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ESE 215 – 103 (Circuit Theory Laboratory)
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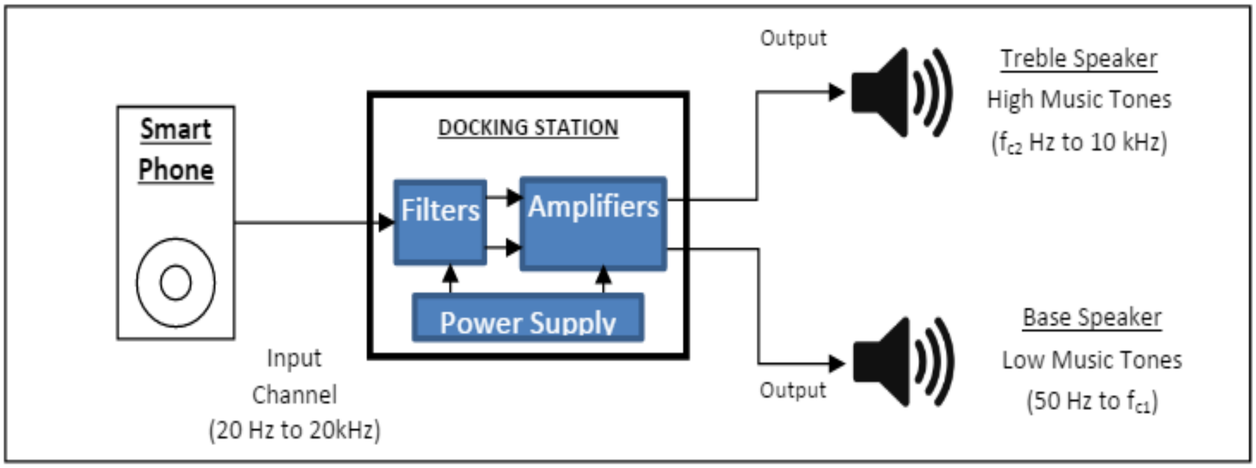
Final Project: An Audio Docking Station for a Smartphone

Section 1: Introduction & Background

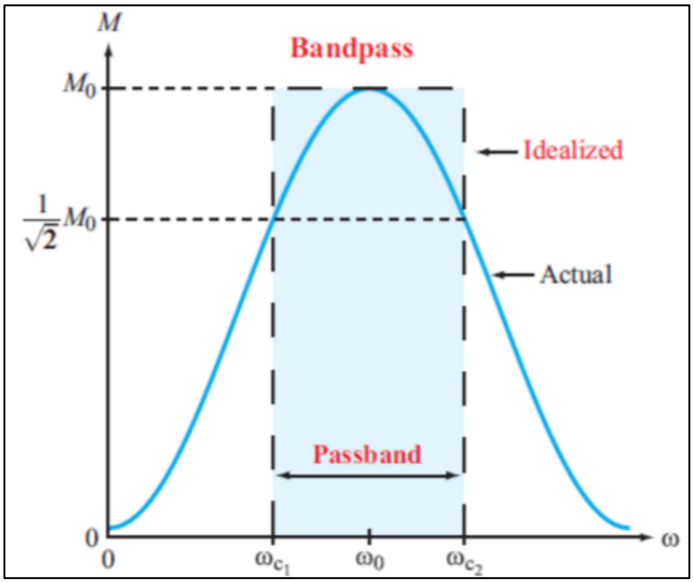
The purpose of this project was to design, build, and test an audio docking station that amplifies and filters out incoming signals from a music source based on their frequencies. For this purpose, high frequencies passed through a treble filter, while low frequencies passed through a bass filter. Then, MOSFET amplifiers increased the amplitude of the frequencies entering from the outputs of the treble and bass filters and transmitted the signals to treble and bass speakers that increased the volume of the music. In **Figure 1.1** below, we show the high-level setup associated with the final project.

Figure 1.1: *Fundamental Diagram of Audio Docking Station Design*



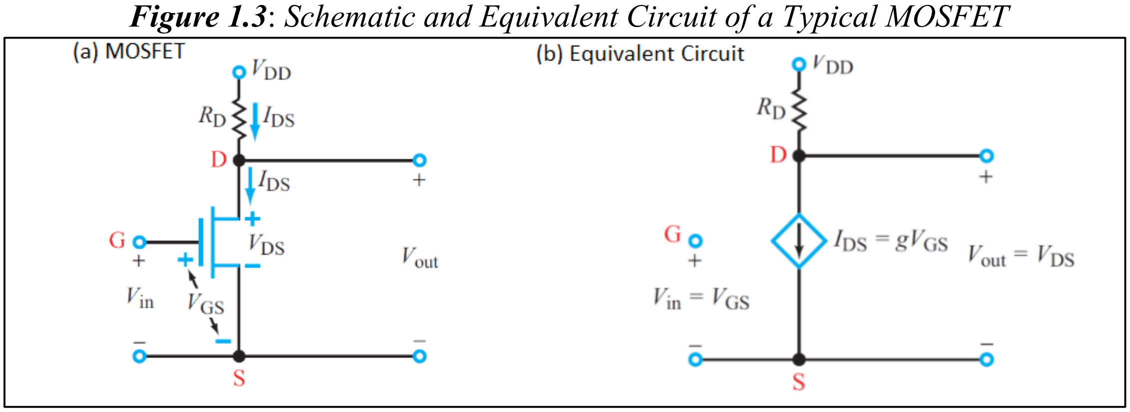
In the filter component of the project, we were to use two *bandpass filters*, preferably active in nature, that served to allow only a certain range of frequencies to pass through them. The treble filter allowed a higher range of frequencies to enter, whereas the bass filter let a lower range of frequencies through. When we want the filters to be *active*, we mean that they should be built using active components, in this case OpAmps, that are powered separately from the filter circuits. We will use our power supply component from the midterm project to power all the active components in the final project. In addition, these filters must be *bandpass* as well, permitting only a small range of frequencies to enter through them. To better understand this, we show the plot of the magnitude of the transfer function associated with bandpass filters as follows:

Figure 1.2: *Plot of the Magnitude of a Typical Bandpass Filter*



The two filters used in this project will have magnitude functions that resemble that of **Figure 1.2**. Only frequencies between the *cutoff frequencies* $\boldsymbol{\omega_{c1}}$ and $\boldsymbol{\omega_{c2}}$ will be able to pass through the filter – hence the term *passband*. The *center frequency* $\boldsymbol{\omega_0}$ is given by the geometric mean of the corner frequencies, namely $\boldsymbol{\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}}$. We can design each of our filters so that only low frequencies will pass through the bass filter and high frequencies through the treble filter by determining what configuration of circuit elements will result in the desired center and cutoff frequencies. The frequencies that do pass through the filter component are subsequently amplified by the MOSFET amplifier prior to reaching the speakers. We reveal the schematic and equivalent circuit of the MOSFET in **Figure 1.3**, where the drain (D), gate (G), and source (S) of the MOSFET are shown.

Figure 1.3: *Schematic and Equivalent Circuit of a Typical MOSFET*



As can be seen from the equivalent circuit of the typical MOSFET, there is a factor g that serves as the gain of the MOSFET. The incoming voltage at the gate V_{GS} will be multiplied by the factor g to get the current set by the MOSFET. Then, the output voltage will be given by $V_{out} = V_{DD} - (gV_{GS})*R_D$, which, by setting appropriate values for V_{GS} and R_D can be used to amplify the signal of V_{GS}. In our circuit, we use voltage division to obtain an adequate value for V_{GS} that is then amplified by the MOSFET to V_{out}. Finally, we used the power supply that was made in the midterm project to power not only the OpAmps in the filter component but also the MOSFETs because they too are active components. Moreover, we had to be especially wary of the power budget of the entire circuit and whether our power supply was capable of providing that magnitude of power.

Part I: Filter Component

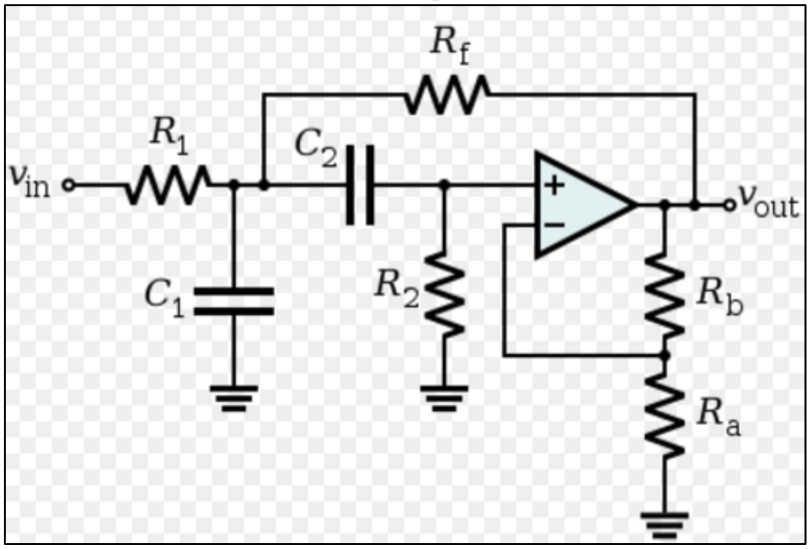
Section 2: Breakdown of Specification

Each of our filters had to take in as input an unamplified audio signal whose voltage varies from 0 to about $300mV_{RMS}$ and whose frequency ranges from 20Hz to 20kHz, i.e. the audio spectrum itself. Having received this signal, the filters were then to separate it based on its frequency – lower frequencies, ranging from 150Hz to 350Hz, became the output of the *bass* filter; whereas higher frequencies, ranging from 8kHz to 10kHz, were the output of the *treble* filter. Both filters needed to have a roll-off rate of at least -20 dB/decade in order to adequately ensure the speedy attenuation of the output signal's amplitude. In addition, the filter type of bass and treble filters had to be the same and had to experience a gain of at least 1 V/V – a higher gain is suitable if it will be properly interfaced with the amplifier component. Finally, the input and output impedances of both filters should be designed so as to maximize voltage transfer from the audio source to the amplifier component block. Keeping these specifications in mind and realizing that the design of one filter largely determined the design of the other (in terms of roll-off rate and filter type), we shall proceed to characterize the fundamental structure of both filters.

Section 3: Prelab Hand Calculations

In this section, we hope to show some of the calculations that served as the theoretical basis for our filter designs. We divided these hand calculations into six subsections, each of which focuses on a particular aspect of the filter component. It is worth mentioning that our choice of filter was a VCVS active bandpass filter for both the bass and treble components, identical in form to the topology shown in **Figure 3.4** below.

Figure 3.4 *Schematic of VCVS Topology, a Sallen-Key Variant*
Source: *Wikipedia*



Transfer Function

After applying the basic techniques of nodal analysis to our treble and bass filters, we were able to arrive at the transfer function that characterizes the gain of each filter. It is given as follows:

$$H(w) = \frac{V_{OUT}}{V_{IN}} = \frac{\left(1+\frac{R_b}{R_a}\right)\frac{w}{R_1C_1}}{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)w \; + j\left(w^2-\frac{R_1+R_f}{R_1R_2R_fC_1C_2}\right)}$$

A detailed derivation of this calculation and the ones which follow may be found in Appendix A.

Order of Filters

In standard form, the transfer function just obtained becomes the following:

$$H(w) = \frac{\left(1+\frac{R_b}{R_a}\right)\frac{1}{R_1C_1}jw}{1+j2\frac{1}{2}\frac{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)}{\sqrt{\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}}\frac{w}{\sqrt{\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}}+\left(\frac{jw}{\sqrt{\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}}\right)^2}$$

From the denominator of the transfer function shown in this form, we realize that each filter is of first-order because of an w term in the numerator and an w^2 term in the denominator, resulting in a net exponent of w. In addition, it becomes a rather straightforward task to deduce the center frequency, corner frequencies, bandwidth, and selectivity in this form, which we do in the next subsection.

Cutoff & Center Frequencies, Bandwidth, and Selectivity

The frequency at which this VCVS bandpass filter is purely real is the center frequency given by

$$w_0 = \sqrt{\frac{R_1 + R_f}{R_1 R_2 R_f C_1 C_2}}\ rad/s$$

This quantity could have been obtained from either the original transfer function (value of w that would make the imaginary component disappear) or the transfer function in standard form (denominator of quadratic term). To get the corner frequencies, we find the magnitude of the transfer function, evaluate the magnitude at the center frequency, and multiply that quantity by $\frac{1}{\sqrt{2}}$. In other words, the magnitude is given as

$$M(w) = \frac{\left(1 + \frac{R_b}{R_a}\right)\frac{w}{R_1 C_1}}{\sqrt{\left[\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}\right) w\right]^2 + \left(w^2 - \frac{R_1 + R_f}{R_1 R_2 R_f C_1 C_2}\right)^2}}$$

And at $\boldsymbol{w_0}$, the *gain* of the filter is simply the magnitude evaluated at $\boldsymbol{w_0}$:

$$M(w_0) = \frac{\left(1 + \frac{R_b}{R_a}\right)\frac{1}{R_1 C_1}}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}}$$

At this point, we realize that, by definition, $\boldsymbol{M(w_{c1}) = M(w_{c2}) = \frac{1}{\sqrt{2}} M(w_0)}$, a fact that allows us to solve for the corner frequencies on either side of the passband. Moreover, these corner frequencies are found to be

$$w_{c1} = \frac{1}{2}\left[\sqrt{\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}\right)^2 + 4\frac{R_1 + R_f}{R_1 R_2 R_f C_1 C_2}} - \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}\right)\right] rad/s$$

and

$$w_{c2} = \frac{1}{2}\left[\sqrt{\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}\right)^2 + 4\frac{R_1 + R_f}{R_1 R_2 R_f C_1 C_2}} + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}\right)\right] rad/s$$

Having found the corner frequencies, finding the bandwidth B and selectivity Q is simple:

$$B = w_{c2} - w_{c1} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}\ rad/s$$

$$Q = \frac{w_0}{B} = \frac{\sqrt{\frac{R_1 + R_f}{R_1 R_2 R_f C_1 C_2}}}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1}}$$

Choice of Gain

As we were using a VCVS configuration, we knew that there would be a tradeoff between bandwidth and gain. Thus, we attempted to keep the bandwidth at about 100Hz for the bass filter and about 2kHz for the treble. This setup nevertheless resulted in a gain of about 3.5dB for the bass filter and 5.2dB for the treble filter, which was then later amplified by the MOSFETs at the end. Had we kept 0dB as a target gain for either filter, our bandwidths would subsequently have been too large for the filters to serve their respective functions.

