

A Monitoring Approach for Surface Reconstruction from 3D Point Cloud

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Abstract

This paper describes the surface reconstruction from 3D point cloud. The vertices of point cloud build a network of triangles using Delaunay Triangulation for 3D surface. DT plays an important role due to its guaranteed quality of triangular mesh generation. The process monitoring is emerging as a valuable tool and it is used to enhance operational efficiency. In this paper Delaunay algorithm monitors the various parameters of mesh generation and evaluates the performance of the algorithm by calculating parameters.

Keywords- 3d point cloud; Delaunay triangulation; surface reconstruction.

1. Introduction

J.D. [1] appeared to be the first researcher proposing a Delaunay-based surface reconstruction algorithm that removes tetrahedral and triangles from the set of Delaunay triangles according to certain geometric rules. The Delaunay based “sculpting” method proposed by Boissonnat tetrahedrizes the input data and progressively removes the non-surface triangles of the tetrahedral according to geometric criterion. The cell subdivision scheme consists of subdividing the bounding space divided by the input data set into disjoint cells and reconstructing the surface from these cells. One typical method is marching cube algorithm [2], [3] proposed by Lorensen in which a travel strategy is used to find the surface from the selected cells through a pre defined lookup table. Another well known method, called alpha shapes [4], introduced by Edelsbrunner and Mücke represents the surface through a finite set of points at different levels of detail. Guo improved the result from alpha shapes using visibility algorithms [5].

Curless and Levoy [6] used least square method to build an initial surface by estimating a local tangent

plane for each 3D point and build a volumetric function for voxel grid. The surface is created by triangulating the zero-set of the signed distance function for laser range data. Some surface reconstruction methods, such as Deformable Models [7], establish the topology of the mesh beforehand. The limitation of these methods concerns target shapes that cannot be obtained by deforming an initial mesh. For example, when the initial mesh is a sphere and the target shape is a torus. Other well established surface reconstruction methods, propose solutions based on geometric techniques [8], [9], [10]. Some of them [9], [10] use Delaunay triangulation as a base for the final triangulation. These methods are complex and time consuming, however Delaunay triangulations guarantees reconstruction features, such as maximizing the minimum angle. Since 3D Delaunay triangulation does not always represent the boundary of the target object, a subset of this triangulation is chosen. For example, in [9], this subset is the crust of the object. Definitions for the Delaunay triangulations of domains in R^2 can be found in the literature [11], however, for immersed surfaces in R^3 , given as 2-manifold meshes, the definition for the Delaunay triangulation is not so clear [12].

Delaunay triangulation of piecewise flat surfaces is presented in [13]. Since the surface reconstruction method presented in this paper produces 2-manifold meshes, it is based on the local Delaunay criterion proposed in [13]. Gopi proposed [14] a method to triangulate the surface points with a localized Delaunay triangulation method which projects the local points to a local plane and Delaunay triangulation in 2D space. Converting point cloud [15], [16] representation into a mathematical surface representation is known as surface reconstruction. In surface reconstruction desired surface is required to interpolate the sample points.

Adamy proposed [17] an umbrella filter algorithm coupled with a topological post-processing based on

linear programming. Gong [18] analyzed the common Delaunay triangulations methods, especially the divide-and-conquer method, a faster algorithm for constructing Delaunay triangulations is presented. It divides the point set by self-adaptive grid, constructs and merges the sub-triangulations Lin presented [19] an improved method based on an intrinsic property of the point set and tried to overcome the limitation of user specified parameters.

Another region growing method [20],[21] by picking triangles from Delaunay triangles. The region growing approaches that the reconstructed surface mesh is highly dependent on the choice of the seed triangle and an appropriate hole-filling post processing method is needed for constructing a closed surface mesh.

More algorithms have been proposed by Edelbrunner [22] known alpha shape algorithms. Andriy [23] proposed that in the absence of external boundaries, the algorithm maintains a Delaunay mesh M ; at any iteration it selects a triangle from the queue of unsatisfactory triangles, then computes the circum center p_i of this triangle and $C(p_i)$ and $dC(p_i)$ is calculated. then all triangles in $C(p_i)$ are deleted from M and the triangles which are obtained by connecting p_i with every edge in $dC(p_i)$ are added to M .

