Review Materials

Propositional Logic

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The Language

- We will study about propositional logic and first order predicate calculus
- Basic Definitions
- Atoms: strings of characters stating with a capital letter. T and F are two distinguished atoms
- Connectives
  - $\cap$, $\wedge$ conjunctions
  - $\cup$, $\vee$ disjunctions
  - $\neg$, ~ negations
  - $\supset$, $\Rightarrow$, $\rightarrow$ logical implications

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Definitions

- The set of objects about which knowledge is being expressed is often called ***Universe of discourse.***
- A ***function*** is some kind of interrelationship among the objects in a universe of discourse.
- A set of functions emphasized in a conceptualization is called the ***functional basis set.***
- A ***relation*** is another kind of interrelationship among the objects in a universe of discourse.
- A set of relations emphasized in a conceptualization is called the ***relational basis set.***

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Definition

- A conceptualization is a triple consisting of a
  universe of discourse,
  a functional basis set,
  and a relational basis set for the universe of discourse.
- Example – class illustration

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Basis of propositions

- A proposition is either true or false
- A conjunction of propositions is true iff all the propositions are true, otherwise it is false.
- A disjunction of proposition is false if all the propositions are false, otherwise it is true.
- A set of propositions can be re-written as conjunction of disjunctions of propositions.
- A truth assignment to proposition is called an interpretation. An interpretation that satisfies a set of propositions Δ is called the model of Δ.
- Examples are in the class

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Syntax of a Well Formed Formula

- Any atom is a well-formed formula (wff)
- If w_1 and w_2 are wffs, so are
  - $w_1 \vee w_2$
  - $w_1 \wedge w_2$
  - $w_1 \supset w_2$
  - $\neg w_1$
- There are no other wff
- Note that

  $w_1 \supset w_2$ is rewritten as $\neg w_1 \wedge w_2$

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Rules of inference

- Modus ponens

$$\frac{\begin{array}{l} w_1 \\ w_1 \supset w_2 \end{array}}{w_2}$$

Infer w_2 from w_1 and $w_1 \supset w_2$

- $w_1 \wedge w_2$ from w_1 and w_2 Introduction of $\wedge$
- $w_1 \wedge w_2$ from $w_2 \wedge w_1$ commutativity
- $w_1 \vee w_2$ from $w_2 \vee w_1$ commutativity
- $w_1 \vee w_2$ from w_1 ($\vee$ introduction)
- w_1 from $w_1 \wedge w_1$ ($\wedge$ elimination)
- w_1 from $\neg(\neg w_1)$ ($\neg$ elimination)

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Basic Definitions (propositional logic)

- Clause
  - A disjunction of propositions
- A set of clauses
  - Conjunction of clauses
- A set of clauses is said to be satisfiable if there exists a model that satisfy the set.
- A set of clauses is said to be valid if the set is true under any possible interpretation or all the interpretations.
- A set is said to be unstaisfiable if there is no model that satisfy the set.

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Binary Resolution Rule

- A clause is a disjunction of literals

Consider

$Cl_1 : C_1 \vee p$ and

$Cl_2 : C_2 \vee \sim p$

- From

$C_1 \vee p$

$C_2 \vee \neg p$

$C_1 \vee C_2$ is inferred

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Proof

- $\Delta \vdash w_n$ proof of w_n from Δ. There exists a derivation of w_n from a set of wffs Δ by applying inference rules.
- W_k is a theorem of Δ if $\Delta \vdash w_k$ of $w_k \in \Delta$

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Semantics

- *Interpretations*: An association of atoms with propositions about the world
- In a given interpretation, the proposition associated with an atom is called *denotation* of that atom.
- Under a given interpretation, an atom has values {True (T), False (F)}

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Satisfiability and Model

- An interpretation satisfies a wff if the wff is assigned the value *true* under the interpretation.
- An interpretations that satisfies a wff is called a *model* of that wff.
- If there is no interpretation that satisfies a wff, it is called *unsatisfiable*.
- If a wff is true under all interpretation, it is called a *valid* wff.

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Equivalence

- DeMorgan's laws
  - $\neg (w_1 \wedge w_2) \equiv \neg w_1 \vee \neg w_2$
  - $\neg (w_1 \vee w_2) \equiv \neg w_1 \wedge \neg w_2$
- Law of contra positive
  - $(w_1 \supset w_2) \equiv (\neg w_2 \supset \neg w_1)$
- If w_1 and w_2 are equivalent, then the following formula is valid
  - $(w_1 \supset w_2) \wedge (w_2 \supset w_1)$

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Entailment

- $\Delta \models w$ states that Δ entails w. All the models that satisfy Δ will also satisfy w.
- if $\Delta \vdash w_k$ by a sound inference mechanism then $\Delta \models w$

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Soundness and completeness

- If for any set of wffs Δ and w, $\Delta \vdash_R w$ implies $\Delta \models w$ we say the inference rule *R* is sound.
- If for any set of wffs Δ and w, it is the case that whenever $\Delta \models w$ there exist a proof of w from using a set of inference rule *R*, we say that *R* is complete.

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P-sat problem

The problem of finding a model for a formula is known as the propositional satisfiability problem.

A disjunction of literals is called a clause.

A formula written as a conjunctions of clauses is said to be in conjunctive normal form (CNF).

3SAT problem consists of clauses of 3 literals each.

3PSAT problem is NP-complete

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3PSAT Problem

- Suppose we have n clauses and the total number of distinct literals are k
- Number of possible models

  2^k
- Given a model time taken to verify whether the model satisfy the set of clauses is

  O(n)

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Example

- You are a stranger to a place and came to a T junction. You know that one branch leads to a crawfish boil and the other branch leads to alligator farm. You would like to enjoy the feast instead of being a fest to alligator.

  Luckily there was a person standing at the T junction helping the people, but he can either say Yes or No answer to your question. Further, you also know that he is either a truth teller or a liar. What single question you can ask the person that leads to the crawfish boil.

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