1. If vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right| = \frac{1}{2}$, $\left|\vec{b}\right| = \frac{4}{\sqrt{3}}$ and $\left|\vec{a} \times \vec{b}\right| = \frac{1}{\sqrt{3}}$, then find $\left|\vec{a} \cdot \vec{b}\right|$.

2. Find K, if $f(x) = \begin{cases} k\sin\left(\frac{\pi}{2}(x+1)\right), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.

3. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\,\vec{b}$ to be a unit vector?

4. Find the distance between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$ and $\vec{\cdot}^{r}(6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$.

5. If A is a square matrix such that $\left|A\right| = 5$, write the value of $\left|AA^T\right|$.

6. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$, find $\left|AB\right|$.

7. If $\mathbf{A} = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $\mathbf{KA} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$ find the values of k and a.

8. Differentiate $(\sin 2x)^x$+$\sin^{-1}\ \sqrt{3}x$ with respect to x.

9. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$with respect to $\cos^{-1} x^2$

10. Find K, if $f(x) = \begin{cases} k\sin\left(\frac{\pi}{2}(x+1)\right), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.

11. Find equation of normal to the curve $ay^2 = x^3$ at the point whose x coordinate is am^2.

12. Find :

$$\int \frac{1 - \sin x}{\sin x(1 + \sin x)}, dx$$

.

13. Find :

$$\int [\log(\log x) + \frac{1}{(\log x)^2}]dx$$

.

14. Evaluate:

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\, dx$$

15. Evaluate :

$$\int_0^1 \cot^{-1}(1 - x + x^2)dx$$

16. Solve the differential equation :

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^3$$

17. Solve the differential equaton : $2ye^{x/y}dx + [y - 2xe^{x/y}]dy = 0$

18. Ishan wants to donate a rectangular plot of land for a school in his village, When he was asked to give dimensions of the plot, he told that if its length is decreased by $50m$ and breadth is increased by $50m$, then its area will remain same, but if length is decreased by $10m$ and breadth is decreased by $20m$, then its area will decrease by $5300m^2$ Using matrices, find the dimensions of the plot. Also give reason why he wants to donate the plot for a school.

19. Prove that $\sin^{-1}(\frac{3}{5}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

20. Solve the equation for x : $\cos(\tan^{-1} x) = \sin(\cot^{-1}(\frac{3}{4})$

21. There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag B.

22. given that vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find P, Q, R, S such that area of triangle is $5\sqrt{6}$ where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$, $\vec{b} = s\hat{i} + 3\hat{k} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$

23. Find the equation of plane passing through the points $A(3, 2, 1)$, $B(4, 2, -2)$ and $C(6, 5, -1)$and hence find the value of λ for which $A(3, 2, 1)$, $B(4, 2, -1)$, $C(6, 5, -1)$ and $D(\lambda, 5, 5)$ are coplanar.

24. Find the coordinates of the point where the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ meets the plane which is perpendicular to the vector $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$and at a distance of $\frac{4}{\sqrt{11}}$ from the origin.

25. Let $f : N \to N$ be a function defined as $f(x) = 4x^2 + 12x + 15$.Show that $f : N \to S$ is invertible(where S is range of f). Find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$.

26. Using properties of determinants, prove that:
$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right).$$

27. using elementary row operations,find the inverse of thefollowing matrix:$\mathbf{A} =$
$$\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

28. Determine the intervals in which the function$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is strictly increasing or strictly decreasing.

29. Find the maximun and minimum values of $f(x) = \sec x + \log \cos^2 x$, $0 < x < 2\pi$.

30. Using integration find the area of the region $\{(x, y) : y^2 \le 6ax \text{ and } x^2 + y^2 \le 16a^2\}$

31. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also,find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

32. A diet is is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available costing ₹5 *perunit* and ₹6 per unit respectively. One unit of food F_1contains 4 *units* of vitamin A and 3 *units* of minerals whereas one unit of food F_2 contains 3 *units* of vitamin A and 6 *units* of minerals. Formulate this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement.

33. Three numbers are selected at random (without replacement) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also find the mean and variance of the distribution.