Expected Input & Output of Each Filter (Sources/Loads)

We expected a gain of about 3.5dB for the bass filter and a gain of about 5.2dB for the treble filter at the respective center frequencies. At the cutoff frequency of the bass filter, we expected 3.5dB – 3dB = 0.5dB gain and at the cutoff frequency of the treble filter, we looked for 5.2dB – 3dB = 2.2dB gain. Since both filters were first-order VCVS bandpass filters, the roll-off rate for both of the filters was to be around -20dB/decade.

Expected DC Power Requirements (Voltage/Current/Power)

As each filter used an active OpAmp component, we decided to use low-noise TL071 OpAmps. These OpAmps have a very low current limit of 2.5mA and, since our power supply delivered $\pm$12V, this meant that the amount of power consumed by either filter would be $P = IV = (2.5mA) * (12V) = 30mW$.

Section 4: Prelab Simulations

Since we were using TL071 OpAmps in both of our filter circuits, we knew they could handle up to Vcc = $\pm$18V; as our power supply delivered $\pm$12V, we had satisfied this voltage constraint. At the same time, the Tl071 OpAmp has a GBWP of 4MHz, which meant that our expected bandwidth would be $BW_{bass} = 4MHz/(10^{\frac{3.5dB}{20dB}}) = 2.67MHz$ for our bass filter and $BW_{treb} = 4MHz/(10^{\frac{5.2dB}{20dB}}) = 2.20MHz$, both of which are more than large ranges of frequency for our designing purposes (our maximum frequency under consideration is only 20kHz). To verify and lend credence to our previous hand calculations, we conducted several simulations on *Circuit Lab*. Here, we present the results of these simulations for both our bass and treble filters and provide brief explanations as to their meaning and significance.

Figure 4.5: *Schematic of Treble Filter*

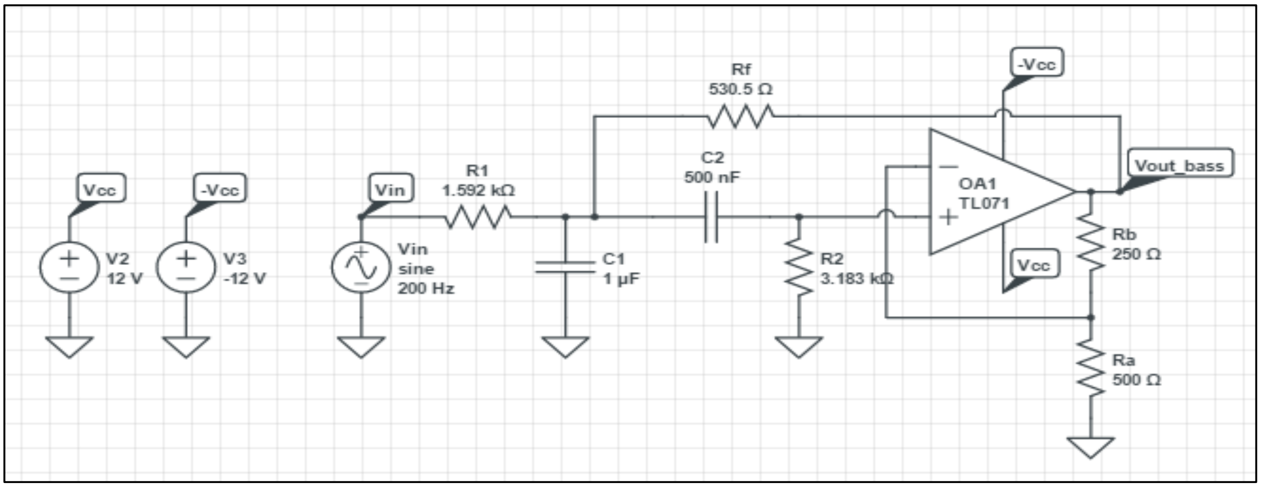
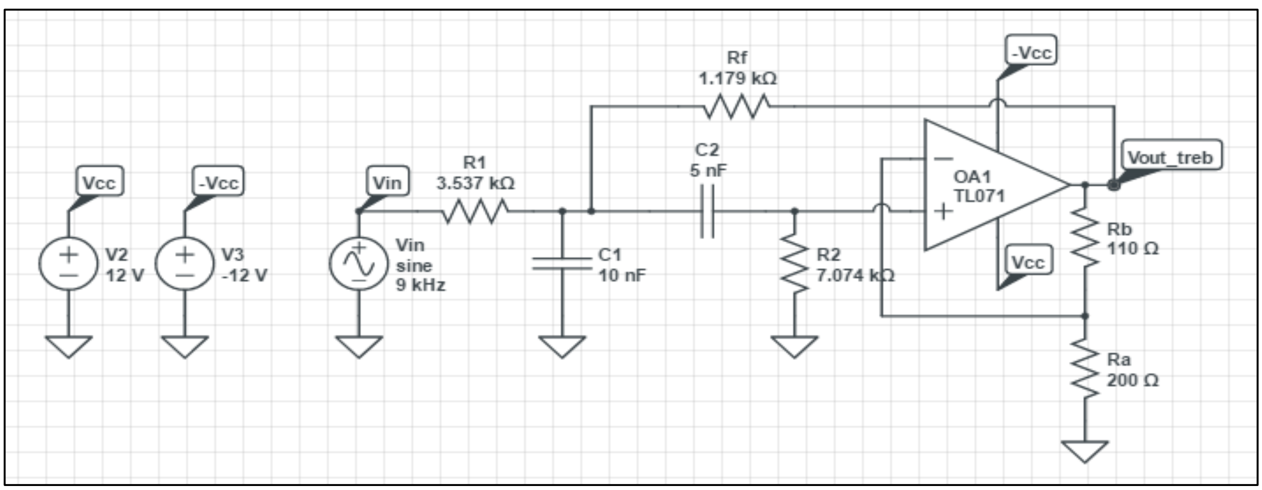


Figure 4.6: *Schematic of Bass Filter*



Each of these diagrams shows the bass and treble filters as they will look like when placed in circuit. The only difference is that both filters are powered by an external $\pm 12V_{DC}$ input when in fact they will be powered by an AC/DC power supply that delivers $\pm 12V_{DC}$.

Figure 4.7: *Semi-Logarithmic Frequency Response for Magnitude and Phase of Bass Filter*

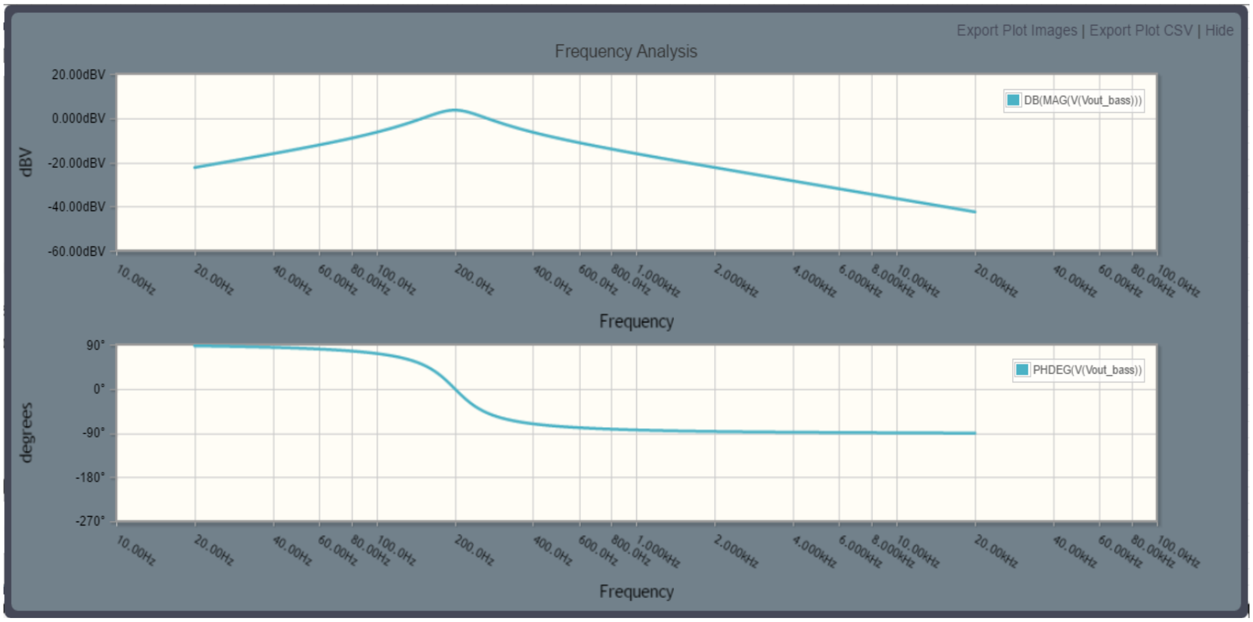
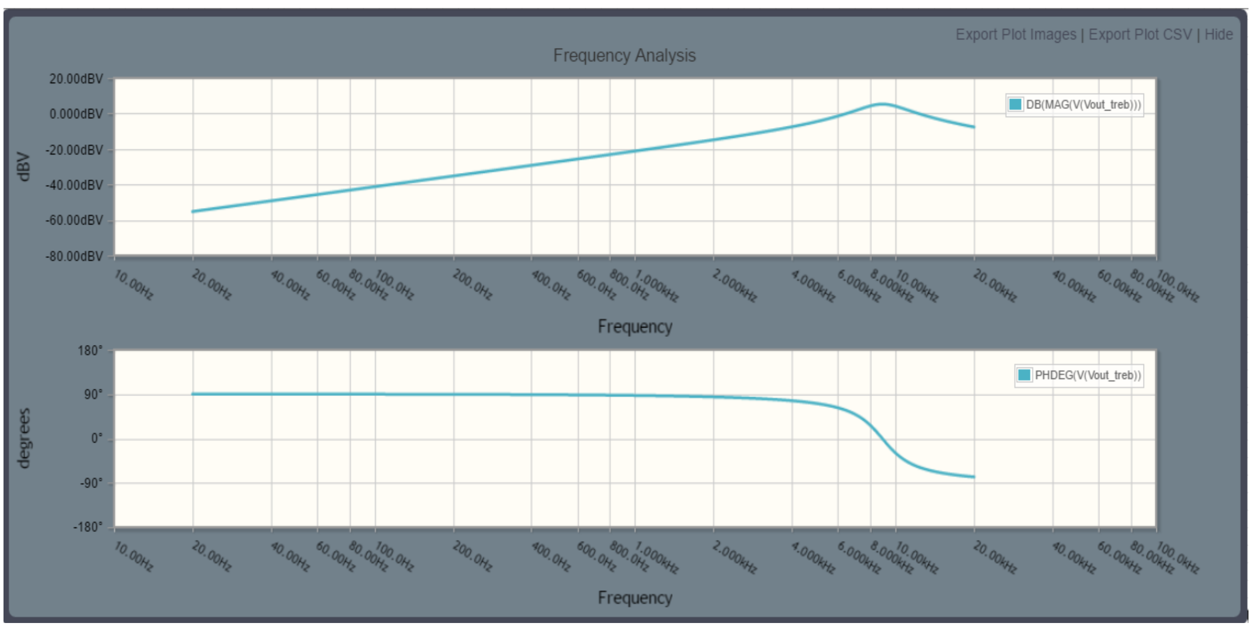


Figure 4.8: *Semi-Logarithmic Frequency Response for Magnitude and Phase of Treble Filter*



These graphs represent the frequency responses of the bass and treble filters with the magnitude of the transfer functions in dB and the phase in degrees. It is necessary to note that these semi-logarithmic graphs are exact in nature; by contrast, Bode plots would serve only as linear approximations to these frequency responses. From these semi-log graphs, we can approximate the roll-off rates of the bass and treble filters as if they were Bode plots:

$$Roll - Off\ Rate\ of\ Bass\ Filter = \frac{-22.43dB - 3.522dB}{\left(\frac{2000Hz}{200Hz}\right)} = -25.95\ dB/decade$$

$$Roll - Off\ Rate\ of\ Treble\ Filter = \frac{-22.30dB - 5.232dB}{\left(\frac{89.34kHz}{8.934kHz}\right)} = -27.53\ dB/decade$$

The roll-off rate of the treble filter could only have been calculated if we knew the gain in dB at ten times the center frequency, namely about 90 kHz. As the upper limit on the audio spectrum is 20 kHz, we had to extrapolate the value of gain in dB at around 90 kHz. Additionally, we further decided to see what the input and output impedances of both of our filters would be. As we were dealing with OpAmps, we somewhat expected high input impedance and low output impedance for the purpose of maximizing voltage transfer. Thus, using the amplitudes of the respective AC voltage and current waveforms, we obtained the following results via Ohm's Law:

$$Bass\ Filter\ Input\ Impedance = \frac{V}{I} = \frac{424.1mV}{2.305\mu A} = 184k\Omega.$$

$$Treble\ Filter\ Input\ Impedance = \frac{V}{I} = \frac{424.1mV}{1.990\mu A} = 213k\Omega.$$

$$Bass\ Filter\ Output\ Impedance = \frac{V}{I} = \frac{620.5mV}{1.274mA} = 487\Omega.$$

$$Treble\ Filter\ Output\ Impedance = \frac{V}{I} = \frac{680.2mV}{2.404mA} = 283\Omega.$$

In the following graphs, we seek to present time-domain simulations of the bass and treble filters at their respective center, left cutoff, and right cutoff frequencies. As usual, we expect each graph to be sinusoidal.

