

SEARCHING AND SORTINGCLASS \Rightarrow IIIQ: Binary Search in a nearly sorted array.

| | | | | | | |
|----|---|----|----|----|----|----|
| 10 | 3 | 40 | 20 | 50 | 80 | 70 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

target = 40

Sol:

sorted \rightarrow

| |
|---|
| 3 |
| 0 |

 10

| |
|----|
| 20 |
| 2 |

 40 50

| |
|----|
| 70 |
| 5 |

 80

i^{th} index \rightarrow Eg: 3 can lie on
 (i-1) (i) (i+1) \rightarrow (-1) (0) (1) indexes.

Eg:

| |
|----|
| 20 |
| 2 |

 20 can be on (2-1) or (2) or (2+1)
 (1) or (2) or (3)
 \rightarrow in ques, 20 is on 3

Eg:

| |
|----|
| 70 |
| 5 |

 70 can be on (5-1) or (5) or (5+1)
 (4) or (5) or (6)

Similar for all elements for an array.

Now,

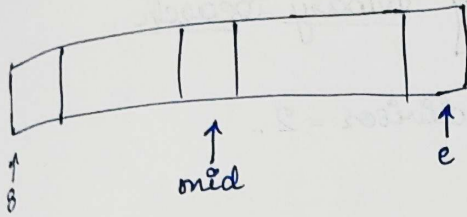
| | | | | | | |
|----|---|----|----|----|----|----|
| 10 | 3 | 40 | 20 | 50 | 80 | 70 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Approach 1: Linear search

for $(0 \rightarrow n) \Rightarrow T.C = O(n)$ $O(n \log n)$ is even more worst than $O(n)$ Approach 2: \rightarrow sort array

\rightarrow Then Binary search $\left. \begin{array}{l} \\ \end{array} \right\} T.C \Rightarrow O(\log n + n \log n)$
 $= O(n \log n)$

Normal sorted



if (arr[mid] == target)
return mid;

↓

if (target > arr[mid])

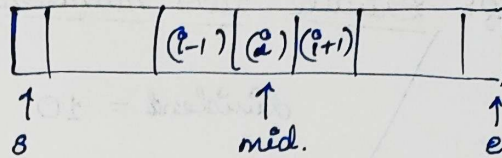
RS $\rightarrow s = mid + 1;$

else if (LS $\rightarrow e = mid - 1;$

Difference in

Binary Searches

Nearly sorted.



if (arr[mid] == target)
return mid;

else if (arr[mid-1] == target)
return mid-1;

else if (arr[mid+1] == target)
return mid+1;

↓

if (target > arr[mid])

$\rightarrow RS \Rightarrow s = mid + 2;$

else

LS $\Rightarrow e = mid - 2;$

NOTE: $mid + 1 \leq arr.size();$
 $mid - 1 \geq start$ or $mid - 1 \geq 0$

↓
more optimised than zero \therefore less checkings.

TC $\rightarrow O(\log n)$

\rightarrow will be same as

Binary search

Q: Divide two numbers using Binary Search.

dividend = 65 and divisor = 4.

find quotient = ?

★ like that square root wala question, first create a search space.

∴ Search space = {0, dividend}

Conditions:

- ① if $(mid \times divisor == dividend) \rightarrow \text{return mid};$
- ② if $(mid \times divisor > dividend) \rightarrow \text{left search}$
[e = mid - 1]
- ③ else $\rightarrow \text{store ans}$
 $\rightarrow \text{right search [s = mid + 1]}$

Dry Run:

65 ← dividend
4 ← divisor

| | | |
|-------|-----|-----|
| 0 | 32 | 65 |
| ↑ | ↑ | ↑ |
| start | mid | end |

~~ans~~ $mid \times divisor = 32 \times 4 = 128 > 65$

↳ Left search.

| | | |
|---|-----|----|
| 0 | 15 | 31 |
| ↑ | ↑ | ↑ |
| s | mid | e |

$15 \times 4 = 60 < 65 \rightarrow \text{store ans}$

ans = 15

→ right search.

| | | |
|----|----|----|
| 16 | 23 | 31 |
| ↑ | ↑ | ↑ |
| s | m | e |

$23 \times 4 = 92 > 65 \rightarrow \text{left}$

$$\begin{array}{ccc}
 16 & 19 & 22 \\
 \uparrow & \uparrow & \uparrow \\
 s & \text{mid} & e
 \end{array}
 \quad \text{mid} \times \text{divisor}$$

$$= 19 \times 4 = 76 > 65$$

→ left.

$$\begin{array}{ccc}
 16 & 17 & 18 \\
 \uparrow & \uparrow & \uparrow \\
 s & \text{mid} & e
 \end{array}
 \quad 17 \times 4 = 68 > 65$$

→ left.

$$\begin{array}{ccc}
 16 & & 16 \\
 \uparrow & & \uparrow \\
 s & & e
 \end{array}
 \quad \begin{array}{c} \text{start} \\ \swarrow \\ \boxed{16} \\ \uparrow \\ \text{mid} \end{array}$$

$$16 \times 4 = 64 < 65$$

→ store ans
ans = 16

→ right search.

