12. What is the group velocity for an electromagnetic wave in water if the refractive index of water at 20 degree celcius is 1.3311 for wavelength = 656.3nm and is 1.3314 for wavelength = 643.8nm...?

A. The group velocity v_g of a wave is defined by:

$$v_g = \frac{\partial \omega}{\partial k} \quad \begin{cases} w \rightarrow \text{angular frequency} \\ k \rightarrow \text{angular wavenumber} \end{cases}$$

Given data in terms of $\lambda_0 \rightarrow$ *vaccum* wavelength, and refractive index n. We have:

$$\lambda_0 = \frac{c}{\nu} = \frac{2\pi c}{\omega}; \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0}$$

where $\lambda, k \rightarrow$ properties in the medium. All except c are variables here. Therefore:

$$\frac{\partial \omega}{\partial k} = \frac{\frac{\partial \omega}{\partial \lambda_0}}{\frac{\partial k}{\partial \lambda_0}}$$

$$\frac{\partial \omega}{\partial \lambda_0} = 2\pi c \left(\frac{-1}{\lambda_0^2} \right)$$

$$\frac{\partial k}{\partial \lambda_0} = 2\pi \left\{ \frac{1}{\lambda_0} \frac{\partial n}{\partial \lambda_0} - \frac{n}{\lambda_0^2} \right\}$$

$$\therefore \boxed{\frac{\partial \omega}{\partial k} = \frac{c}{n - \lambda_0 \frac{\partial n}{\partial \lambda_0}}}$$

Substituting the values,

$$\frac{\partial \omega}{\partial k} = \frac{3 \times 10^8 \ ms^{-1}}{1.3311 - 656.3 \left(\frac{1.3314 - 1.3311}{643.8 - 656.3} \right)} = \underline{\underline{2.2274 \times 10^8 \ ms^{-1}}}$$

Bonus: The phase velocity v_p of a wave is defined by:

$$v_p = \frac{\omega}{k}$$

Q13. A spherical soap bubble of radius 0.01 m is formed inside another soap bubble of radius 0.02 m. The radius of single bubble which will have an excess pressure equal to the difference in pressure between the inside of the inner bubble and the outside of the large bubble is equal to?

A.

$$P_{out} - P_{in} = \frac{4S}{r_1} + \frac{4S}{r_2} = \frac{4S}{R}$$

$$\therefore R = \frac{r_1 r_2}{r_1 + r_2} = \boxed{0.0067 \ m}$$

Q14. Consider the superposition of two sinusoidal waves given by: y1 = 4sin(3x - 2t) cm and y2 = 4sin(3x+2t) cm. The maximum displacement of the resultant motion at x = 2.3 cm is?

A.

$$y = y_1 + y_2 = 4\{sin(3x - 2t) + sin(3x + 2t)\} = 4\{2sin(3x)cos(2t)\}\ cm$$
$$\therefore y_{max}(2.3) = 8 \mid \sin(3 \times 2.3) \cdot 1 \mid = \boxed{4.63\ cm}$$

Q15. In photographing the Sun's spectrum, it was found that the yellow spectral line (A = 5890Å) in the spectra obtained from the left and right edges of the sun is displaced by 0.08 Å. The linear velocity of rotation of the solar disk is?

A. When $v << c$,

$$\lambda' = \lambda\left(1 - \frac{v}{c}\right) \implies v = c\frac{\lambda - \lambda'}{\lambda} = c\frac{\Delta\lambda}{\lambda}$$

Given $2\Delta\lambda = 0.08$Å. Therefore,

$$v = 3 \times 10^8 ms^{-1} \times \frac{0.04}{5890} = \boxed{2.037\ kms^{-1}}$$

Q16. A soap bubble 10 cm in radius with a wall thickness of 3.3*10^-6 cm is charged to a potential of 100V. The bubble bursts and falls as a spherical drop. The potential of drop is?

A. Volume of the bubble = Volume of the drop:

$$\tau = 4\pi R^2 t = \frac{4}{3}\pi r^3$$
$$\implies r = \left(3R^2 t\right)^{\frac{1}{3}} = \left(3 \times 10^2 \times 3.3 \times 10^{-6}\right)^{\frac{1}{3}} = 0.1\ cm$$
$$i.e.\ r = \frac{R}{100}$$

We have:

$$V_r = \frac{Q}{4\pi\epsilon_0 r} = \frac{Q}{4\pi\epsilon_0 R} \cdot 100 = V_R \times 100 = \boxed{10000 \text{ Volts}}$$

Q17. A sample of uranium emitting alpha particles of 4.18 MeV is placed near an ionization chamber. Assume that only 10 particles enter the chamber per second and the energy required to create an ion-pair by an alpha particle is 35 eV. What will be the current produced?

