# LPV- MPC Controller For Autonomous vehicle

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12-8-2023

**ABSTRACT**

This project presents the development of a Model Predictive Controller (MPC) using a Linear Parameter-Varying (LPV) approach for the trajectory tracking of an autonomous vehicle. The primary objective is to accurately follow predetermined trajectories while ensuring optimal performance through the minimization of a cost function J. The cost function is designed considering the vehicle's steering wheel angle and applied acceleration as control inputs. Our system is characterized as a multi-input multi-output (MIMO) system, incorporating six states and four outputs, with two control inputs.

In the realm of autonomous vehicles, precise trajectory tracking is crucial for safety and efficiency. Our approach employs an LPV MPC, a versatile control strategy that can handle system dynamics varying with time or operating conditions, making it particularly suitable for the nonlinear and time-varying nature of vehicle dynamics. The controller is meticulously designed to respect the constraints imposed on both the states and the control inputs, ensuring robust and safe operation under various driving scenarios.

The effectiveness of the proposed LPV MPC is demonstrated through simulations involving three distinct example trajectories, encompassing varying degrees of complexity and practical driving scenarios. These simulations are pivotal in evaluating the controller's performance in terms of trajectory adherence, handling dynamics, and responsiveness to the changing conditions. The results from these simulations exhibit the controller's proficiency in maintaining the desired trajectory, highlighting its potential applicability in real-world autonomous driving systems.

This project not only contributes to the advanced control strategies in autonomous vehicle technology but also lays a foundation for future research in enhancing the reliability and efficiency of autonomous vehicular control systems.

# INTRODUCTION

]The cornerstone of this project lies in the development of a Linear Parameter-Varying Model Predictive Controller (LPV-MPC) for an autonomous vehicle. This advanced control system is meticulously designed to regulate key vehicular functions: the steering angle and the applied acceleration. These control inputs are pivotal in navigating the autonomous vehicle with precision and adaptability in a variety of driving conditions. To realistically model the vehicle's dynamics while maintaining computational simplicity, a bicycle model is employed as a surrogate for the actual vehicle model. This choice balances the need for accurate representation of vehicle kinematics against the computational intensity typically associated with more complex models.

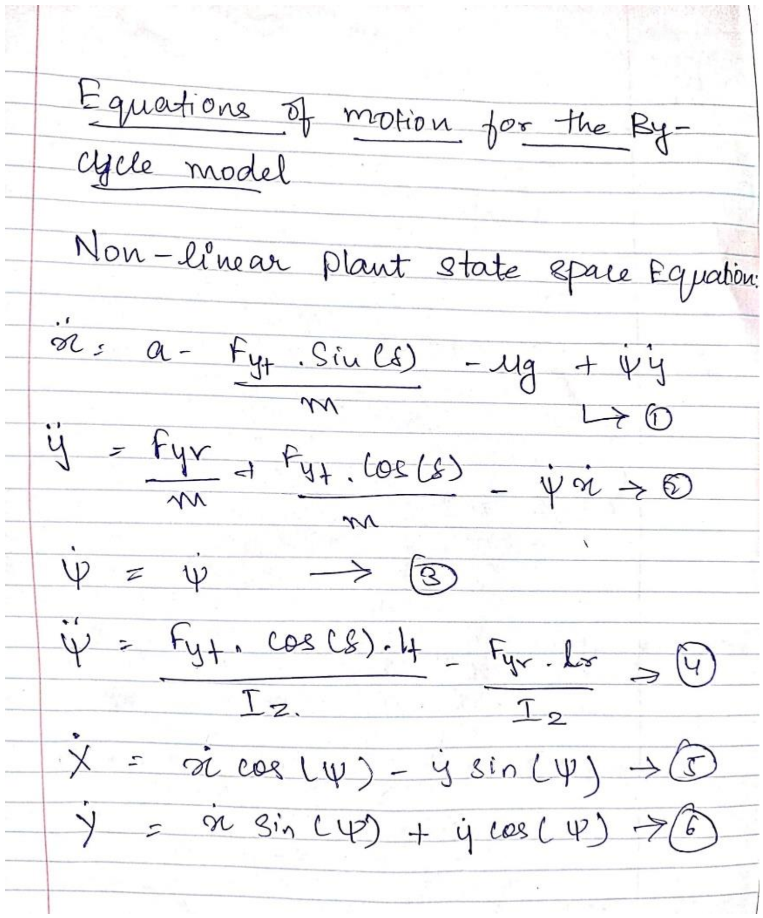
The kinematic model of the bicycle is elegantly crafted with a set of states, inputs, and outputs that encapsulate the vehicle's dynamic behavior. The model is characterized by:

- **States**:
  - $\dot{X}$: Longitudinal velocity
  - $\dot{Y}$: Lateral velocity
  - $\Psi$: Yaw angle
  - $X$: Inertial X position
  - $Y$: Inertial Y position
- **Inputs**:
  - Steering angle
  - Applied acceleration
- **Outputs**:
  - Longitudinal velocity ($\dot{X}$)
  - Yaw angle ($\Psi$)
  - Inertial X position ($X$)
  - Inertial Y position ($Y$)

These components form the bedrock of the project, providing a structured framework for the LPV-MPC to function effectively. The controller is tasked with the crucial role of interpreting these states and inputs to generate optimal outputs, thereby ensuring the autonomous vehicle adheres to its intended path with utmost accuracy and efficiency. The integration of this model into the LPV-MPC framework marks a significant stride in autonomous vehicle control, paving the way for enhanced performance and safety in autonomous driving systems.

Kinematic Modelling:

The equations of motion are derived, and the Final equations of Motion are as in the Figure 1. We Have Nonlinear Model and we are using a Linear MPC for the trajectory tracking Problem So we are using LPV to the put the Non linear model in the controller.



Equations of motion for the By-cycle model

Non-linear plant state space Equation:

$$\ddot{x} = a - \frac{F_{yf} \cdot \sin(\delta)}{m} - \mu g + \dot{\psi}\dot{y} \rightarrow (1)$$

$$\ddot{y} = \frac{F_{yr}}{m} + \frac{F_{yf} \cdot \cos(\delta)}{m} - \dot{\psi}\dot{x} \rightarrow (2)$$

$$\dot{\psi} = \dot{\psi} \longrightarrow (3)$$

$$\ddot{\psi} = \frac{F_{yf} \cdot \cos(\delta) \cdot l_f}{I_z} - \frac{F_{yr} \cdot l_r}{I_z} \rightarrow (4)$$

$$\dot{X} = \dot{x}\cos(\psi) - \dot{y}\sin(\psi) \rightarrow (5)$$

$$\dot{Y} = \dot{x}\sin(\psi) + \dot{y}\cos(\psi) \rightarrow (6)$$

Figure 1: Nonlinear Model

LPV (Linear Parameter Varying)

Linear Parameter-Varying (LPV) systems represent a powerful approach in control engineering, particularly for managing systems with nonlinear dynamics through a linear control framework. The LPV framework is especially effective for systems where the dynamics can change with time or under varying operating conditions. It bridges the gap between the robustness of linear control techniques and the flexibility required to handle nonlinearities.

In the context of an LPV Model Predictive Controller (LPV-MPC), the approach involves approximating the nonlinear behavior of a system with a set of linear models, each valid for a specific operating regime. This is particularly pertinent for complex systems like autonomous vehicles, where direct application of linear MPC might be inadequate due to the inherent nonlinear dynamics. The LPV-MPC controller operates by first identifying the current operating regime of the vehicle and then selecting the appropriate linear model that best represents the system's dynamics in that regime.

Now lets Derive the LPV Matrices:

To compute the LPV Matrix

$$\ddot{x} = \underbrace{\boxed{-\frac{\mu g}{\dot{x}}}}_{A_{11}} \dot{x} + \underbrace{\boxed{\frac{C_{\alpha f} \cdot \sin(\delta)}{M \dot{x}}}}_{A_{12}} \dot{y} + \underbrace{\left(\left(\frac{C_{\alpha f} \cdot \sin(\delta) \cdot l_f}{m \dot{x}}\right) + \dot{y}\right)}_{A_{14}} \dot{\psi} - \underbrace{\frac{C_{\alpha f} \cdot \sin(\delta)}{m}}_{B_{11}} \delta + \underbrace{1}_{B_{12}} \cdot a$$

The $\ddot{x}$ is simplified for the LPV

Fig 2 : Double dot calculation

Now lets factor and simplify the $\dot{y}$ to form the LPV matrix.

$$\ddot{y} = \underbrace{-\frac{(C_{\alpha r} + C_{\alpha f} \cdot \cos(\delta))}{m \dot{x}}}_{A_{22}} \dot{y} + \underbrace{\left(-\frac{(C_{\alpha f} \cdot \cos(\delta) \cdot l_f - C_{\alpha r} \cdot l_r)}{m \dot{x}} - \dot{x}\right)}_{A_{24}} \dot{\psi} + \underbrace{\frac{C_{\alpha f} \cdot \cos(\delta)}{m}}_{B_{21}} \delta$$

