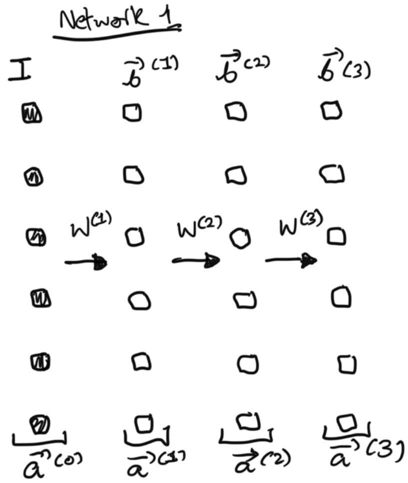
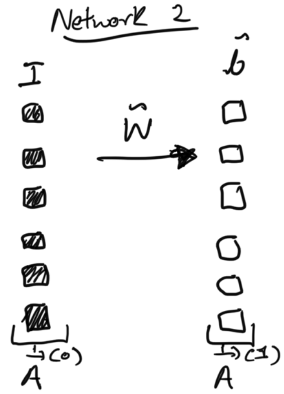
Formulating Equivalent Neural Networks





For Network 1, the equations that summarise the network are:

$$\boxed{\begin{aligned} \vec{a}^{(1)} &= W^{(1)}\vec{a}^{(0)} + \vec{b}^{(1)} && \text{Eq. 1} \\ \vec{a}^{(2)} &= W^{(2)}\vec{a}^{(1)} + \vec{b}^{(2)} && \text{Eq. 2} \\ \vec{a}^{(3)} &= W^{(3)}\vec{a}^{(2)} + \vec{b}^{(3)} && \text{Eq. 3} \end{aligned}}$$

Writing $\vec{a}^{(2)}$ in terms of $\vec{a}^{(0)}$:

$$\vec{a}^{(2)} = W^{(2)}\left(W^{(1)}\vec{a}^{(0)} + \vec{b}^{(1)}\right) + \vec{b}^{(2)}$$

$$\boxed{\vec{a}^{(2)} = W^{(2)}W^{(1)}\vec{a}^{(0)} + \left(W^{(2)}\vec{b}^{(1)} + \vec{b}^{(2)}\right)}$$

Eq. 4

Writing $\vec{a}^{(3)}$ in terms of $\vec{a}^{(0)}$:

$$\vec{a}^{(3)} = W^{(3)}\left(W^{(2)}W^{(1)}\vec{a}^{(0)} + W^{(2)}\vec{b}^{(1)} + \vec{b}^{(2)}\right)$$

$$\boxed{\vec{a}^{(3)} = W^{(3)}W^{(2)}W^{(1)}\vec{a}^{(0)} + W^{(3)}W^{(2)}\vec{b}^{(1)} + W^{(3)}\vec{b}^{(2)}}$$

Eq. 5

Now, if two neural networks are equivalent,

$$\left(\vec{a}^{(0)} = \vec{A}^{(0)}\right) \text{ and } \left(\vec{a}^{(3)} = \vec{A}^{(1)}\right)$$

Therefore, $\underline{\hat{W} = W^{(3)}W^{(2)}W^{(1)}}$ and

$$\underline{\hat{b} = W^{(3)}W^{(2)}\vec{b}^{(1)} + W^{(3)}\vec{b}^{(2)}}$$

DIMENSIONAL ANALYSIS

1. $\hat{W}$ is a 6x6 matrix. $W^{(1)}, W^{(2)}, W^{(3)}$ are 6x6 matrices and their product must be the same dimensions.

2. $\hat{b}$ is 6x1 matrix.

$$\underbrace{(W^{(3)}W^{(2)})_{6\times6}\,\vec{b}^{(1)}_{6\times1}}_{6\times1} + \underbrace{W^{(3)}_{6\times6}\,\vec{b}^{(2)}_{6\times1}}_{6\times1}$$