A. Number of ion pairs created per second $= \frac{4.18}{35} \times 10^6 \times 10/sec = 1.194 \times 10^6/sec$ $\therefore$ Current $= 1.194 \times 10^6/sec \times 1.6 \times 10^{-19} C = \boxed{1.91 \times 10^{-13} A}$

Q18. X-rays of wavelength 0.24 nm are Compton scattered and the scattered beam is observed at an angle of 60deg relative to the incident beam. The Compton wavelength of the electron is 0.00243 nm. The kinetic energy of scattered electrons in eV is ____. [JAM 2015: Section-C, Q8]

A.

$$\Delta\lambda = \lambda_c(1-\cos\phi) = \lambda_c(1-\cos(60\,\text{deg})) = \frac{\lambda_c}{2}$$
$$\therefore \Delta E = h\Delta\nu = \frac{-hc\Delta\lambda}{\lambda^2} \quad \left\{\because \nu = \frac{c}{\lambda}\right\}$$

Energy lost by the photon = Energy gained by the electron;

$$K_e = -\Delta E = hc\frac{\Delta\lambda}{\lambda^2} = \frac{13.6eV}{R}\cdot\frac{\lambda_c}{2\lambda^2}$$
$$= \frac{13.6eV \times 0.00243\ nm}{1.097\times 10^7\ m^{-1}\times 2\times(0.24\ nm)^2} = \boxed{26.15eV}$$

Q19. X-ray diffraction of a cubic crystal gives an intensity maximum for Bragg angle of 20° corresponding to the (110) plane. The lattice parameter of the crystal is ____nm. (Consider wavelength of X-ray = 0.15 nm) **[JAM 2016: Section-C, Q56]**

A.

$$2d\sin\theta = n\lambda \implies d = \frac{\lambda}{2\sin\theta} \quad \{n=1$$
$$= \frac{0.15\ nm}{2\times\sin(20\,\text{deg})} = 0.219\ nm$$

Given $(h\ k\ l) = (1\ 1\ 0)$; We have:

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}} \implies a = d\sqrt{h^2+k^2+l^2} = 0.219\ nm\times\sqrt{2} = \boxed{0.31\ nm}$$

Q20. An unstable particle of rest energy 1000 MeV decays into a meu -meson and a neutrino with a mean life 10^-8sec, when at rest. If the particle has a momentum of 1000 MeV/c the mean decay distance is given by?

A. Method 1 - Long cut:

$$\text{Given}\quad E_0 = m_0c^2 = 1000\ MeV \implies m_0 = 1000\frac{MeV}{c^2}$$
$$\text{Momentum,}\quad p = \gamma m_0 v \implies 1000\ \frac{MeV}{c} = \gamma v\times 1000\ \frac{MeV}{c^2}$$
$$\implies \frac{1}{\gamma} = \frac{v}{c} \implies \frac{1}{\gamma^2} = 1-\frac{v^2}{c^2} = \frac{v^2}{c^2}$$
$$\therefore \frac{v^2}{c^2} = \frac{1}{2} \implies v = \frac{c}{\sqrt{2}}$$
$$\text{And}\quad \gamma = \frac{1}{\sqrt{1-\frac{1}{2}}} = \sqrt{2}$$

$$\therefore t' = \gamma t = \sqrt{2} \times 10^{-8} s$$

$$\text{Mean decay distance} = vt' = \frac{3 \times 10^8 ms^{-1}}{\sqrt{2}} \times \sqrt{2} \times 10^{-8} = \boxed{3\,m}$$

Method 2 - Short cut:

$$\text{Given } E_0 = pc = 1000\ MeV; \quad \therefore E^2 = E_0^2 + (pc)^2 = 2E_0^2$$

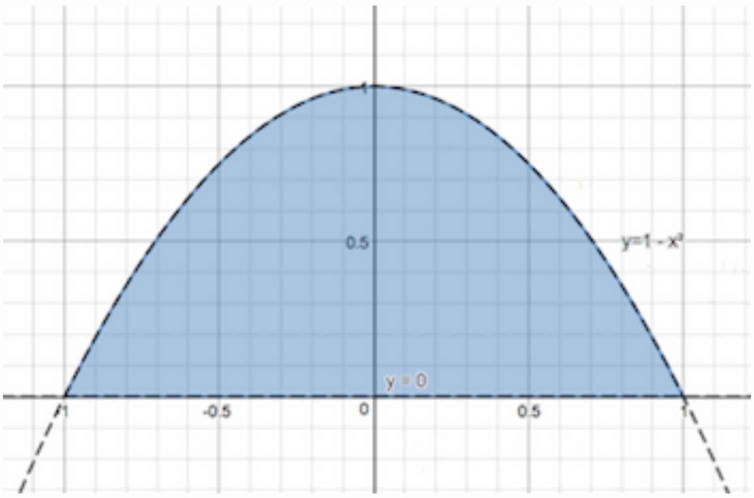
$$\implies E = \sqrt{2}E_0 \implies \gamma = \frac{E}{E_0} = \sqrt{2}$$