The incremental [24] surface reconstruction methods build up the object surface using the surface-oriented properties of the input points. Aiming to speed up the computation in the simulation, Xiao [25] proposed a revised Delaunay algorithm which makes a good balance of quality of tetrahedra, boundary preservation and time complexity, with many improved methods. Delaunay algorithm and the revised Delaunay Algorithm is processed based on the simulation criteria. Sheng proposed a mesh growing [26] process in an efficient way that takes input as an unorganized set of points.

The main idea [27] by Wang to build triangular mesh by finding new edges circularly. Most of the calculations are concentrated on searching the discrete points coincident with the requirements from a large number of data points. Therefore, the efficiency of building triangular mesh would be greatly improved if the technique of searching points could be mended. Yung presented a Delaunay [28] based algorithm driven by Umbrella Facet matching. A full umbrella was constructed for every point from its Delaunay triangle set and then optimizing the umbrella into a fully matched umbrella which gives the guarantee for generation of a topologically correct mesh without the need for hole-filling post-processing.

This Paper is organized as follows: Section 2 describes the surface reconstruction algorithm. Section 3 describes the flow of surface reconstruction process. Section 4 gives the results and readings of different parameters. Finally we conclude in section 5. This paper describes

the surface reconstruction from 3D point cloud. The vertices of point cloud build a network of triangles using Delaunay Triangulation for 3D surface. It plays an important role due to its guaranteed quality of triangular mesh generation. The process monitoring is emerging as a valuable tool and it is used to enhance operational efficiency. In this paper Delaunay algorithm monitors the various parameters of mesh generation and evaluates the performance of the algorithm by calculating parameters.

2. Surface Reconstruction Algorithm

In this section, we consider a surface reconstruction process from point cloud. Surface reconstruction is to approximate a shape from the coordinates of a point cloud of the shape. The point cloud is called a point sample, or simply a sample of the shape. The specific shape that we will deal with curves in two dimensions & surfaces in three dimensions.

2.1. Two Dimensions

Let P be a set of non degenerate points in the plane R^2 . The non degeneracy condition for the point set P is the affine hull of any l points from P with $1 \leq l \leq k$ is homeomorphic to R^{l-1} and no $k+2$ points are co-spherical.

2.1.1. Voronoi Diagrams

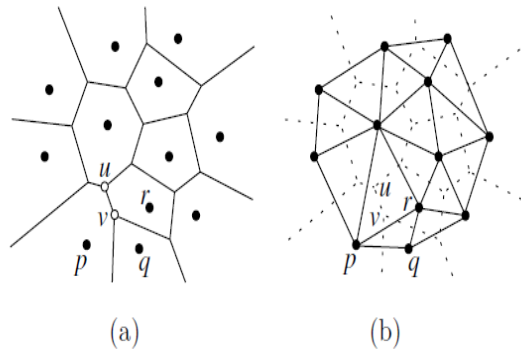
The Voronoi cell V_p for each point $p \in P$ is given as

$$V_p = \{x \in R^2 \mid d(x, P) = \|x - p\|\}$$

In words, V_p is the set of all points in the plane that have no other point in P closer to it than p . For any two points p, q the set of points closer to p than q are demarked by the perpendicular bisector of the segment pq . This means the Voronoi cell V_p is the intersection of the closed half-planes determined by the perpendicular bisectors between p and each other point $q \in P$. An implication of this observation is that each Voronoi cell is a convex polygon since the intersection of convex sets remains convex.

Voronoi cells have Voronoi faces of different dimensions. A Voronoi face of dimension k is the intersection of $3 - k$ Voronoi cells. This means a k -dimensional Voronoi face for $k \leq 2$ is the set of all points that are equidistant from $3 - k$ points in P . A zero-dimensional Voronoi face, called Voronoi vertex is equidistant from three points in P , whereas a one-dimensional Voronoi face, called Voronoi edge contains points that are equidistant from two points in P . A Voronoi cell is a two-dimensional Voronoi face. The

Voronoi diagram is the cell complex formed by Voronoi faces.



**Figure 1. (a) The Voronoi diagram
(b) The Delaunay triangulation.**

Figure 1(a) shows a Voronoi diagram of a point set in the plane where u and v are two Voronoi vertices and uv is a Voronoi edge. Some of the Voronoi cells may be unbounded with unbounded edges. It is a straightforward consequence of the definition that a Voronoi cell V_p is unbounded if and only if p is on the boundary of the convex hull of P . In Figure 1, V_p and V_q are unbounded and p and q are on the convex hull boundary.

