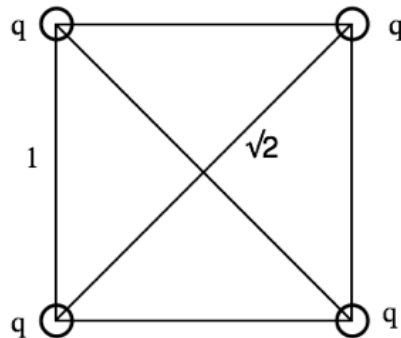


1. If 3.6 gm of uranium is to be completely converted to energy, How many joules of energy will be obtained from it?

A. $E = mc^2 = 3.24 \times 10^{14} \text{ J}$

2. The energy stored in a system of four identical charges of $4 \times 10^{-9} \text{ C}$ at the corners of a square of side 1 m is...?

A.



$$E = \frac{1}{4\pi\epsilon_0} \sum_i \sum_j \frac{q_i q_j}{r_{ij}} = \frac{q^2}{4\pi\epsilon_0} \left(4 \times \frac{1}{1} + 2 \times \frac{1}{\sqrt{2}} \right)$$

$$= 9 \times 10^9 (4 \times 10^{-9})^2 (4 + \sqrt{2}) \approx 7.796 \times 10^{-7} \text{ J}$$

3. Intensity of sound is increased by a factor of 20. By how many decibels is the sound level increased?

A. $= 10 \times \log(20) = 13.01 \text{ dB}$ (Intensity \propto Power)

4. An electron and a positron at rest annihilate each other and two gamma ray photons are emitted. The wavelength of gamma rays is given by ?

A.

$$E_\gamma = \frac{2m_e c^2}{2} = 0.511 \text{ MeV}$$

$$\lambda = \frac{c}{\nu} = \frac{ch}{E_\gamma} = 2.43 \times 10^{-12} \text{ m}$$

5. What is the amount of energy released when 10 micrograms of uranium-235 undergoes fission, if energy released per fission is 200 MeV?

$$N = \frac{m}{M} N_A = \frac{10 \mu\text{g}}{235 \text{ g}} \times 6.022 \times 10^{23} = 2.56 \times 10^{16}$$

$$\therefore E = N \times 200 \text{ MeV} = 5.12 \times 10^{18} \text{ MeV} = 820 \text{ kJ}$$

6. A radio frequency carrier of frequency 8 MHz is amplitude modulated by an audio signal frequency of 20 KHz. The modulated signal consist of frequencies...?

∴ Ans: 8 MHz, $8+0.02 = 8.02$ MHz, $8-0.02 = 7.98$ MHz

Here $\mu = A_m/A_c$ is the *modulation index*; in practice, μ is kept ≤ 1 to avoid distortion.

Using the trigonometric relation $\sin A \sin B = \frac{1}{2} (\cos(A-B) - \cos(A+B))$, we can write $c_m(t)$ of Eq. (15.4) as

$$c_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m) t - \frac{\mu A_c}{2} \cos(\omega_c + \omega_m) t \quad (15.5)$$

Here $\omega_c - \omega_m$ and $\omega_c + \omega_m$ are respectively called the lower side and upper side frequencies. The modulated signal now consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from, known as side bands. The frequency spectrum of the amplitude modulated signal is shown in Fig. 15.9.

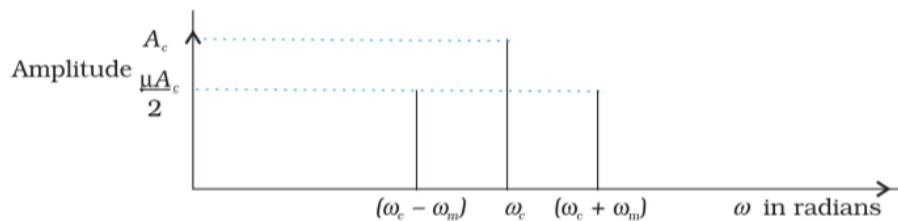


FIGURE 15.9 A plot of amplitude versus ω for an amplitude modulated signal.

7. If a particle of mass 1.67×10^{-27} kg is confined to move in a box of length 10^{-14} m and infinite depth, the minimum kinetic energy of the particle would be?