$$\text{Using equation for } \gamma \text{, obtain } v = \frac{c}{\sqrt{2}}$$

$$\therefore \text{Mean distance } = vt' = v\gamma t = \frac{c}{\sqrt{2}} \cdot \sqrt{2} \cdot 10^{-8} = \boxed{3\,m}$$

Q21. The shape of a dielectric lamina is defined by the two curves $y = 0$ and $y = 1 - x^2$. If the charge density of the lamina $\sigma = 15y\ C/m^2$, then the total charge on the lamina is ___C. **[JAM 2016: Section-C, Q53]**

A.



$$Q = \int_s \sigma da = \int_{-1}^{1}\int_0^{1-x^2} 15y\ dydx = 15\int_{-1}^{1} \frac{(1-x^2)^2}{2} dx$$

$$= \frac{15}{2}\int_{-1}^{1} 1 - 2x^2 + x^4\ dx = \frac{15}{2}\left[x - \frac{2x^3}{3} + \frac{x^5}{5}\right]_{-1}^{1} = \boxed{8\text{ C}}$$

Q22.

A hemispherical shell is placed on the $xy-$ plane centered at the origin. For a vector field $\vec{E} = (-y\hat{e}_x + x\hat{e}_y)/(x^2 + y^2)$, the value of the integral $\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a}$ over the hemispherical surface is __________ π.

($d\vec{a}$ is the elemental surface area. $\hat{e}_x, \hat{e}_y$, and $\hat{e}_z$ are unit vectors in Cartesian $-$ coordinate system.)

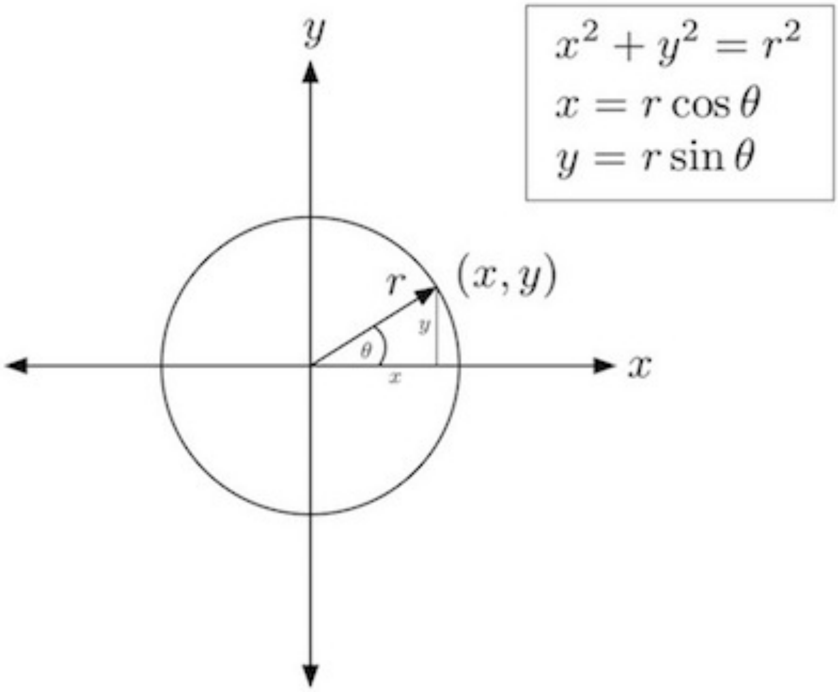
[JAM 2016: Section-C, Q52]

A.

$$\text{Stokes theorem} \implies \int_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = \oint \vec{E} \cdot d\vec{l}$$

$$\vec{E} \cdot d\vec{l} = \left[\frac{-y}{x^2+y^2}\hat{e_x} + \frac{x}{x^2+y^2}\hat{e_y}\right] \cdot [dx\,\hat{e_x} + dy\,\hat{e_y}] = \frac{-y\,dx}{x^2+y^2} + \frac{x\,dy}{x^2+y^2}$$

Here, the boundary of the hemisphere is a circle in the xy plane. So, convert to polar form:



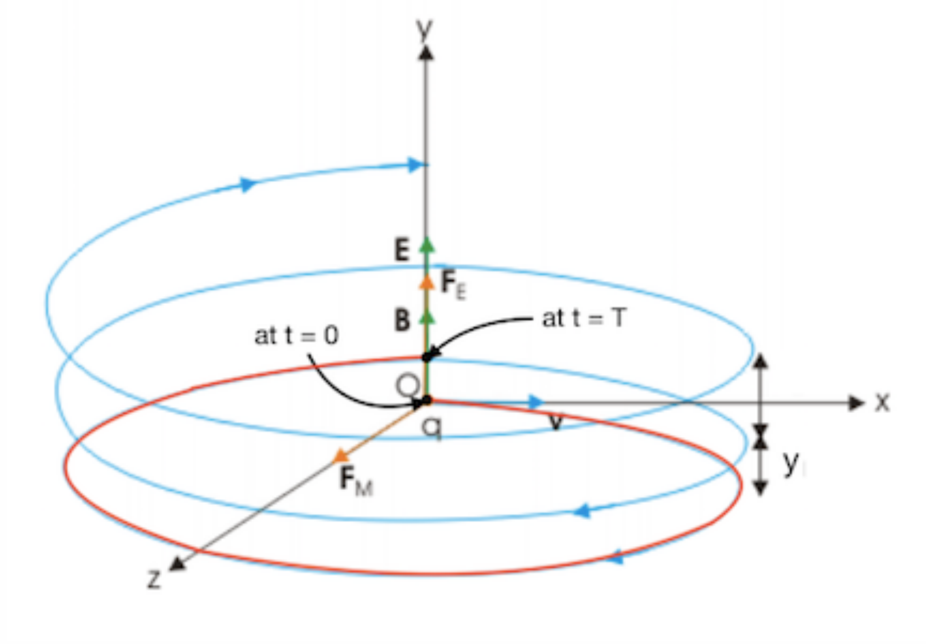
$$\frac{-y\,dx}{x^2+y^2} = \frac{-r\sin\theta \cdot {}^- r\sin\theta\,d\theta}{r^2} = \sin^2\theta\,d\theta.\ \text{Similarly,}\ \frac{x\,dy}{x^2+y^2} = \cos^2\theta\,d\theta$$

$$\implies \vec{E} \cdot d\vec{l} = (\sin^2\theta + \cos^2\theta)\,d\theta = d\theta.$$

$$\therefore \oint \vec{E} \cdot d\vec{l} = \int_0^{2\pi} d\theta = \boxed{2\pi}$$

Q23. A positively charged particle, with a charge q, enters a region in which there is a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$, both directed parallel to the positive y- axis. At $t = 0$, the particle is at the origin and has a speed v_0 directed along the positive x- axis. The orbit of the particle, projected on the x-z plane, is a circle. Let T be the time taken to complete one revolutionof this circle. The y-coordinate of the particle at $t = T$ is given by? **[JAM 2015: Section-A, Q17]**

A. Circular motion about y-axis due to $\vec{B}$ $+$ acceleration along y-axis due to $\vec{E}$ $\implies$ HELIX:



$$\text{Cyclotron equation,}\;\; mv = qBR. \;\; \because v = R\omega = \frac{2\pi R}{T},$$

$$m\frac{2\pi R}{T} = qBR \implies T = \frac{2\pi m}{qB} \qquad \ldots (1)$$

$$\text{Acceleration along y-axis,}\;\; a = \frac{F}{m} = \frac{qE}{m}.$$

$$\therefore\;\; y = \frac{1}{2}aT^2 = \frac{1}{2}\frac{qE}{m}\frac{4\pi^2 m^2}{q^2B^2} = \boxed{\frac{2\pi^2 mE}{qB^2}}$$

Q24. A 1 W point source at origin emits light uniformly in all the directions. If the units for both the axes are measured in centimetre, then the Poynting vector at the point (1,1,0) in W/cm² is? **[JAM 2016: Section-A, Q2]**

A. Magnitude of the Poynting vector:

$$|\vec{S}| = S = \frac{\text{Power}}{\text{Area}} = \frac{1W}{4\pi r^2}$$

Radius of the sphere passing through $(1,1,0)$:

$$r = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$$

$$\therefore\;\; S = \frac{1W}{4\pi \cdot 2} = \frac{1}{8\pi} Wcm^{-2}$$

Direction of the Poynting vector at $(1,1,0)$:

$$\hat{r}_{(1,1,0)} = \frac{\vec{r}}{r} = \frac{1 \cdot \hat{e_x} + 1\hat{e_y}}{\sqrt{2}}$$

$$\therefore \vec{S} = S\hat{r} = \boxed{\frac{1}{8\pi\sqrt{2}}(\hat{e}_x + \hat{e}_y)}$$

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