2.1.2. Delaunay Triangulations

There is a dual structure to the Voronoi diagram, called the Delaunay triangulation. The Delaunay triangulation of P is a simplicial complex

$$\text{Del } P = \{ \sigma = \text{Conv } T \mid \bigcap_{p \in T \subseteq P} V_p \neq \emptyset \}$$

In words, $k + 1$ points in P form a Delaunay k -simplex in $\text{Del } P$ if their Voronoi cells have nonempty intersection. We know that $k + 1$ Voronoi cells meet in a $(2 - k)$ dimensional Voronoi face. So, each k -simplex in $\text{Del } P$ is dual to a $(2 - k)$ dimensional Voronoi face. Thus, each Delaunay triangle pqr in $\text{Del } P$ is dual to a Voronoi vertex where V_p , V_q , and V_r meet, each Delaunay edge pq is dual to a Voronoi edge shared by Voronoi cells V_p and V_q , and each vertex p is dual to its corresponding Voronoi cell V_p . In Figure 1(b), the Delaunay triangle pqr is dual to the Voronoi vertex v and the Delaunay edge pr is dual to the Voronoi edge uv . In general, when μ is a dual Voronoi face of a Delaunay simplex σ we say $\mu = \text{dual } \sigma$ and conversely $\sigma = \text{dual } \mu$. A circumscribing ball of a simplex σ is a ball whose boundary contains the vertices of the simplex. The smallest circumscribing ball of σ is called its diametric ball. A triangle in the plane has only one circumscribing ball, namely the diametric one. However, an edge has

infinitely many circumscribing balls among which the diametric one is unique, namely the one with the edge as diameter.

A dual Voronoi vertex of a Delaunay triangle is equidistant from its three vertices. This means that the center of the circumscribing ball of a Delaunay triangle is the dual Voronoi vertex. It implies that no point from P can lie in the interior of the circumscribing ball of a Delaunay triangle. These balls are called Delaunay. A ball is empty if its interior does not contain any point from P . Clearly, the Delaunay balls are empty. A triangle is in the Delaunay triangulation if and only if its circumscribing ball is empty. The triangle emptiness property of Delaunay triangles also implies a similar emptiness for Delaunay edges.

2.2. Three Dimensions

We chose the plane to explain the concepts of the Voronoi diagrams and the Delaunay triangulations in the previous subsection. However, these concepts extend to arbitrary dimensions. We will mention these extensions for three dimensions which will be important for later expositions. Voronoi cells of a point set P in \mathbb{R}^3 are three-dimensional convex polytopes some of which are unbounded. There are four types of Voronoi faces: Voronoi vertices, Voronoi edges, Voronoi facets, and Voronoi cells in increasing order of dimension starting with zero and ending with three. Four Voronoi cells meet at a Voronoi vertex which is equidistant from four points in P .

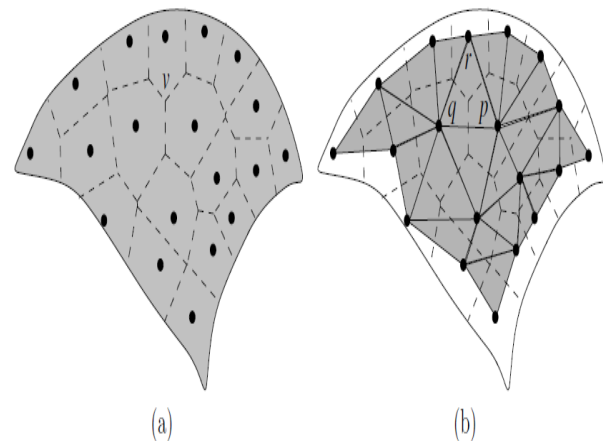


Figure 2. (a) The restricted Voronoi diagram and (b) The restricted Delaunay Triangulation.