A. 2 equations und: Particle in a box (Beiser):

$$E_1 = \frac{\hbar^2 \pi^2}{2mL^2} \quad \Rightarrow \quad E_1 = 3.286 \times 10^{-13} \text{ J} = 2.05 \text{ MeV}$$

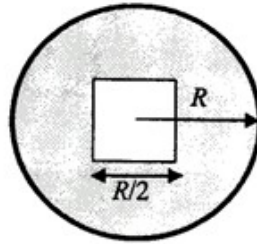
Uncertainty principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \text{Gives } \Rightarrow \quad \frac{1}{2} m v^2 \approx \frac{\hbar^2}{2mL^2}$$

Again \cong same

8.

- Q.12 Consider a uniform thin circular disk of radius R and mass M . A concentric square of side $R/2$ is cut out from the disk (see figure). What is the moment of inertia of the resultant disk about an axis passing through the centre of the disk and perpendicular to it?



4

- (A) $I = \frac{MR^2}{4} \left[1 - \frac{1}{48\pi} \right]$
 (B) $I = \frac{MR^2}{2} \left[1 - \frac{1}{48\pi} \right]$
 (C) $I = \frac{MR^2}{4} \left[1 - \frac{1}{24\pi} \right]$
 (D) $I = \frac{MR^2}{2} \left[1 - \frac{1}{24\pi} \right]$

$$I = I_{\text{disk}} - I_{\text{hole}} = \frac{M_{\text{disk}} R^2}{2} - \frac{M_{\text{hole}} (a^2 + b^2)}{12} \quad \left\{ \text{here } a = b = \frac{R}{2} \right.$$

$$= \frac{MR^2}{2} - \frac{M_{\text{hole}} R^2}{24}$$

$$\text{Here, } M_{\text{hole}} = M \times \frac{A_{\text{hole}}}{A_{\text{disk}}} = M \times \frac{R^2}{4} \cdot \frac{1}{\pi R^2} = \frac{M}{4\pi}$$

$$\therefore I = \frac{MR^2}{2} - \frac{MR^2}{24 \times 4\pi} = \frac{MR^2}{2} \left[1 - \frac{1}{48\pi} \right] \quad (b)$$

9. The vector $A = 5i + 6j$ is rotated through an angle 45° about the z -axis in the anticlockwise direction. The resultant vector is

A.

In **linear algebra**, a **rotation matrix** is a **matrix** that is used to perform a **rotation** in **Euclidean space**. For example, using the convention below, the matrix

$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

rotates points in the xy -plane counterclockwise through an angle θ about the origin of the **Cartesian coordinate system**. To perform the rotation using a rotation matrix R , the position of each point must be represented by a **column vector** \mathbf{v} , containing the coordinates of the point. A rotated vector is obtained by using the matrix multiplication $R\mathbf{v}$.

$$v = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 11 \end{bmatrix} \quad \mathbf{v} = -1/\sqrt{2} \mathbf{i} + 11/\sqrt{2} \mathbf{j}$$

10. An open organ pipe is closed suddenly. As a result, the frequency of the second overtone of the closed pipe becomes higher by 100 vibrations/sec than that of the first overtone of the original pipe. The fundamental frequency of the open pipe is **A.**

$$\left(2 + \frac{1}{2}\right) \frac{v}{2L} = 2 \frac{v}{2L} + 100$$

$$\frac{5v}{4L} - \frac{v}{L} = 100$$

$$\frac{v}{L} = 4 \times 100 = 400$$

$$\therefore \text{Fundamental freq of open pipe} = \frac{v}{2L} = 200 \text{ vibrations/sec}$$

11. A whistle of frequency 540 Hz rotates in a horizontal circle of radius 2 m at an angular speed of 15.0 rad/s. The maximum and minimum frequencies heard by a listener, standing a long distance away at rest from the centre of the circle are respectively **A.**

$$v = v_0 \left(1 - \frac{v_s}{v}\right) \quad (15.51)$$

For a source approaching the observer, we replace v_s by $-v_s$ to get

Speed of sound in air = 330m/s
fmax = 589 Hz and fmin = 491.
