# Stochastic Gravitational Wave Background: Sensitivity of GW detectors

Build-a-detector workshop

June 03, 2021

## 1 Upper limits

Using the equation for $\sigma_{\hat{\Omega}_\alpha}$ given in the presentation,

$$\sigma^2_{\hat{\Omega}_\alpha} \equiv \left[ \sum_f \sigma^{-2}_{\hat{\Omega}_\alpha}(f) \right]^{-1}$$

where

$$\sigma^2_{\hat{\Omega}_\alpha}(f) \approx \frac{1}{2T\Delta f} \frac{P_1(f)P_2(f)}{\gamma^2_{12}(f)S^2_\alpha(f)} \qquad \text{and} \qquad S_\alpha(f) = \frac{3H^2}{10\pi^2} \frac{1}{f^3} \left( \frac{f}{f_{ref}} \right)^\alpha$$

estimate $\sigma_{\hat{\Omega}_\alpha}$ for Hanford-Livingston detector pair (HL) for both $\alpha = 0$ and $\alpha = 2/3$.

- Get power spectral densities $P_1(f)$ and $P_2(f)$ from the representative amplitude spectral densities of H (link) and L (link) during O3 run. The first columns in those linked files are frequencies and the second columns are amplitude spectral densities (Note: Square the amplitude spectral densities to get power spectral densities).
- For $\gamma_{12}$ use the data from this link. The first column in that file is frequency and the second column is $\gamma_{12}(f)$.
- For T (in secs) use 160 days. This is the approximate livetime of the recent SGWB analysis reported in [1].

- For Hubble Constant H, use $0.67 \times 3.24 \times 10^{-18}$ (km/s)/Mpc.

Multiply the values you get for $\sigma_{\hat{\Omega}_\alpha}$ by 2 to get 95% C.L. upper limits on $\hat{\Omega}_\alpha$ reported in reference [1]. Compare the 95% C.L. you get with the numbers reported in Table II of reference [1] under O3 column.

# 2 Signal-to-noise ratios (SNRs)

The predicted $\Omega_\alpha$ for BBH and BNS mergers (after combining all frequencies) are $\Omega_{BBH}(25\text{Hz}) = 5.0^{+1.7}_{-1.4} \times 10^{-10}$ and $\Omega_{BNS}(25\text{Hz}) = 2.1^{+2.9}_{-1.6} \times 10^{-10}$ [1].

- Just using the maximum likliehood values of $\Omega_{BBH}$ and $\Omega_{BNS}$ (neglecting the uncertainties) and using $\sigma_{\hat{\Omega}_{2/3}}$ from the first part, calculate SNRs for BBH and BNS gravitational wave background during O3 run.

- By setting $\gamma_{12} = 1$ in the first part, estimate $\sigma_{\hat{\Omega}_{2/3}}$ and then calculate SNRs again. How much do they compare to the previous estimates?

- Calculate the BBH and BNS gravitational wave background SNRs for A+ design of Hanford and Livingston LIGO detectors and for Einstein Telescope (ET). The amplitude spectral densities for A+ and ET can be obtained from this link (download the zip file). For A+ use the $\gamma_{12}$ from the file link given in the first part and for ET use $\gamma_{12} = -0.35$ for all frequencies. For both A+ and ET, use the same power spectral denstities for $P_1(f)$ and $P_2(f)$ i.e., $P_1(f) = P_2(f)$.

# References

[1] KAGRA Collabration, LIGO Scientific Collaboration, and VIRGO Collaboration, *Upper Limits on the Isotropic Gravitational-Wave Background from Advanced LIGO's and Advanced Virgo's Third Observing Run* (2021). https://arxiv.org/abs/2101.12130.