Figure 4.9: *Time-Domain Simulation of Bass Filter at Center Frequency*

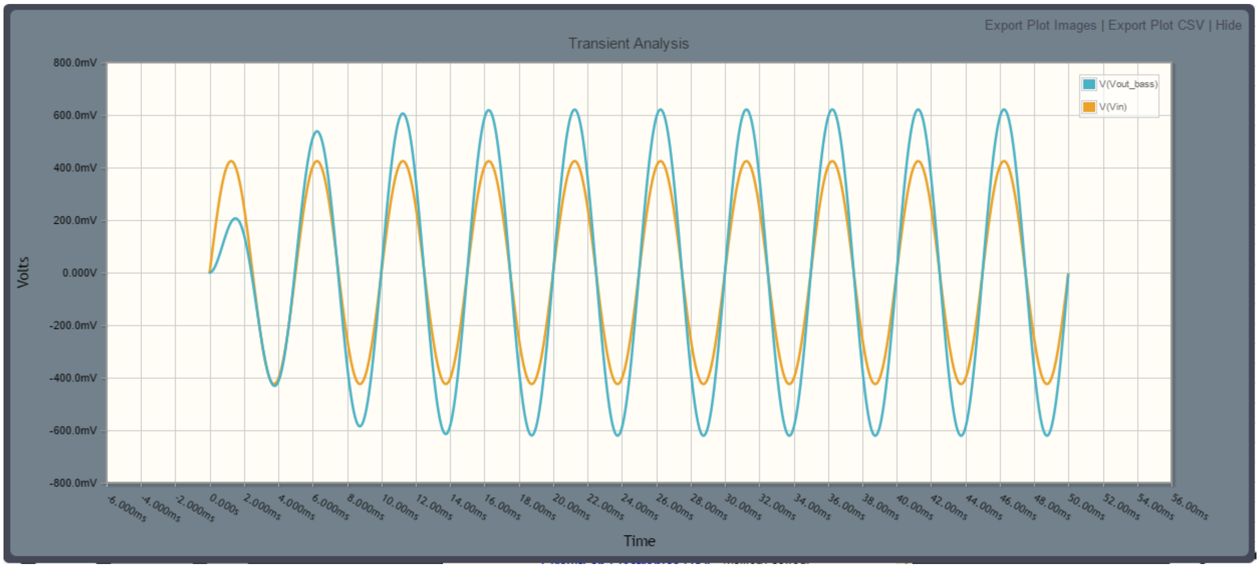


Figure 4.10: *Time-Domain Simulation of Treble Filter at Center Frequency*

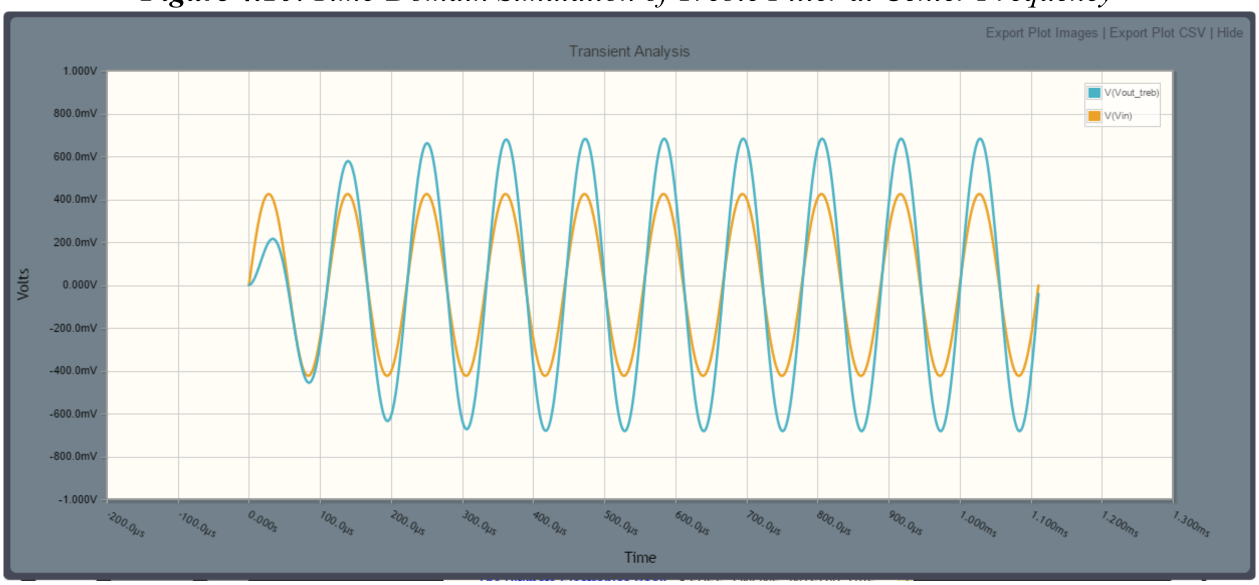


__Figure 4.11__: Time-Domain Simulation of Bass Filter at Left Cutoff Frequency

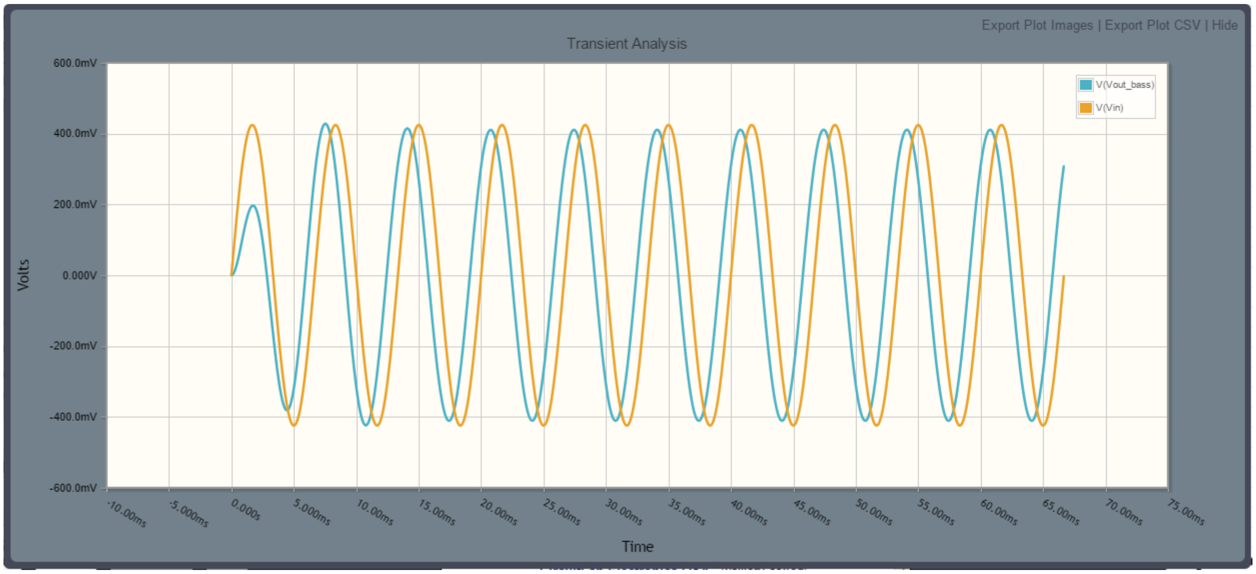


__Figure 4.12__: Time-Domain Simulation of Bass Filter at Right Cutoff Frequency

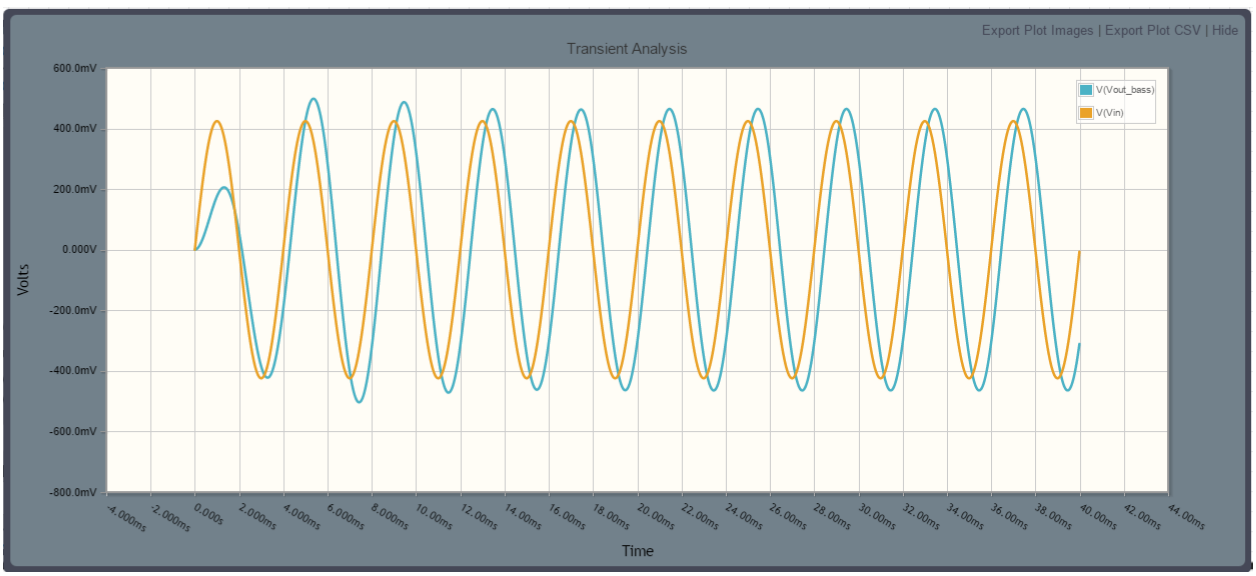


Figure 4.13: *Time-Domain Simulation of Treble Filter at Left Cutoff Frequency*

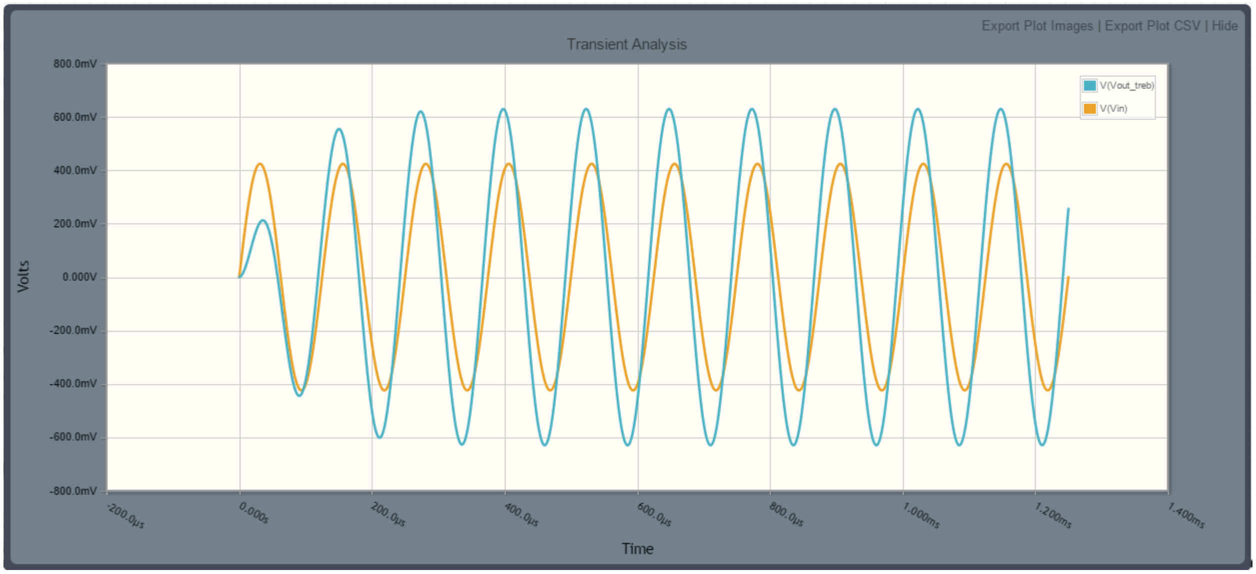
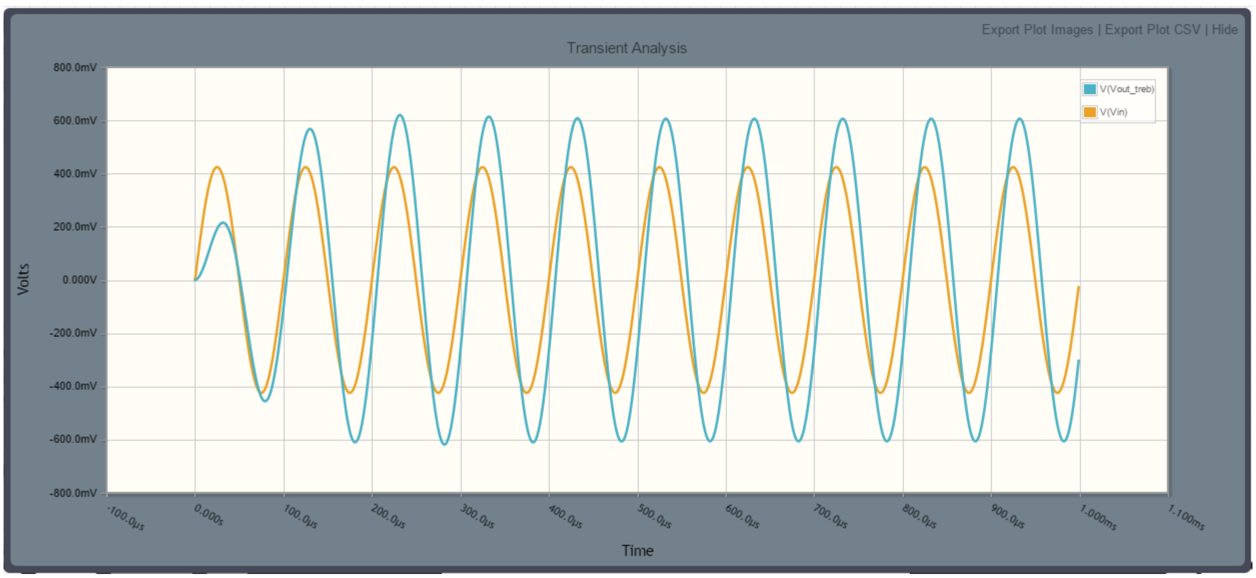


Figure 4.14: *Time-Domain Simulation of Treble Filter at Right Cutoff Frequency*



Lastly, we show the expected DC power requirements from our voltage sources at the voltage rails of each of the filters in the following set of simulations:

Figure 4.15: *DC Simulation of the Voltage at the Bass Filter OpAmp Rails*

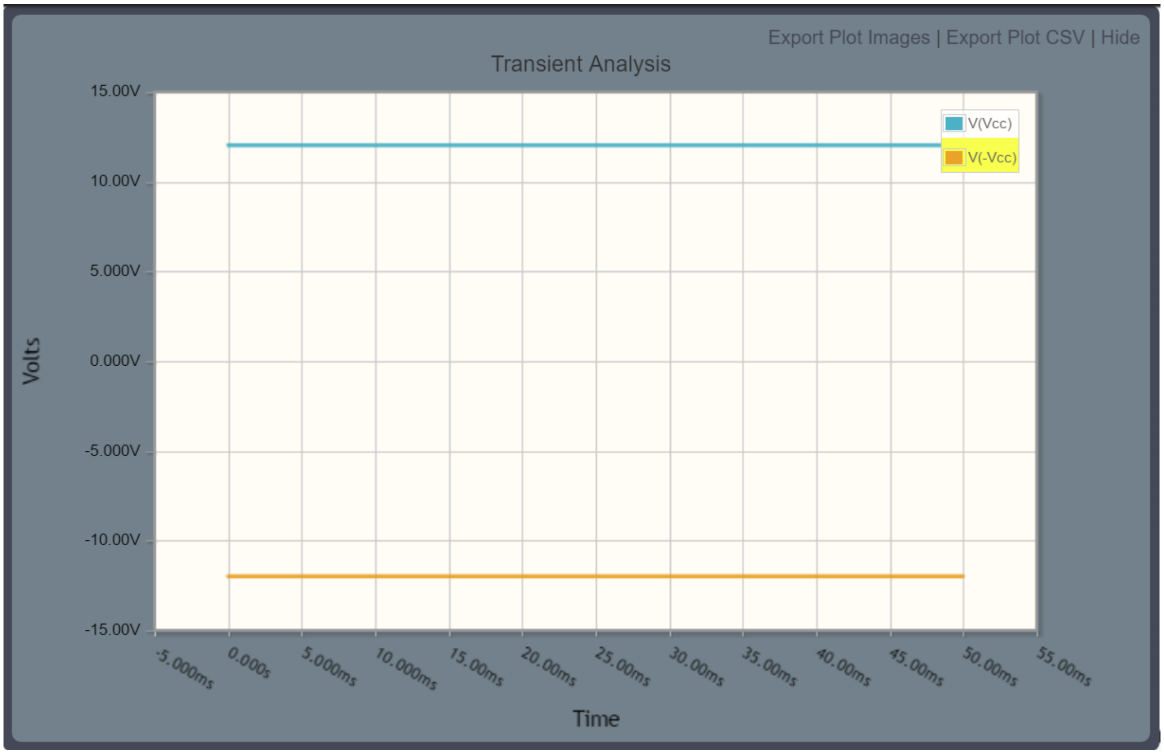


Figure 4.16: *DC Simulation of the Current at the Bass Filter OpAmp Rails*

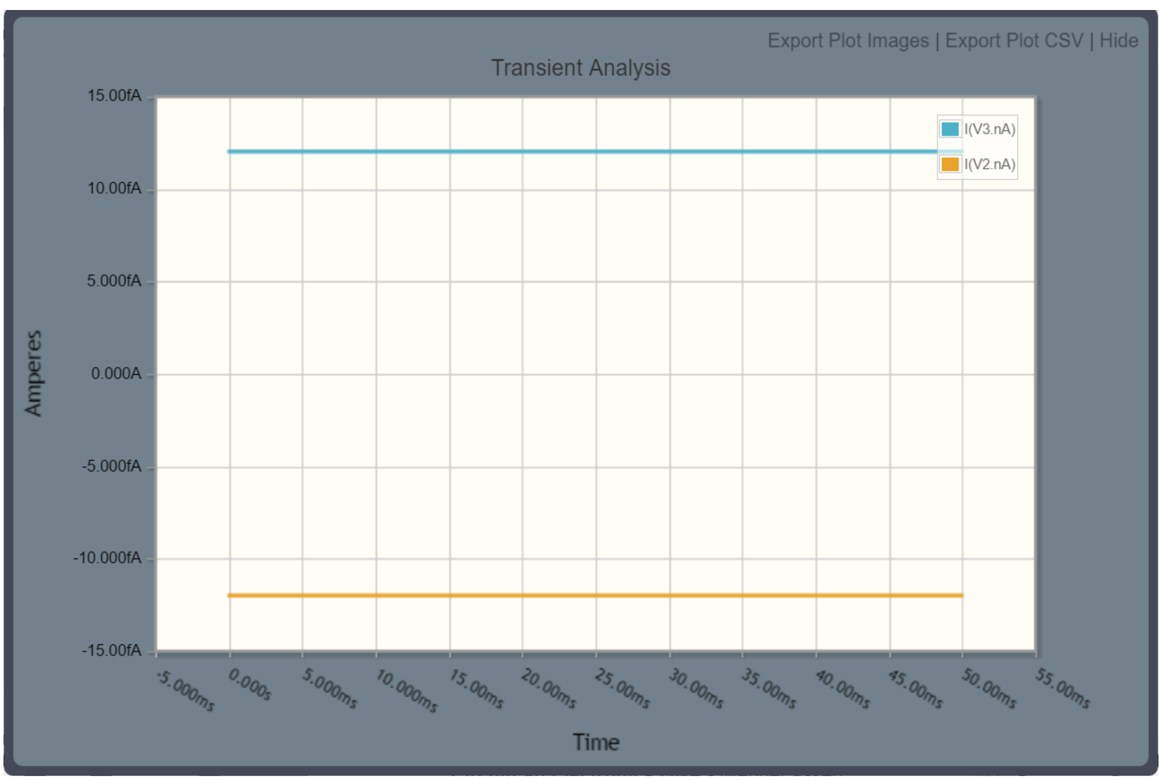


Figure 4.17: *DC Simulation of the Power at the Bass Filter OpAmp Rails*

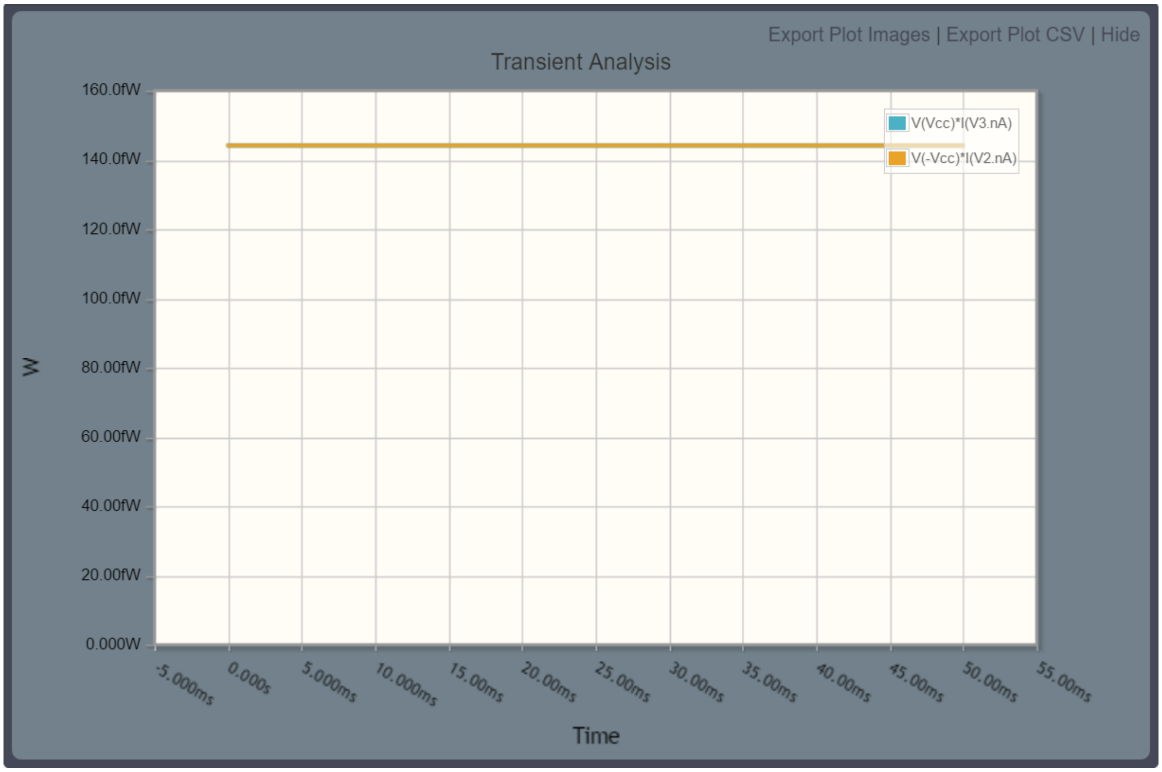


Figure 4.18: *DC Simulation of the Voltage at the Treble Filter OpAmp Rails*

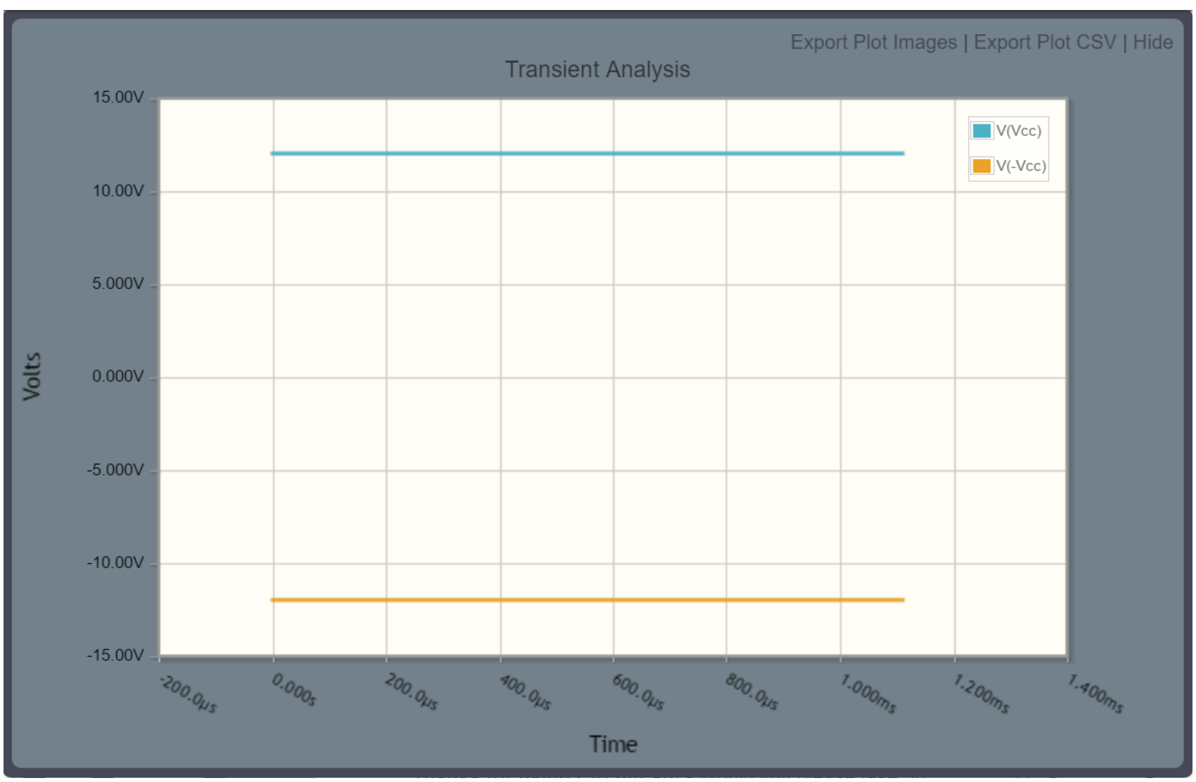


Figure 4.19: *DC Simulation of the Current at the Treble Filter OpAmp Rails*

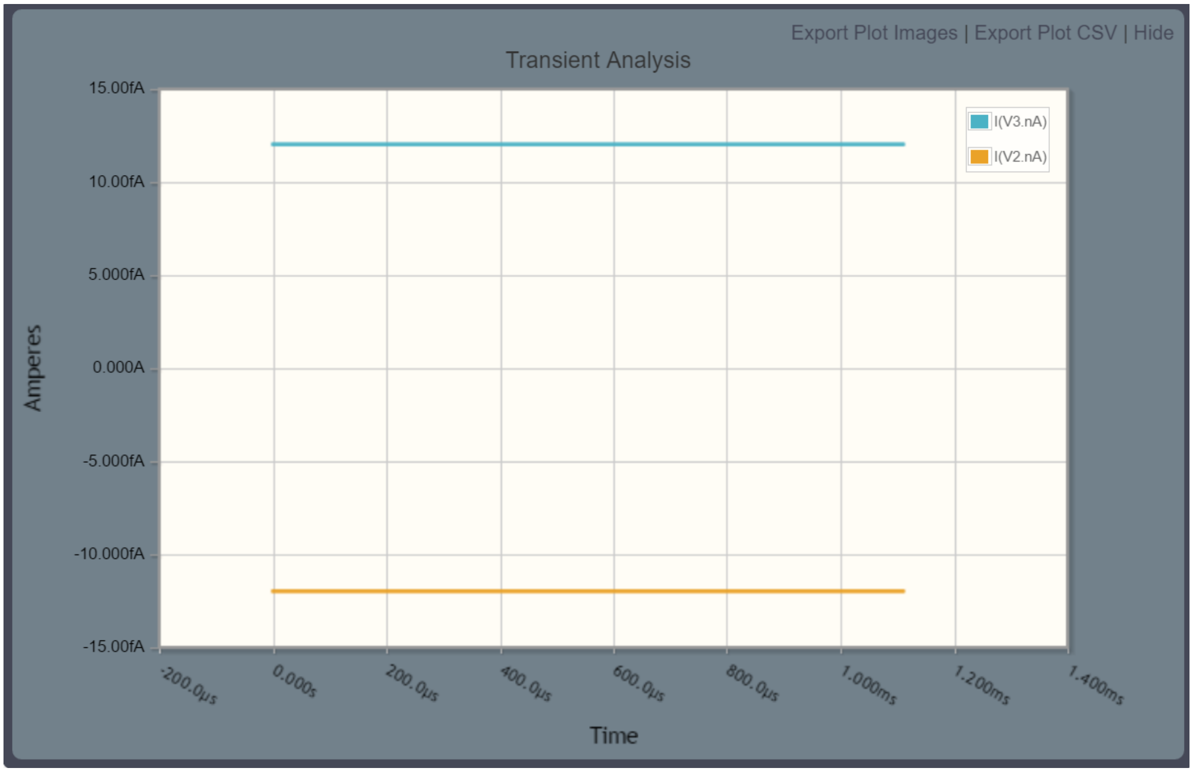
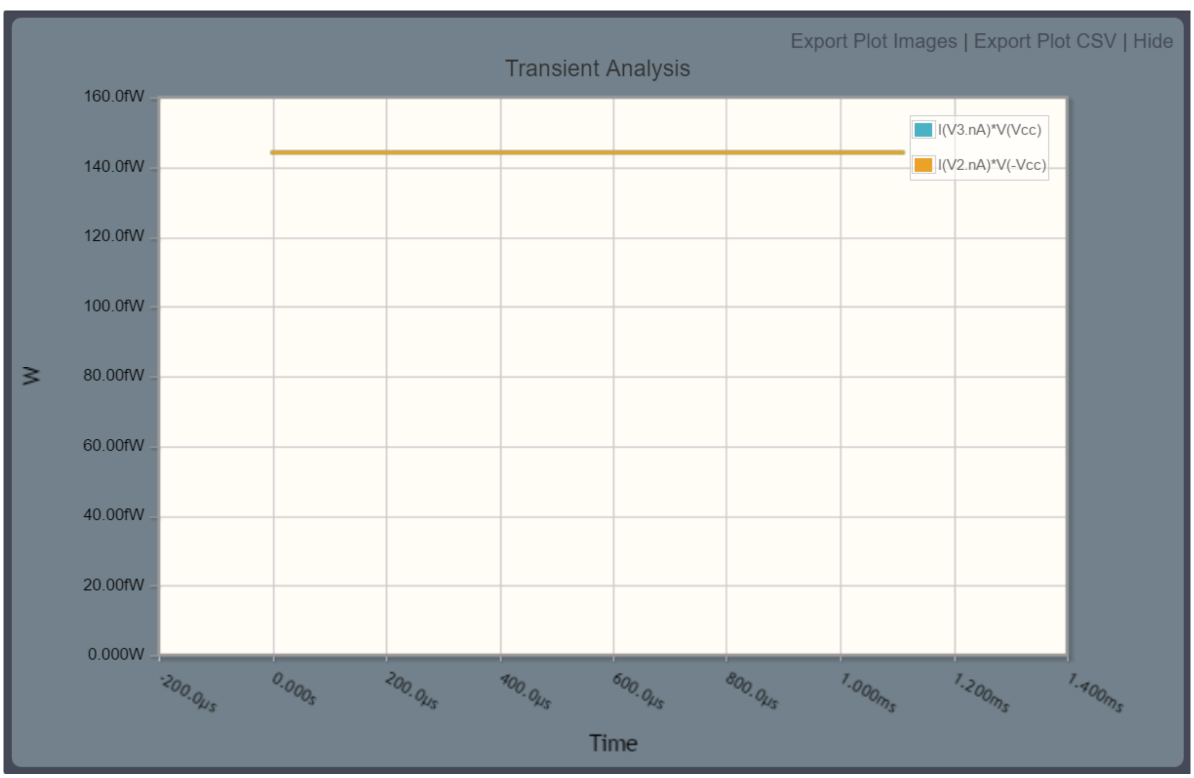


Figure 4.20: *DC Simulation of the Power at the Treble Filter OpAmp Rails*



Section 5: Experimental Setup

In order to ensure the proper functionality of our filters, we used several pieces of lab equipment to test and debug each filter's circuit. In particular, we employed an oscilloscope with probes, a digital multimeter (DMM), a function generator, an external power supply, an audio source, and two op-amps. The details of this equipment are given as follows:

Table 5.1: *Lab Equipment of Filter Component with Specifications*

Equipment	Manufacturer & Model Number (if given)
D300 MHz, 4 Analog Channel + 16 Logic Channel Input Oscilloscope	Agilent MSO7034B
500 MHz Passive Probe	Agilent 10073D
Digital Multimeter (DMM)	HP 34401A
30 MHz Function/Arbitrary Waveform Generator	Agilent 33521A
Triple Output DC Power Supply	HP E3631A
Audio Source (iPhone 6s)	A1633
Low-Noise JFET-Input General-Purpose OpAmp	TL071

In **Figures 4.5** and **4.6**, all of this equipment comes into play. My iPhone 6s, on full volume, served as the audio source, supplying approximately $300V_{RMS}$ to each filter. To ensure maximum voltage transfer and a gain of at least 1 V/V, a TL071 OpAmp was part of both filters and received ±Vcc from the Triple Output DC Power Supply. For testing and verifying the proper functionality of each of our filters, we employed both an oscilloscope and DMM. That