$$s = 17 \text{ and } e = 16$$

$$s = \text{mid} + 1 \Rightarrow \boxed{s = 17}$$

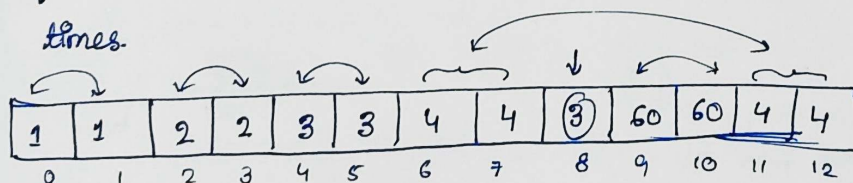
→ Rek jao;!

Q: Find the Odd Occurring Element in an Array.

→ all elements occur even number of times except one.

→ all repeating occurrence of elements appears in pairs, and pairs are not adjacent. (there cannot be more than 2 consecutive occurrence of any pair)

→ find the element that appears odd number of times.



| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|----|----|---|---|---|
| 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 3 | 60 | 60 | 4 | 4 | X |
|---|---|---|---|---|---|---|---|---|----|----|---|---|---|

* XOR approach $\begin{cases} a \wedge a = 0 \\ a \wedge b = 1 \end{cases}$

→ $1 \wedge 1 \wedge 2 \wedge 2 \wedge 3 \wedge 3 \wedge 4 \wedge 4 \wedge 3 \wedge 60 \wedge 60 \wedge 4 \wedge 4$
 → ans = 3, but T.C = $O(n)$

★ IMP BINARY SEARCH ★

→ very less type of questions

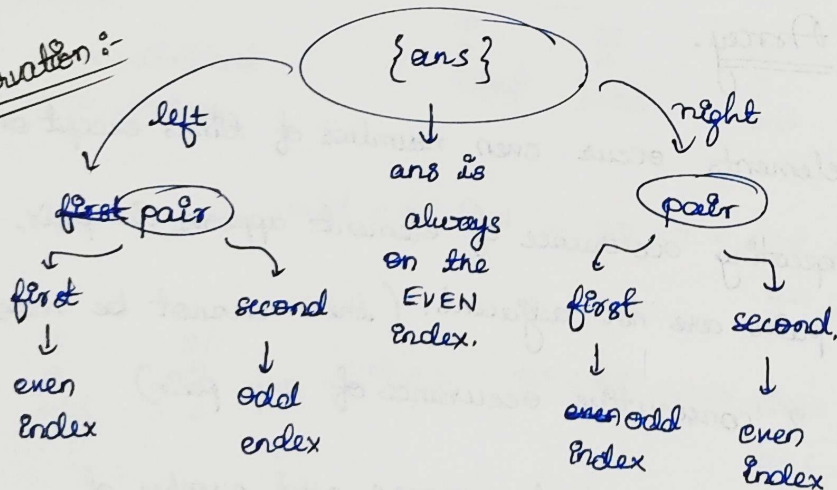
→ i) Classic questions : BS, lower bound, upper bound, total occurrence, pivot element, peak ele, search in rotated sorted array, sq. root.

ii) Find out in Search space : Apply BS for a range.

iii) Index ke andar Observation : This question.

| | | | | | | | | | | | | | |
|------|-----|------|-----|------|-----|------|-----|------|-----|------|-----|------|---|
| 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 3 | 60 | 60 | 4 | 4 | X |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| even | odd | even | odd | even | odd | even | odd | even | odd | even | odd | even | |

Observation:-



$$s=0, e=n-1, mid = s + (e-s)/2 \text{ or } s+e/2.$$

while ($s \leq e$) { *only single element array*

if ($s == e$) \rightarrow return s ;

if ($mid \% 2 == 0$)
 { // Even case
 []
 }

if ($a[mid] == a[mid+1]$)
 // it means we are on
 // the left side of ans
 { // right search.
 } $s = mid + 1$;

~~if~~ (~~else~~)
 { // odd case
 []
 }

else
 { $e = mid$;
 }

① $mid \rightarrow$ even.

[] = []
 $(mid) == (mid+1)$
Even odd

\rightarrow Left me khada hu
 \rightarrow search right.

$s = mid + 1$;

ans 1
 [] \neq []
 $(mid) \neq (mid+1)$
Even odd

ans 1 or
 Left me
 hoga

\rightarrow ans mid bhi ho sakta
 hai, ya fir left me
 bhi ho sakta hai.

$\therefore e = mid$

② $mid \rightarrow$ odd

compare
 [] == []
 $(mid-1) == (mid)$
Even odd

\rightarrow left side me khada hu.

\hookrightarrow ans Right me hoga.