Three Voronoi cells meet at a Voronoi edge, and two Voronoi cells meet at a Voronoi facet. The Delaunay triangulation of P contains four types of simplices dual to

each of the four types of Voronoi faces. The vertices are dual to the Voronoi cells, the Delaunay edges are dual to the Voronoi facets, the Delaunay triangles are dual to the Voronoi edges, and the Delaunay tetrahedra are dual to the Voronoi vertices. The circumscribing ball of each tetrahedron is empty. Conversely, any tetrahedron with empty circumscribing ball is in the Delaunay triangulation. Further, each Delaunay triangle and edge has an empty circumscribing ball. Conversely, an edge or a triangle belongs to the Delaunay triangulation if there exists an empty ball circumscribing it. The number of edges, triangles, and tetrahedra in the Delaunay triangulation of a set of n points in three dimensions can be $O(n^2)$ in the worst case. By duality the Voronoi diagram can also have $O(n^2)$ Voronoi faces. Both of the diagrams can be computed in $O(n^2)$ time and space.

We can define the restricted Voronoi diagram and its dual restricted Delaunay triangulation for a point sample on a surface in R^3 in the same way as we did for a curve in R^2 . Figure 2 shows the restricted Voronoi diagram and its dual restricted Delaunay triangulation for a set of points on a surface. The triangle pqr is in the restricted Delaunay triangulation since $V_p|_{\Sigma}$, $V_q|_{\Sigma}$, and $V_r|_{\Sigma}$ meet at a common point v .

Surface reconstruction in 3D is motivated by availability of modern scanning devices that can generate a point sample from the surface of a geometric object. There are some applications medical imaging, geographic data processing, and drug designs, can take advantage of the scanning technology to produce samples and then compute a digital model of a geometric shape with reconstruction algorithms.

3. Implementation

To monitor the different parameters of surface reconstruction instructions are used. Then all the parameters are defined with their functions and give the result by execution of the Delaunay triangulation.

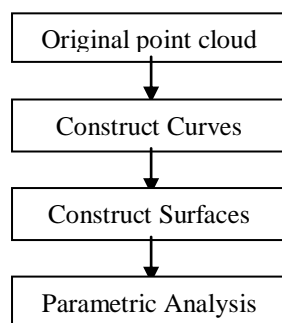


Figure 3. Flow of Surface Reconstruction

We will deal with curves in two dimensions & surfaces in three dimensions.

4. Results and Discussions

This section presents the results of the experiments carried out by Delaunay Algorithm. Firstly we present the visual results of the experiments. Then, we present the parametric readings for the reconstructed meshes.

4.1. Data Objects

The data object is the Vertebra. The results are depicted in Figure 6. The results show the original point cloud, surface reconstruction using Delaunay and Performance evaluation graph. Figure 5 is formed by the points. Figure 6 is formed by the triangulation algorithm. Figure 4 shows the Performance evaluation graph of Delaunay Algorithm.

4.2. Parameters and Computations

To demonstrate the efficiency monitoring algorithm is being optimized. It monitors the various parameters of mesh generation and evaluates the performance of Delaunay surface reconstruction algorithm. As the results are depicted in Figure 4 and Figure 6, corresponding settings and execution times are presented in Table 1.

For analysing the triangulation efficiency time monitoring is a good approach. It describes the details of surface reconstruction and computes the execution times.

Table 1. Computations using Delaunay Algorithm

Parameters	Delaunay Algorithm
Scatter Added	0.0899 s
Triangulation	7.4211 s
Triangle Connectivity	3.9252 s
Circumcenters Tetraedrons	0.2881 s
Intersection factor	0.1722 s
Walking through whole scatter	21.1016 s
Total Time using Delaunay	33.0330 s

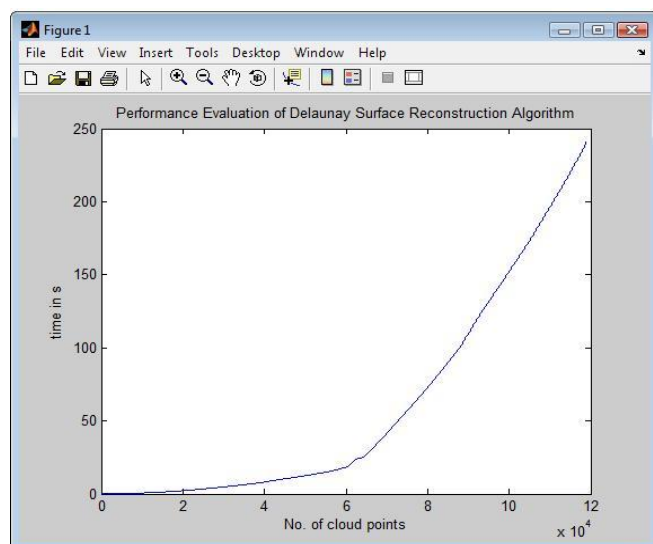


Figure 4. Performance Evaluation of Delaunay algorithm

5. Conclusion

In this paper Delaunay algorithm optimizes the mesh reconstruction system from 3D point cloud and it presents the corresponding settings and execution times. Delaunay triangulation algorithm plays an important role due to its guaranteed quality of mesh generation. It proposes optimization of mesh reconstruction system from 3D cloud point. Some applications medical imaging, geographic data processing, and drug designs, can take advantage of the technology to compute the digital model of a geometric shape with reconstruction algorithms.