is, while the oscilloscope determined the sinusoidal current and output voltage graphs of each filter, the DMM cross-checked their AC magnitude to guarantee quality measurements. In addition, the waveform generator was helpful in debugging because, unlike with our iPhone 6s audio source, we could modulate the frequency of the generator to occur at the center and cutoff frequencies of both filters and then measure those responses with the oscilloscope.

Section 6: Experimental Procedure

In order to ensure that our filters behaved in a manner that we expected, we performed a great deal of debugging. To this end, we made ample use of the oscilloscope and probes, DMM, and waveform generator. In particular, we replaced our audio source with the waveform generator at the same voltage ($300V_{RMS}$) and at specific frequencies, namely the center and cutoff frequencies. Having attached the waveform generator, we employed the oscilloscope and probes to determine the sinusoidal time-domain graphs at the output voltage of each filter and compare them to the time-domain graphs from online simulation. By noting the parallels between the experimental oscilloscope graphs and the simulated graphs, we were able to address any anomalies that occurred in our setup. Through the use of the both the oscilloscope and the DMM, we found the output voltage of each filter at its respective center frequency, which allowed us to deduce the gain of each filter in dB, as well as the total current drawn by each circuit. As a result, we were able to measure the DC power consumption as $P_{bass} = I_{bass}V_{bass} = (1.268mA) * (12V) = 15.2mW$ for the bass filter and $P_{treb} = I_{treb}V_{treb} = (2.379mA) * (12V) = 28.5mW$.

Section 7: Experimental Results

For comparison with our hand calculations and time-domain simulations, we collected detailed graphs of the frequency responses of our filters in our actual circuit. Using an oscilloscope, waveform generator, and DC power supply, we were able to acquire the following diagrams:

Figure 7.21: *Time-Domain Graph of Bass Filter at Center Frequency*

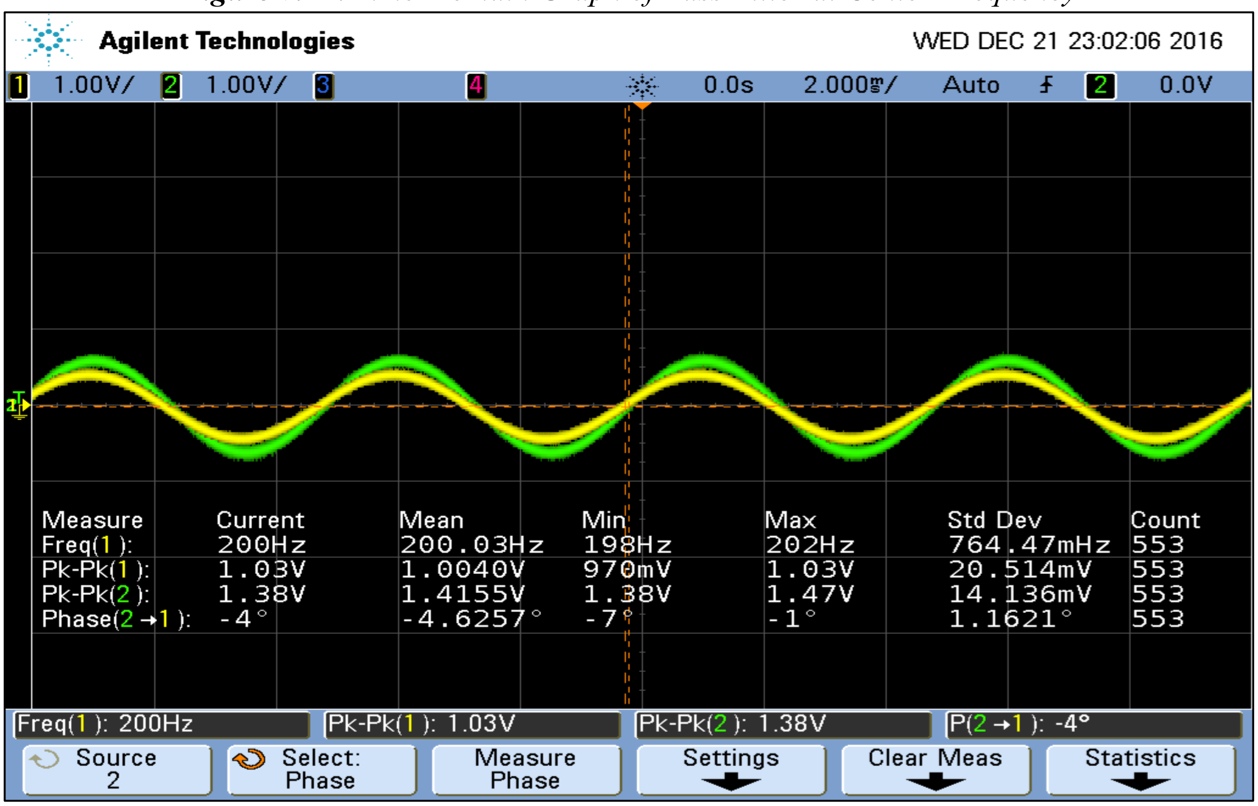


Figure 7.22: *Time-Domain Graph of Treble Filter at Center Frequency*

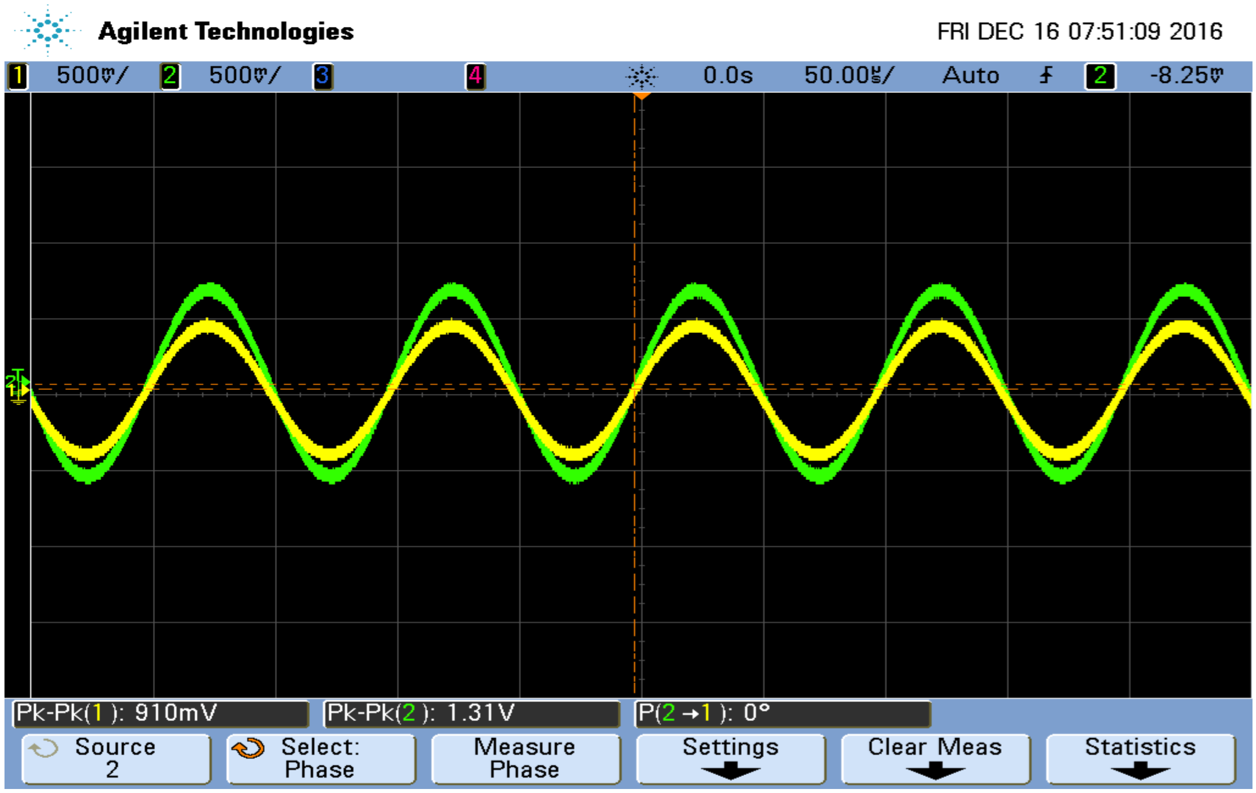


Figure 7.23: *Time-Domain Graph of Bass Filter at Left Cutoff Frequency*

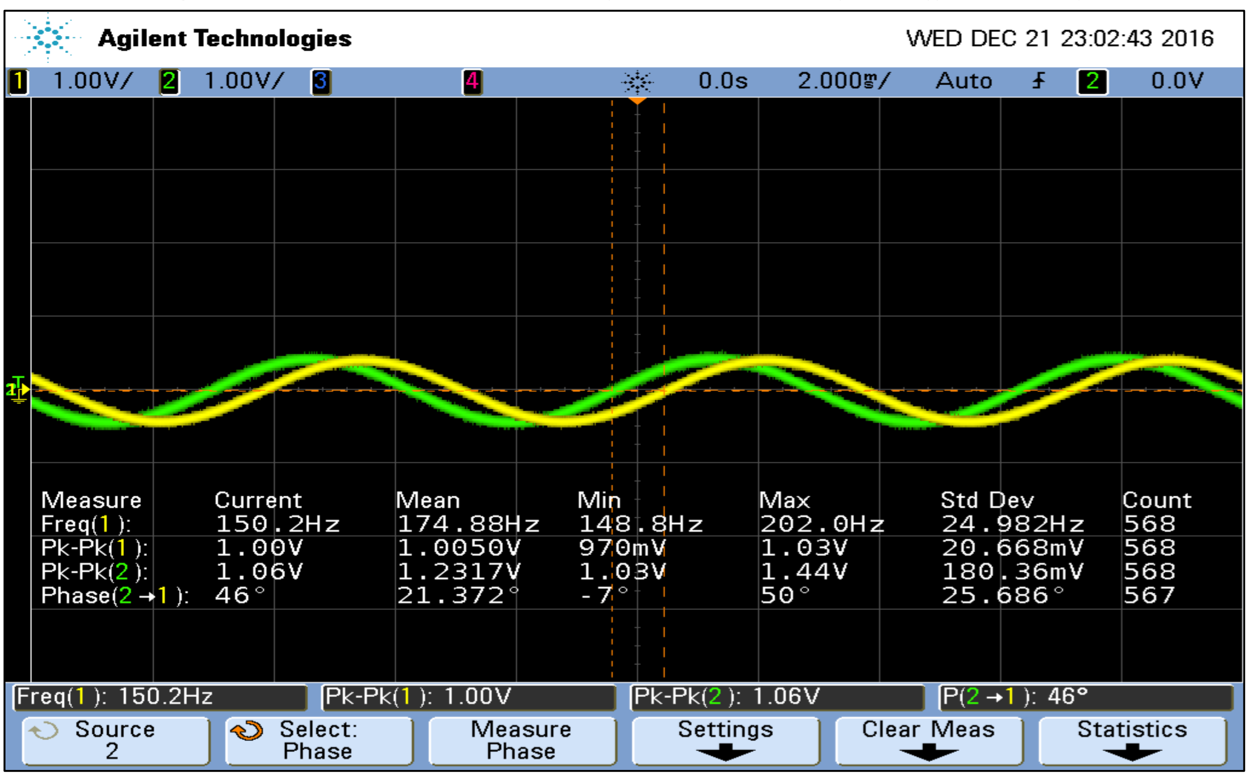


Figure 7.24: *Time-Domain Graph of Bass Filter at Right Cutoff Frequency*

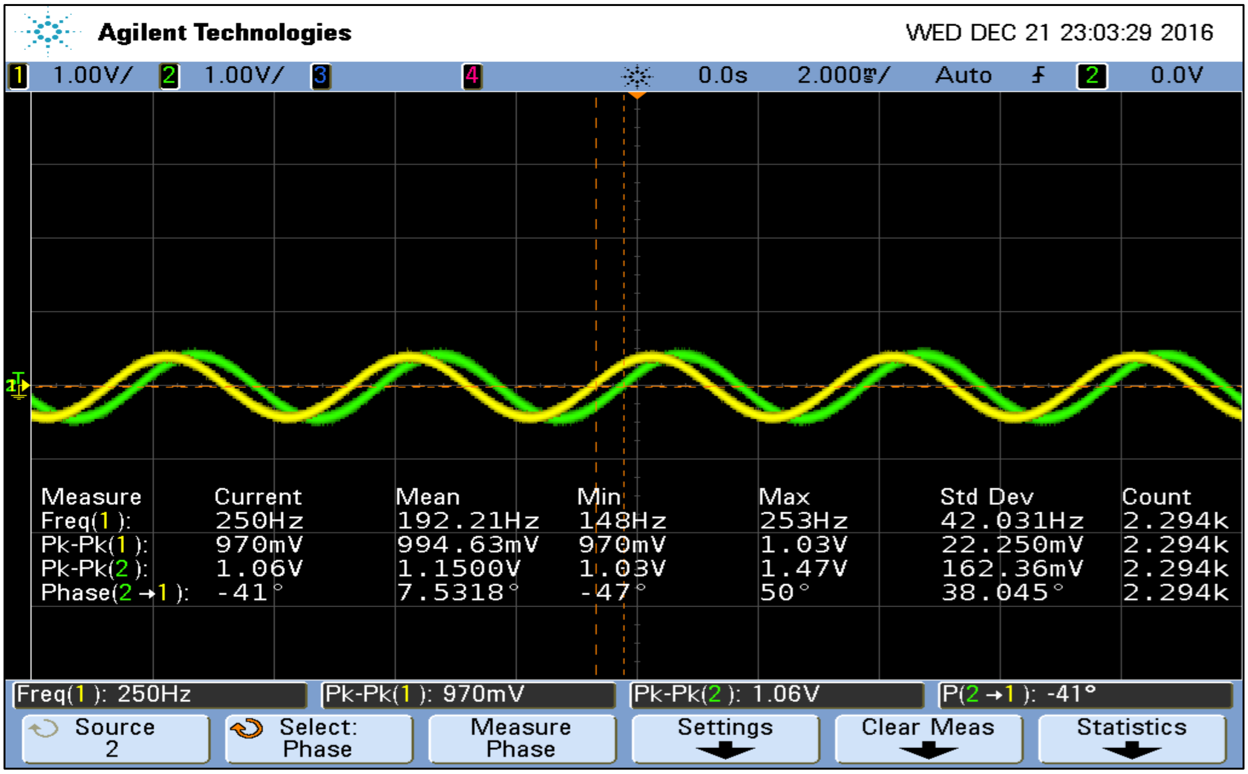


Figure 7.25: *Time-Domain Graph of Treble Filter at Left Cutoff Frequency*

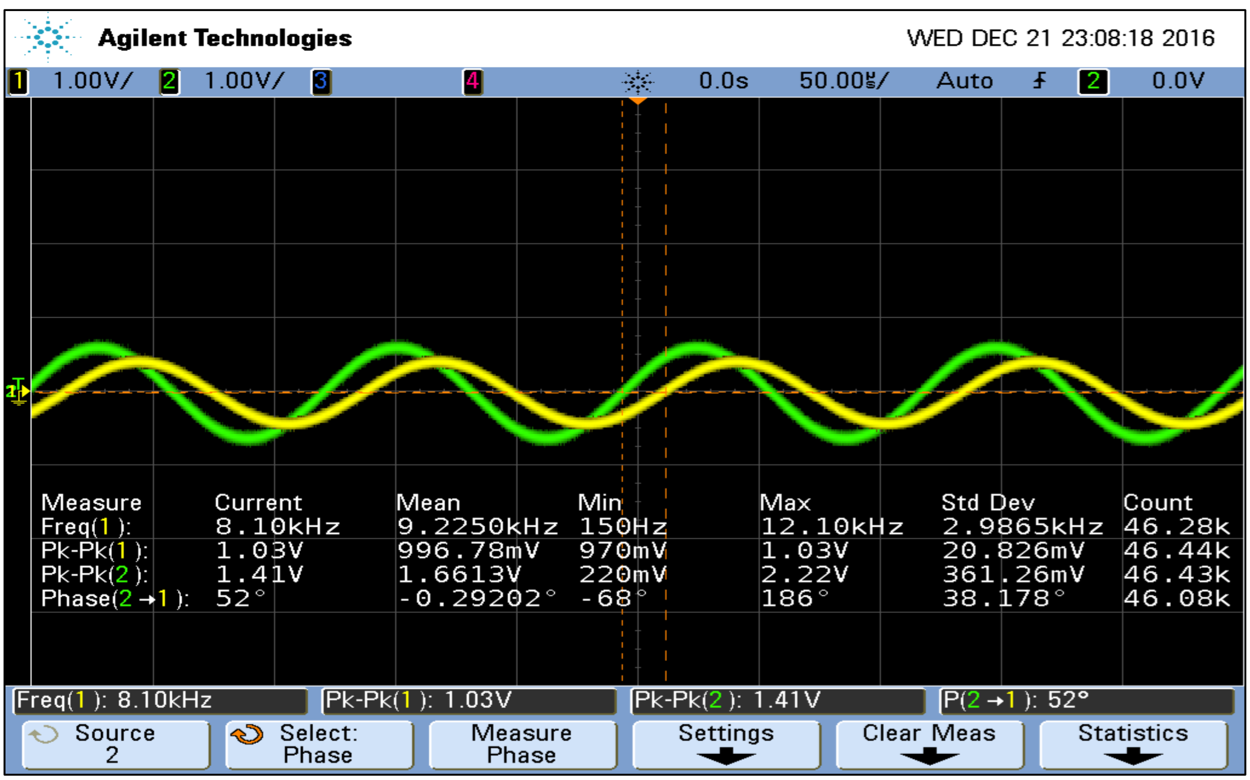


Figure 7.26: *Time-Domain Graph of Treble Filter at Right Cutoff Frequency*

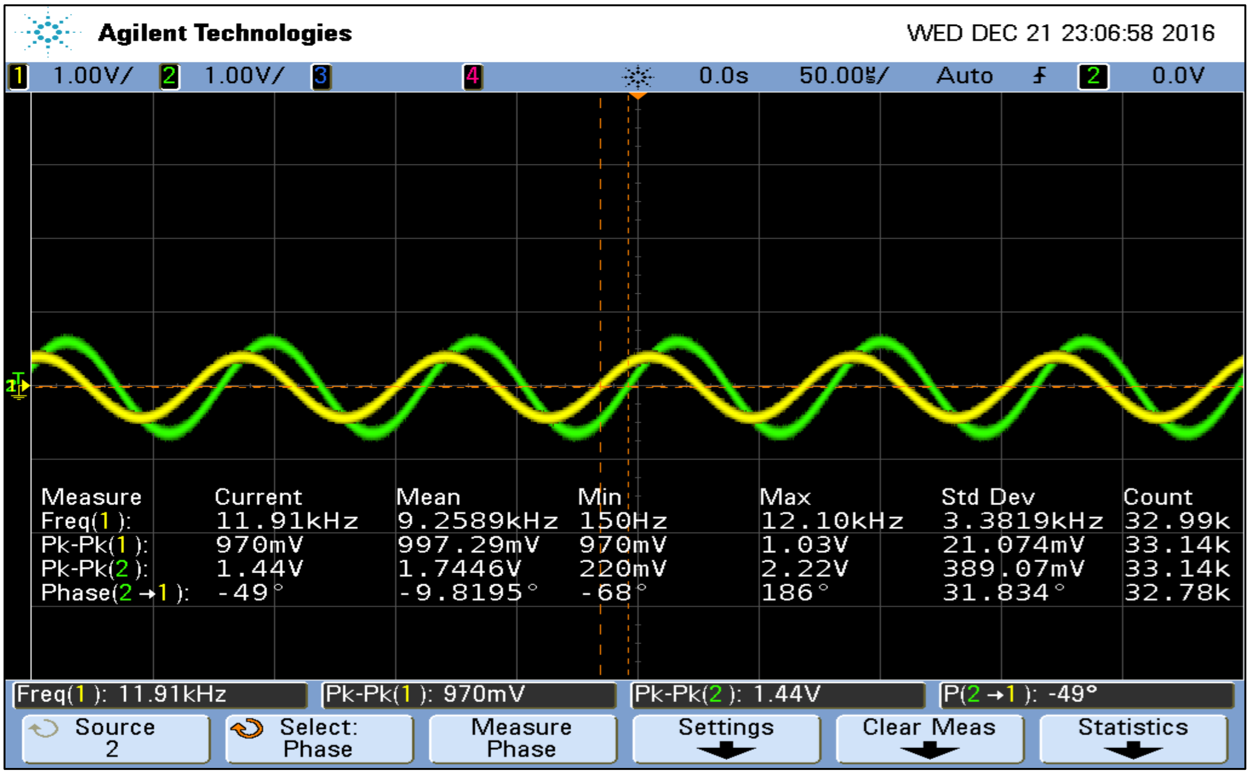


Figure 7.27: *Bode Plot of Magnitude for Bass Filter*

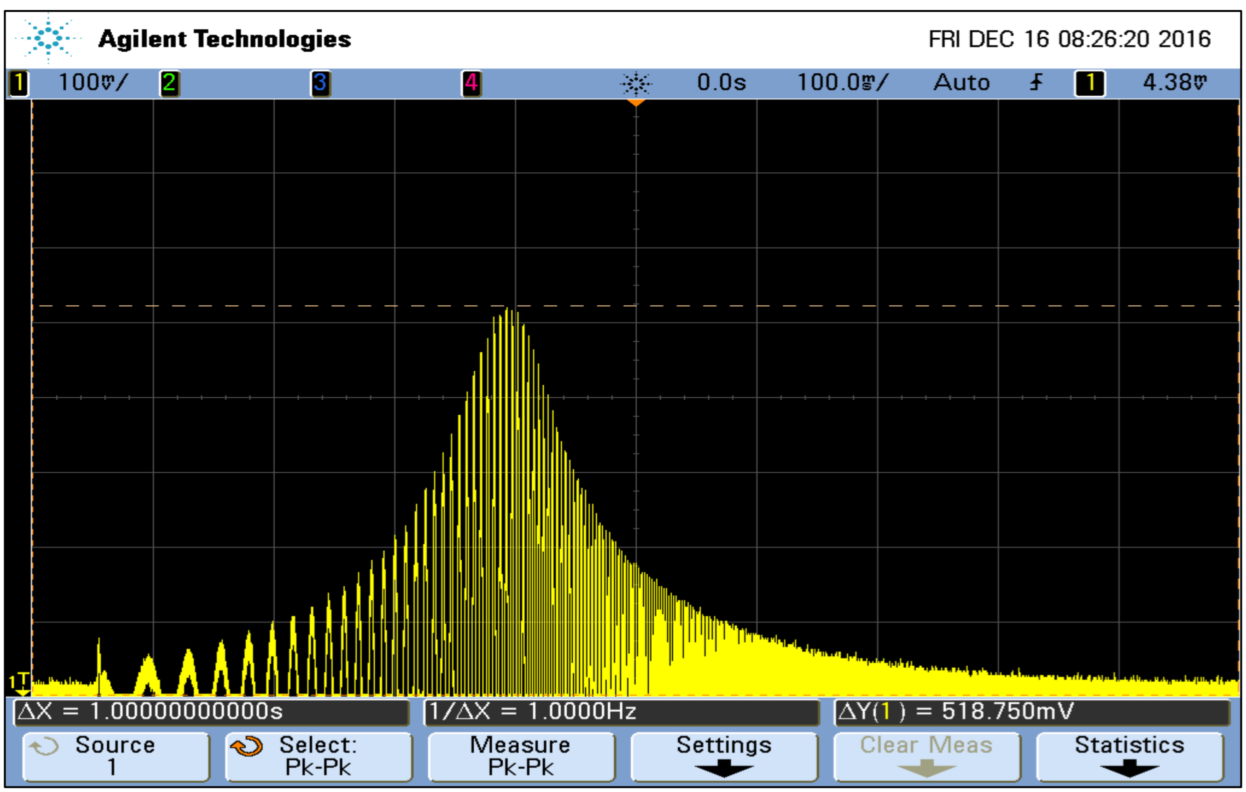
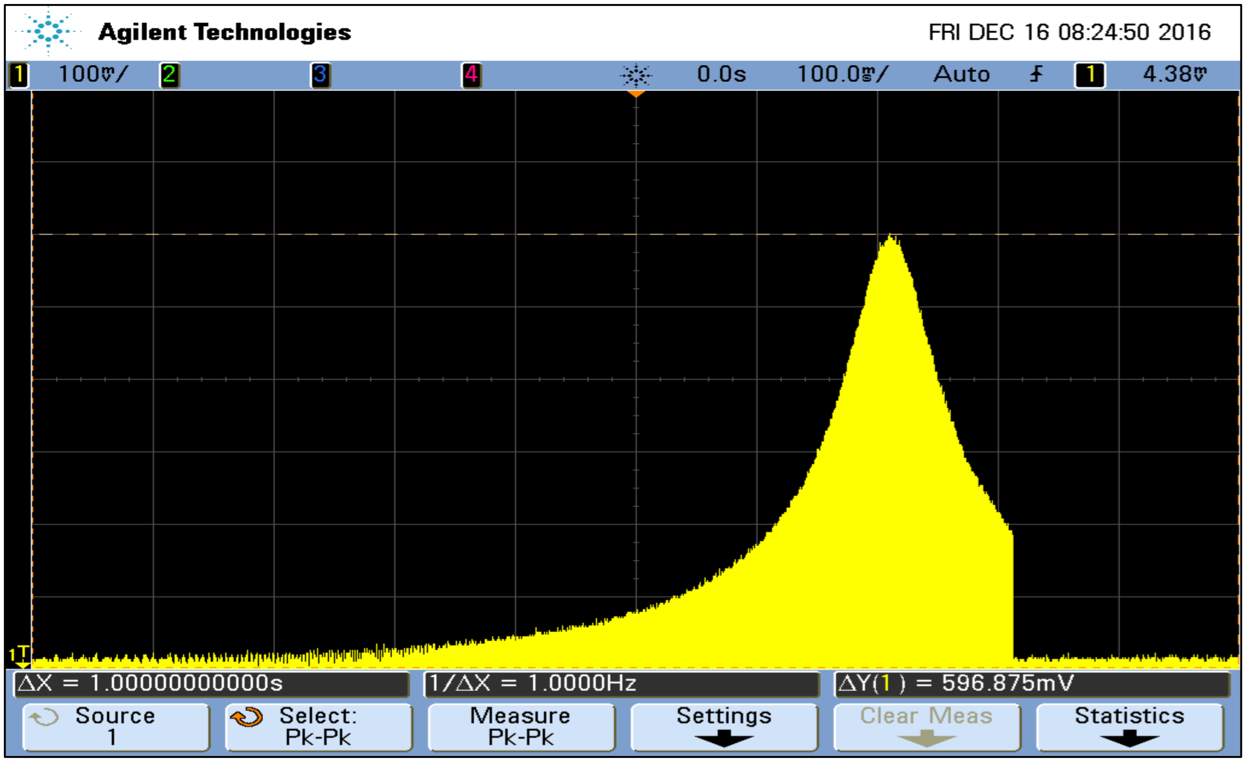


Figure 7.28: *Bode Plot of Magnitude for Treble Filter*



Section 8: Error Analysis

Surprisingly, our bass filter's performance was superior to the results we obtained from the treble filter, as we were able to achieve a bandwidth ($\boldsymbol{B}$) of approximately 100 Hz with the bass filter ($\boldsymbol{B(Hz) = f_{c2} - f_{c1} = 250Hz - 150Hz = 100Hz}$). With regards to the treble filter, we aimed for a bandwidth of 2kHz and more specifically cutoff frequencies of 8kHz and 10kHz. As evident from ***Figure 7.25***, the left cutoff frequency essentially meets this expectation as it is 8.10kHz, but the right cutoff frequency depicted in ***Figure 7.26*** is noticeably high at 11.91kHz, nearly 2000Hz greater than our desired value. Thus, the treble filter's bandwidth $\boldsymbol{B = 11.91kHz - 8.10kHz = 3.81kHz}$. In retrospect, we could have attempted to decrease this bandwidth by increasing the order of the filter. Currently, as illustrated in the circuit schematics (***Figure 4.5*** and ***Figure 4.6***) both filters are first order; increasing the order of the treble filter would effectively increase the roll-off rate by 20dB/decade, consequently raising the quality factor ($\boldsymbol{Q}$). By definition, increasing the quality factor would result in a narrower bandwidth, so we could use this method to obtain the desired bandwidth of 2kHz for the treble filter. Furthermore, the 5.2dB gain from our treble filter exceeded the anticipated value of 0dB; we could decrease the gain by implementing an inverting op-amp to counteract this gain.

Nevertheless, both filters functioned well as we tested them (prior to building our own amplifier) by using the provided amplifier to play music from our mobile devices.

Section 9: Conclusion

Ultimately, both the bass and treble filters produced high quality sound when connected to both the given amplifier and our own. While the bass filter did a better job than the treble, they limited the range of sound to certain frequencies and worked well given the music we inputted. Nevertheless, the music we played via our iPhone 6s likely did not drive the filters to the extent that another, heavier input might—so while there were no audible signs of error, we could have built a more optimal filter that accounted for audio sources beyond daily devices such as a mobile phone. However, for the purpose of this assignment, our filter did an incredible job allowing specific frequencies to pass, resulting in excellent quality sound. Our bass filter performed especially well, as we were able to limit its bandwidth to 100Hz, meeting the extra credit requirement. Furthermore, the gain of the bass filter was nearly 0dB, just as desired! Overall, we succeeded in building filters that prepared us for the next step of the audio docking station: the amplifier.

Part II: Amplifier Component

Section 10: Breakdown of Specification

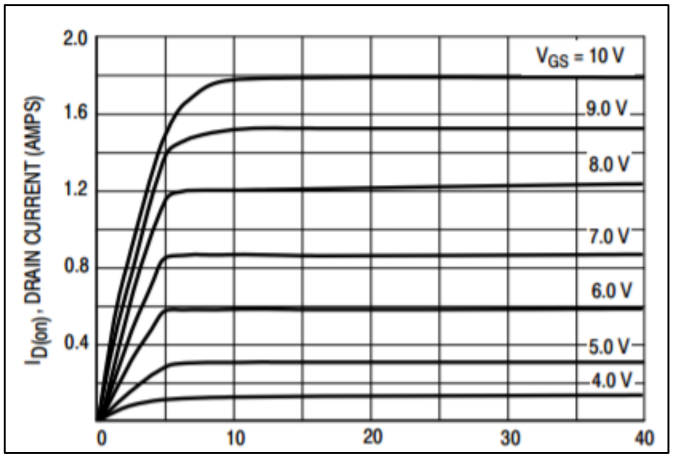