Now for $\dot{\psi} = \underbrace{1}_{A_{34}} \cdot \dot{\psi}$

Now for $\ddot{\psi}$

$$\ddot{\psi} = \underbrace{-\frac{(C_{\alpha f} \cdot \cos(\delta) \cdot l_f - C_{\alpha r} \cdot l_r)}{I_z \dot{x}}}_{A_{42}} \dot{y} - \underbrace{\frac{(C_{\alpha f} \cdot \cos(\delta) \cdot l_f^2 + C_{\alpha r} \cdot l_r^2)}{I_z \dot{x}}}_{A_{44}} \dot{\psi} + \underbrace{\frac{C_{\alpha f} \cdot \cos(\delta) \cdot l_f}{I_z}}_{B_{41}} \delta$$

Figure 3: LPV continuation

Now for $\dot{x}$ and $\dot{y}$

we have:

$$\dot{X} = \overbrace{\cos(\psi)}^{A_{51}}\dot{x} - \overbrace{\sin(\psi)}^{A_{52}}\dot{y}$$

$$\dot{Y} = \underbrace{\sin(\psi)}_{A_{61}}\dot{x} \quad \underbrace{\cos(\psi)}_{A_{62}}\dot{y}$$

Now lets write the LPV-MATRIX

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \\ \dot{X} \\ \dot{Y} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11} & A_{12} & 0 & A_{14} & 0 & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & 0 \\ 0 & 0 & 0 & A_{34} & 0 & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & 0 \\ A_{51} & A_{52} & 0 & 0 & 0 & 0 \\ A_{61} & A_{62} & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \psi \\ \dot{\psi} \\ X \\ Y \end{bmatrix}}_{\vec{x}}$$

$$+ \underbrace{\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & 0 \\ 0 & 0 \\ B_{41} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B} \overbrace{\begin{bmatrix} \delta \\ a \end{bmatrix}}^{u}$$

Fig 4 : Final LPV Matrices

Now after completing LPV Matrices we Derive the Cost function by substituting the A, B and C matrices and our cost function looks like this :

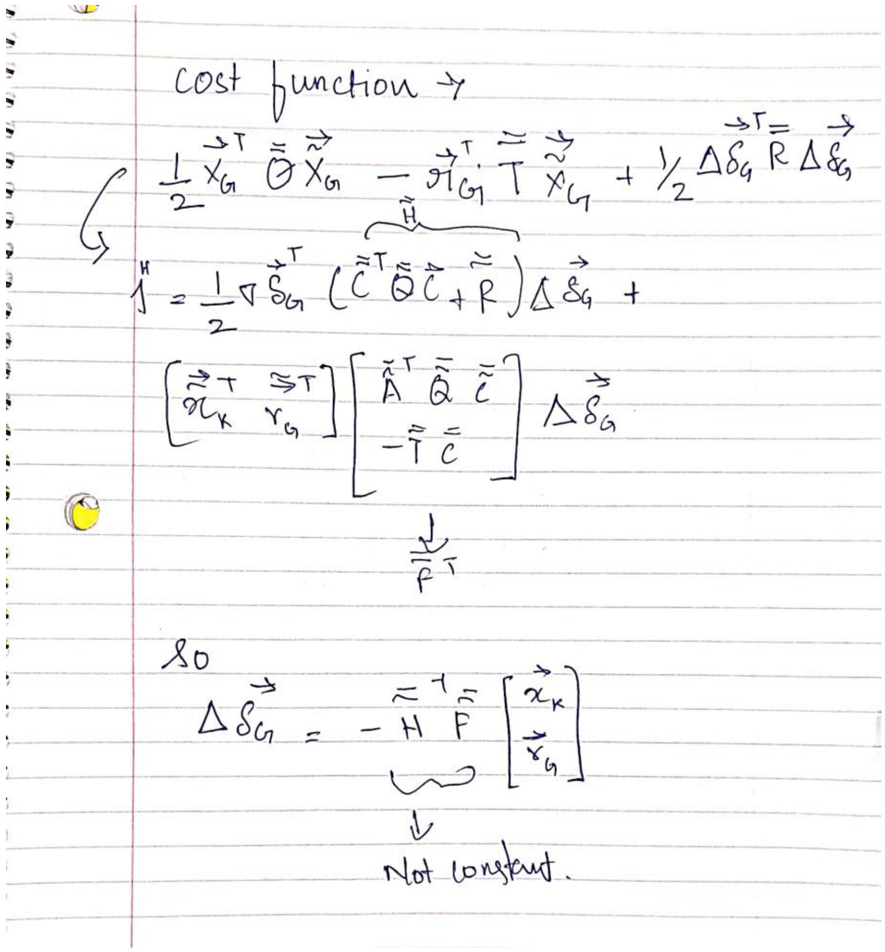



Fig 5 : Cost Function

We substitute all of those to formulate our final controller model and solve the cost function using a qp solver.