\hookrightarrow Right search.

ans 1
 even [] \neq odd [X]
 $(mid-1) \neq (mid)$
 (because ans can't be on
 odd index)
 ans 1 or
 right me hoga

Left search

\therefore Right search

$e = mid - 1$;

$s = mid + 1$;

why, because last time
 we had checked $mid+1$,
 not this time.

Summary:

if (n % 2 == 0)

if (~~mid~~ $a[\text{mid}] == a[\text{mid}+1]$)

$$g = mid + 2;$$

else

$e = mid;$

else

3

if ($a[mid] == a[mid-1]$)

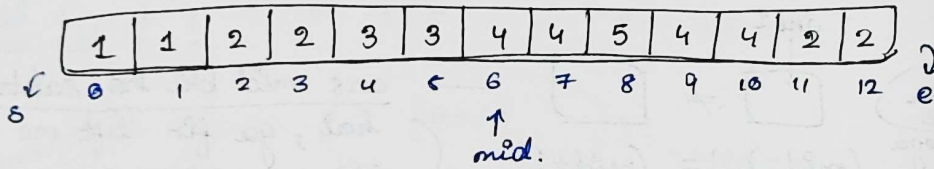
$$B = mid + 1;$$

else

$$e = \text{mid} - 1; \quad a$$

3

Dry Run:

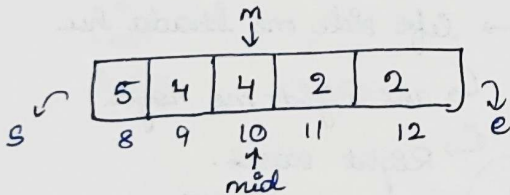


* mid is even

$$\boxed{4} = \boxed{4} \rightarrow \text{night} \rightarrow 8 = \text{mid} + 2$$

mid $\text{mid} + 1$
 $\text{even}(6)$ $\text{odd}(7)$

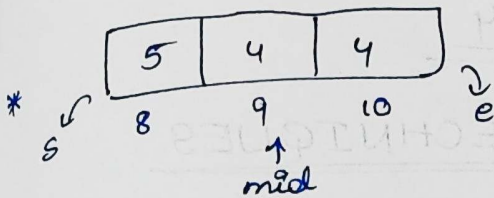
$= 6 + 2$

$$= 6 + 2$$
$$8 = 8$$


* mid is even

$\boxed{4} \neq \boxed{2} \rightarrow e = \text{mid}$
 mid mid+1
 even(10) odd(11)

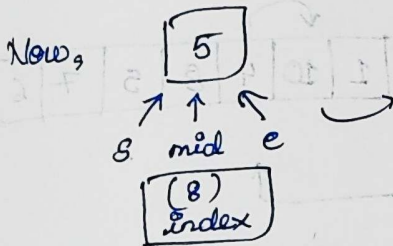
$$e = 10$$



* mid is odd

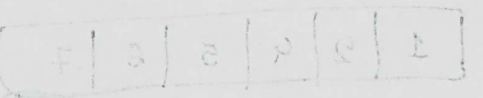
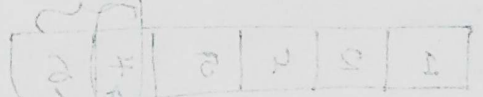
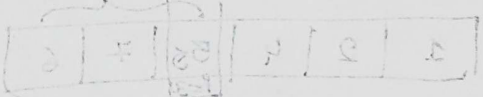
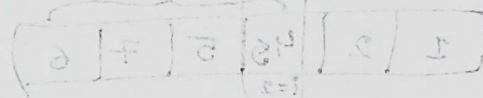
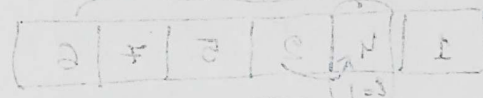
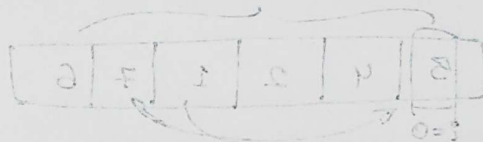
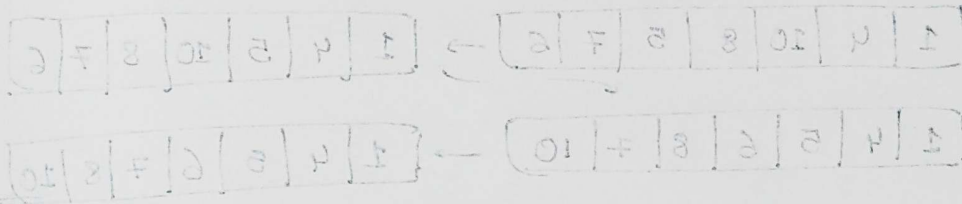
$$\boxed{5} \neq \boxed{4} \rightarrow \text{left search.}$$

$(\text{mid}-1)$ (mid) $e = \text{mid} - 1$
 even(8) odd(9) $e = 9 - 1$
 $\boxed{e = 8}$



$s == e \rightarrow \text{return mid};$

or $(\text{mid}, s \text{ or } e)$



we need to check the last element