TWENTY-SECOND INTERNATIONAL OLYMPIAD,1981

1. P is a point inside a given triangle $ABC.D, E, F$ are the feet of the perpendiculars from P to the lines BC, CA, AB respectively. Find all P for which $\frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}$ is least.

2. Let $1 \leq r \leq n$ and consider all subsets of r elements of the set $\{1, 2, ..., n\}$. Each of these subsets has a smallest member. Let $F(n, r)$ denote the arithmetic mean of these smallest numbers; prove that $F(n, r) = \frac{n+1}{r+1}$

3. Determine the maximum value of $m^3 + n^3$, where m and n are integers satisfying $m, n \epsilon \{1, 2, ..., 1981\}$ and $\left(n^2 - mn - m^2\right)^2 = 1$

4. (a) For which values of $n > 2$ is there a set of n consecutive positive integers such that the largest number in the set is a divisor of the least common multiple of the remaining $n - 1$ numbers?

   (b)For which values of $n > 2$ is there exactly one set having the stated property?

5. Three congruent circles have a common point O and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incenter and the circumcenter of the triangle and the point O are collinear

6. The function $f(x, y)$ satisfies

   (1) $f(0, y) = y + 1$,
   (2) $f(x + 1, 0) = f(x, 1)$,
   (3) $f(x + 1, y + 1) = f\left(x, f(x + 1, y)\right)$,
   for all non-negative integers x, y. Determine $f(4, 1981)$.