In this part of the project, we employed a two-stage MOSFET amplifier to amplify the signal received from the output of each of the filters. We chose a two-stage amplifier due its highly desirable functionality as a voltage-gain MOSFET followed by a current-gain MOSFET so that the voltage delivered to the 8Ω load at either channel would be as large as possible without greatly compromising the current. As such, we aimed to provide at least 0.25W to the 8Ω outputs at the center frequency of the input signals, which would require a DC offset. That is, a DC offset needed to be applied to the incoming signals so that the first MOSFET amplifier would turn on, i.e. so that $V_{GS} > V_T$, where V_T is threshold voltage of the MOSFET. We set this DC offset by altering the voltage-divider configuration of the first MOSFET so that the bias point would be sufficiently large to drive the speaker attached to the amplifier. Finally, the estimated amount of current delivered to the 8Ω speaker would be $P = I^2R$, or $I = \sqrt{\frac{0.25W}{8\Omega}} = 0.177A$.

Section 11: Prelab Hand Calculations

When we looked up the value of the threshold voltage V_T on the datasheet of the BS170 MOSFET, we found that we needed to cross a value of 2.1V in order to turn the MOSFET on.

Thus, we would need to have a bias in place that exceeded the gate threshold value of 2.1V. We used the following characteristic graph of the I_D vs. V_{GS} to assist us in choosing a bias point:

Figure 11.29: *Characteristic Curves of I_D vs. V_{GS} for the BS170 MOSFET*
Source: *BS170 MOSFET Datasheet*



From the output voltage of the filter component, we aimed to bring down the voltage to a V_{GS} that was sufficiently greater than V_T. Using biasing resistors, we developed a voltage divider for the first MOSFET in the two-stage system, selecting values of 33kΩ and 10kΩ. This choice of resistors led to a bias given by the following voltage division equation:

$$V_{GS} = 12V * \frac{10k\Omega}{10k\Omega + 33k\Omega} = 2.79V.$$

In order to prevent the clipping effect that often distorts signals that are amplified by a MOSFET, we had to bias our second MOSFET in the two-stage system by 6V. To this end, we made the second the set of biasing resistors take on the values of 1kΩ and 110Ω, thereby producing an offset close to 6V.

We estimate the peak power delivered to the 8Ω load at the center frequencies of either filter to be given by the product of the expected output voltage and the expected amount of current delivered. As such, I expect the peak power to be $P = IV = (0.354A) * (2.79V) = 0.988W$.

We find that DC coupling capacitors have the effect of preventing DC signals from passing across them and of allowing most AC signals with frequencies greater than 0Hz to pass through. As a result, we employed these coupling capacitors freely in our circuit in both stages of the two-stage MOSFET amplifier so as to remove the DC offset associated with each set of biasing resistors. Thus, the output waveforms across the 8Ω speaker would only be AC in nature with no significant DC portion.

The expected voltage will be our 12V input voltage that would derive from our power supply. Since each filter is expected to deliver 0.176A, this implies, by the principle of homogeneity, that the total amount of current deliver to both of the 8Ω loads will simply be twice the amount delivered to one of them, i.e. 0.177A * 2 = 0.354A. Thus, the expected amount of power will be given by $P = IV = (0.354A) * (12V) = 4.25W$, which is an incredible amount that our own power supply most likely will not be able to deliver. As a result, we used the external power supplies provided in lab to meet this high DC power requirement.

Section 12: Prelab Simulations

For this part of the project, we used BS170 N-Channel Enhancement MOSFETs to serve as the amplifier component largely because they are able to produce a high output gain. We note that the MOSFET has a threshold voltage of $V_T = 2.1V$, so we would have to bias the gate voltage above that to turn the MOSFET on in the first place.

