T = 0$$

$$\Rightarrow 100 - \frac{5T}{2} = 0 \Rightarrow T = \frac{100 \times 2}{5} = \boxed{40^\circ\text{C}}$$