**QP Solver**

Quadratic Programming (QP) solvers are a fundamental component in the field of optimization, particularly in applications where efficiency and precision are paramount. QP solvers are designed to find the minimum of a quadratic objective function subject to linear constraints. The objective function typically takes the form of a quadratic expression, where the goal is to optimize (either minimize or maximize) this functions within the bounds of specified linear equalities and inequalities.

In practical applications, QP solvers are integral to various complex problems, notably in fields like finance for portfolio optimization, in machine learning for support vector machines, and in control systems for Model Predictive Controllers (MPC).

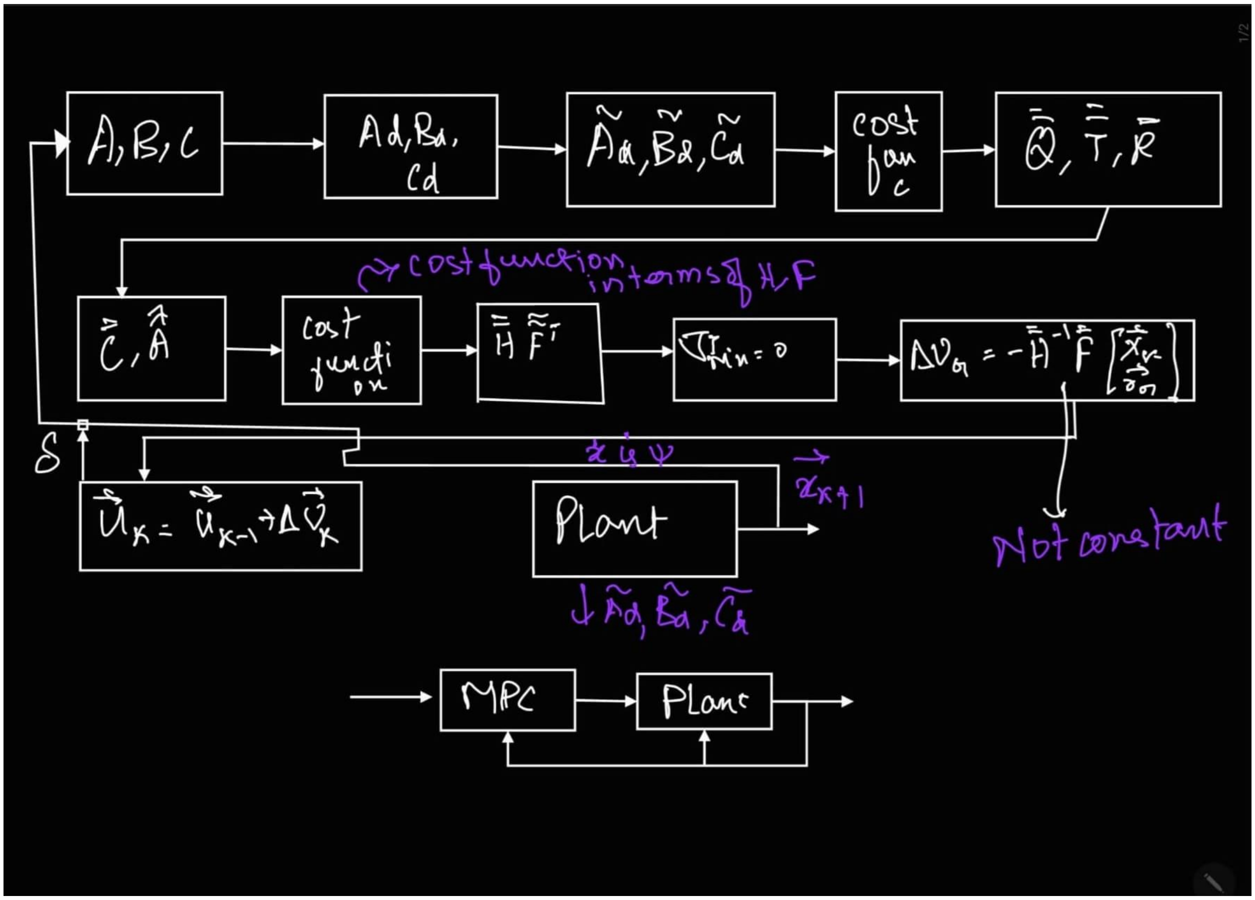



Fig 6 : Controller formulation and Framework