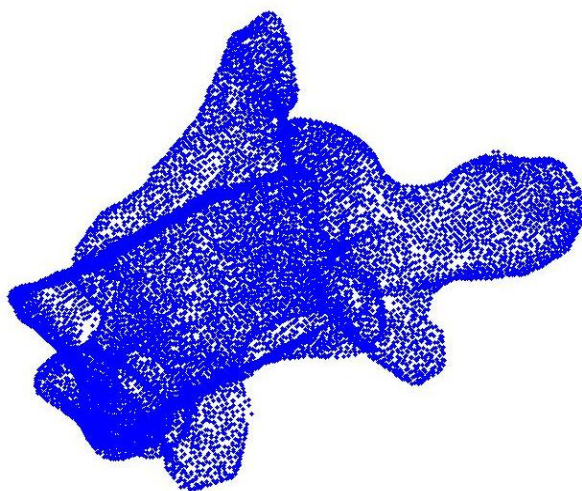


Figure 5. Surface reconstruction using Original point cloud.

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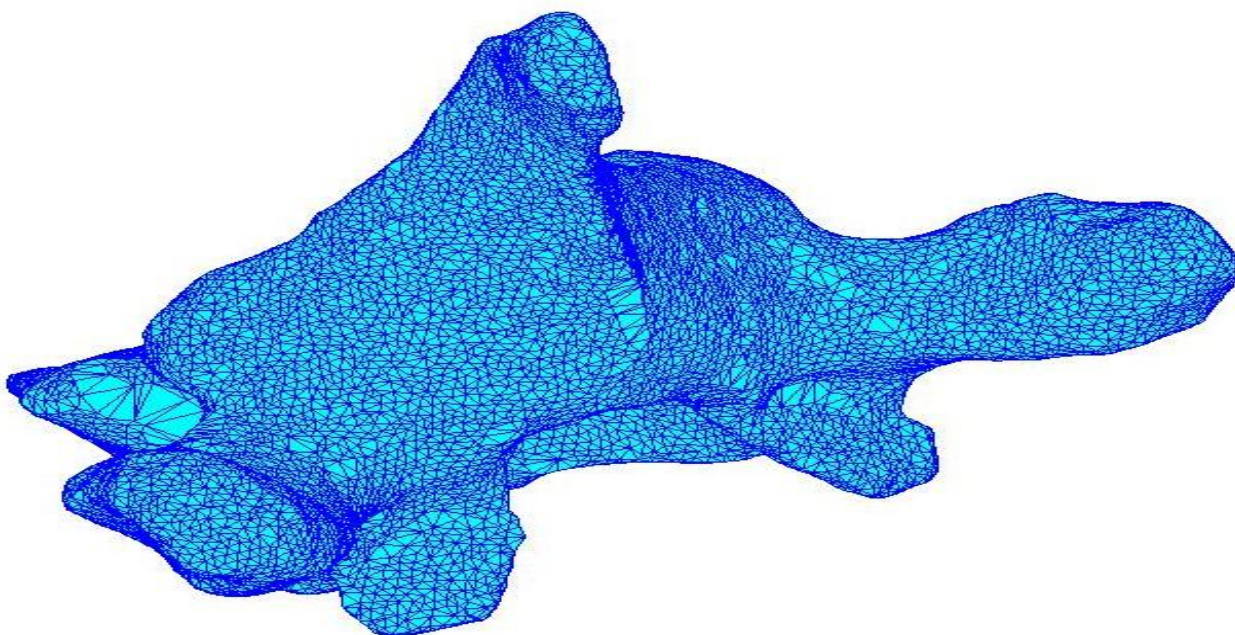


Figure 6. Surface reconstruction using Delaunay Algorithm