Figure 12.30: *DC Sweep Simulation of I_D vs. V_{GS} Curve*

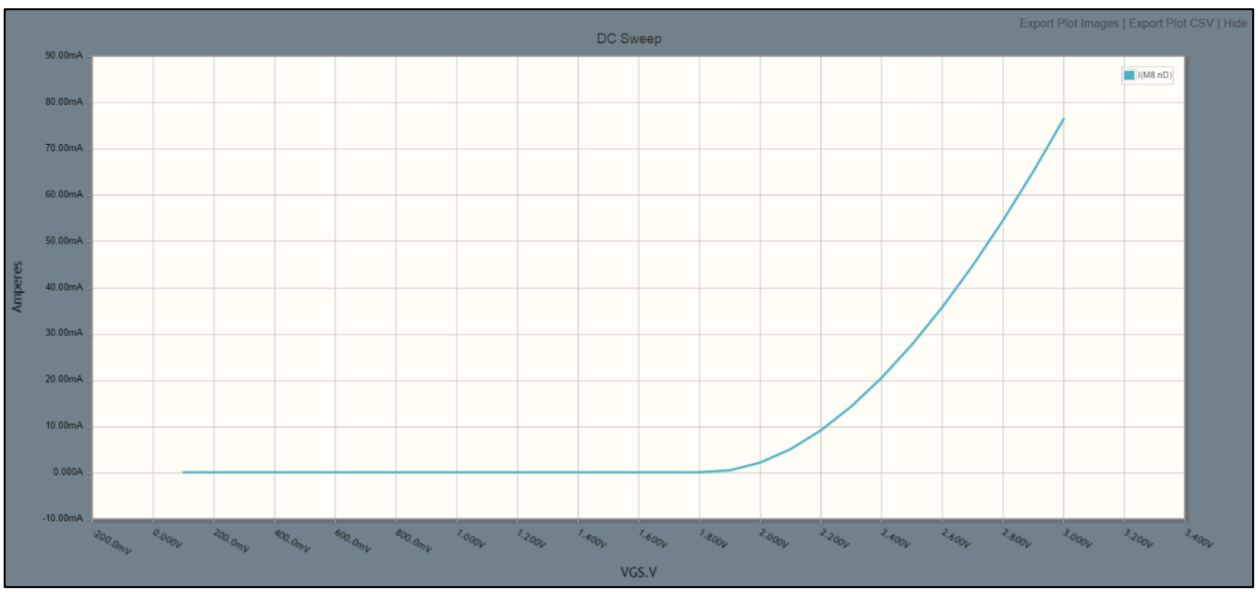


Figure 12.31: *DC Simulation of Transistor at Calculated Bias Point*

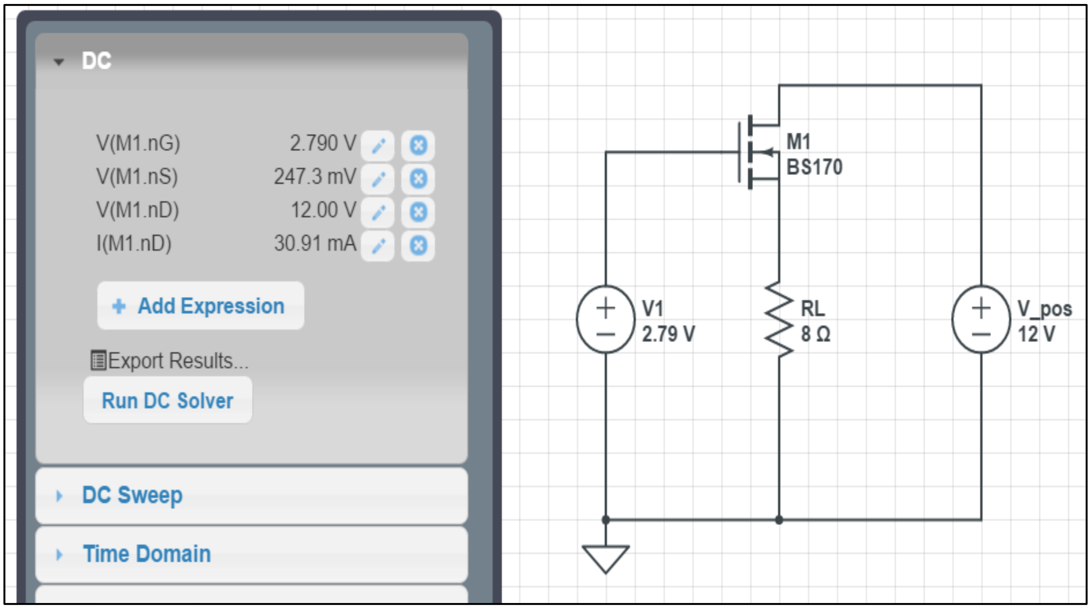


Figure 12.32: *DC Simulation of Transistor with Bias Resistors*

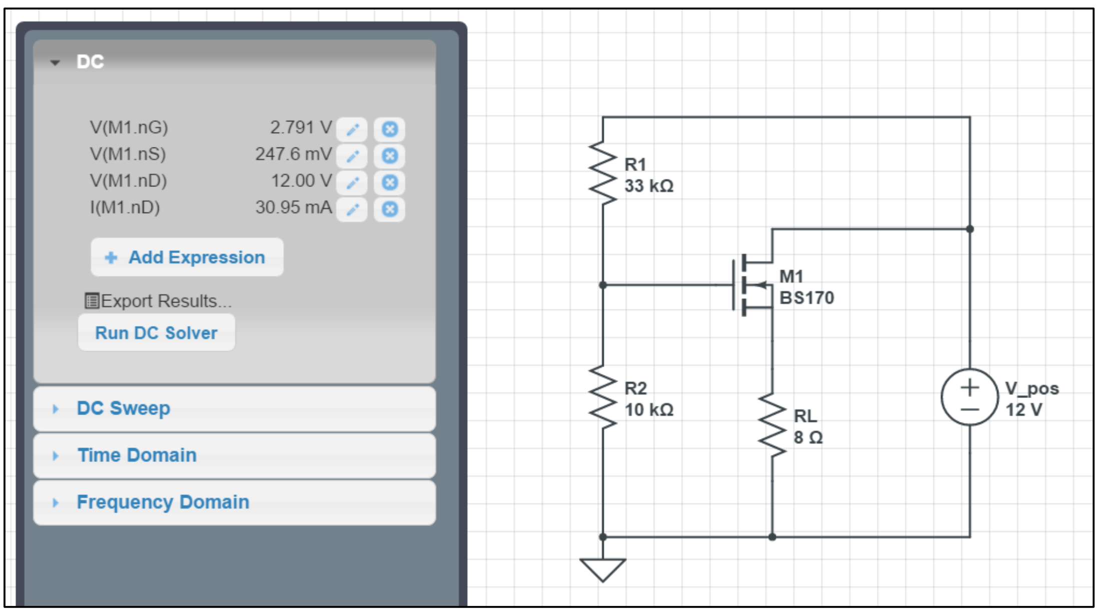


__Figure 12.33__: Bode Plot of Amplifier Component

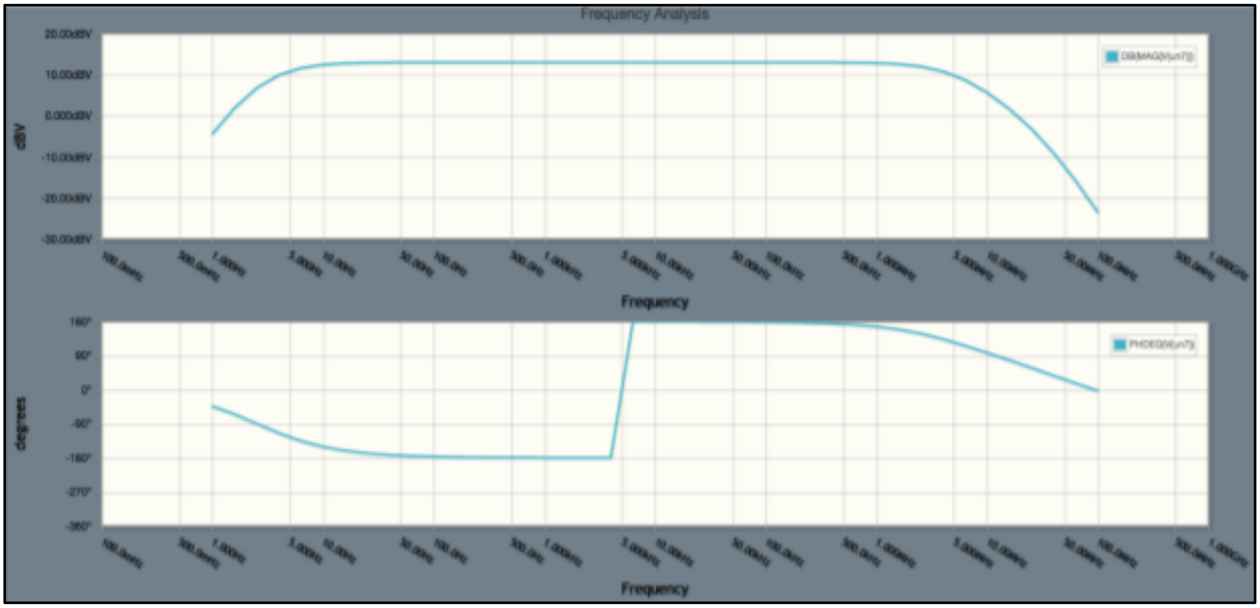


__Figure 12.34__: Time-Domain Simulation of Peak Power at Center Frequency of Bass Filter

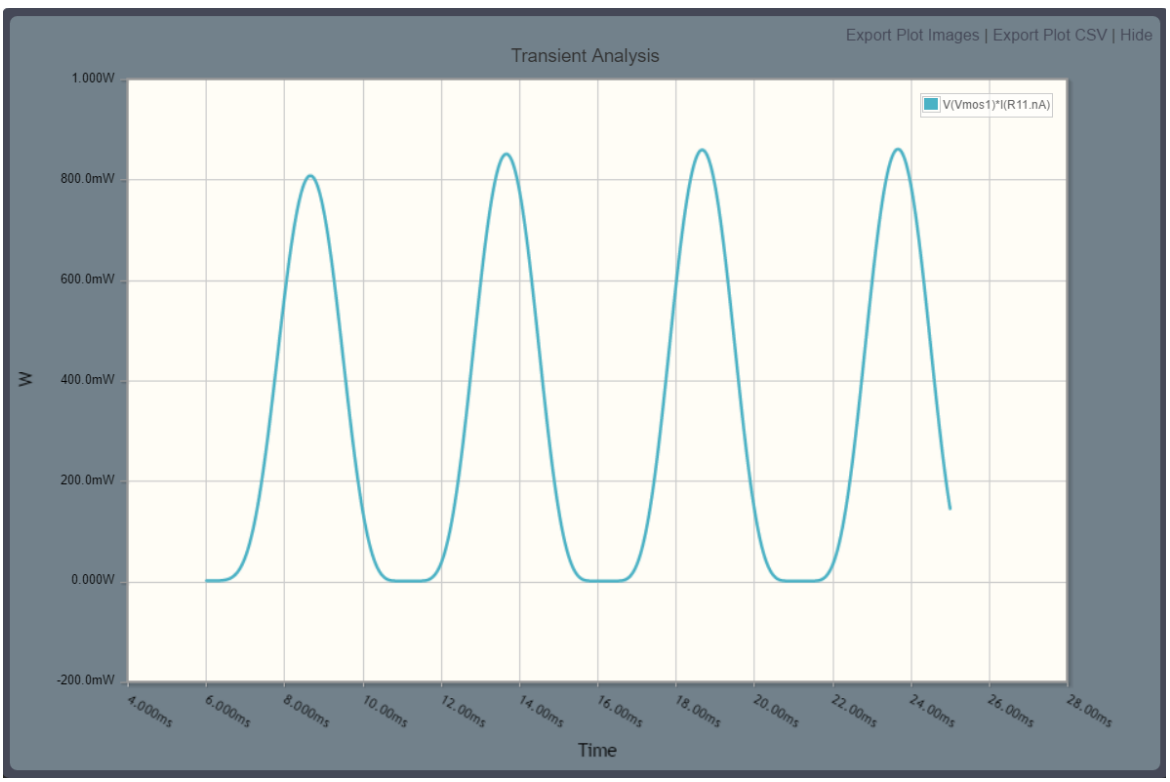
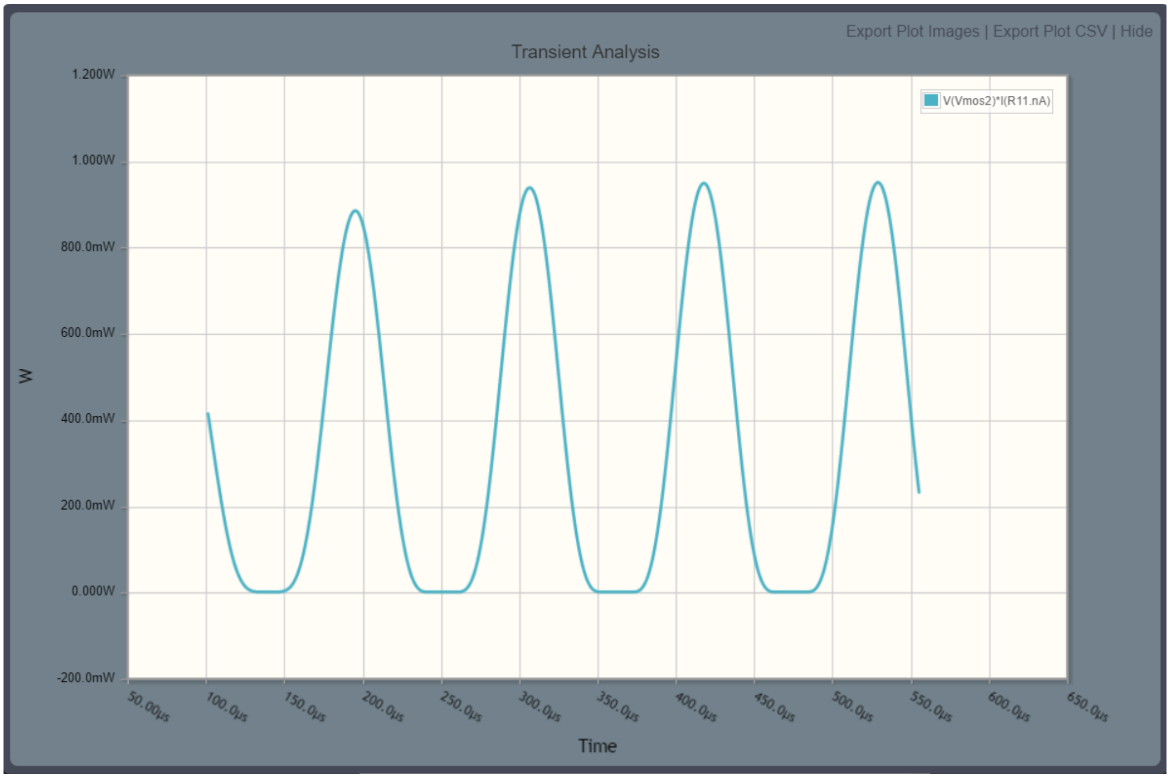


Figure 12.35: *Time-Domain Simulation of Peak Power at Center Frequency of Treble Filter*



Section 13: Experimental Setup

In order to guarantee the orderly functionality of our amplifier component, we employed lab equipment that would assist us in designing and debugging the two MOSFET amplifiers that we were to create. Just as before, we used an oscilloscope with probes, DMM, function generator, and external power supply. However, we initially employed another power supply for the express purpose of testing and debugging. In addition, the current passing through the second MOSFET in the two-stage system often became quite high, so to mitigate its impact, we put three 47Ω resistors in parallel after the 8Ω speaker output. This configuration reduced the magnitude of current flowing through the MOSFET and speaker output. Hence, the details of the equipment put into use in the amplifier component of the project is given by **Table 5.2** below.

Table 13.2 *Lab Equipment of Amplifier Component with Specifications*

Equipment	Manufacturer & Model Number (if given)
N-Channel Small Signal Enhancement MOSFET	BS170
Single Output DC Power Supply 0-30V, 5A	Agilent U8002a

In **Figures 13.36** and **13.37** shown below for the bass and treble filters respectively, we notice the setup of the amplifier component with respect to the rest of the circuit:

Figure 13.36: *Schematic of Amplifier Component Connected to the Bass Filter Component*

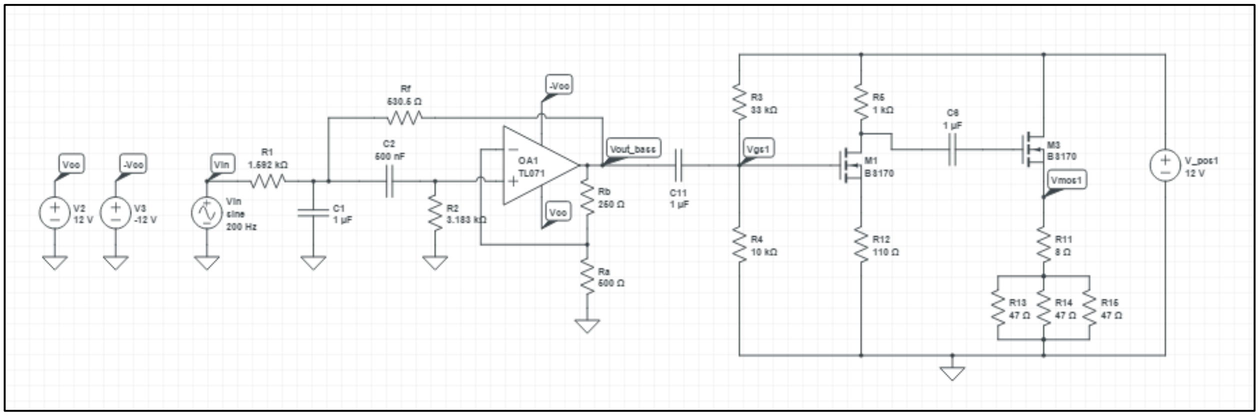
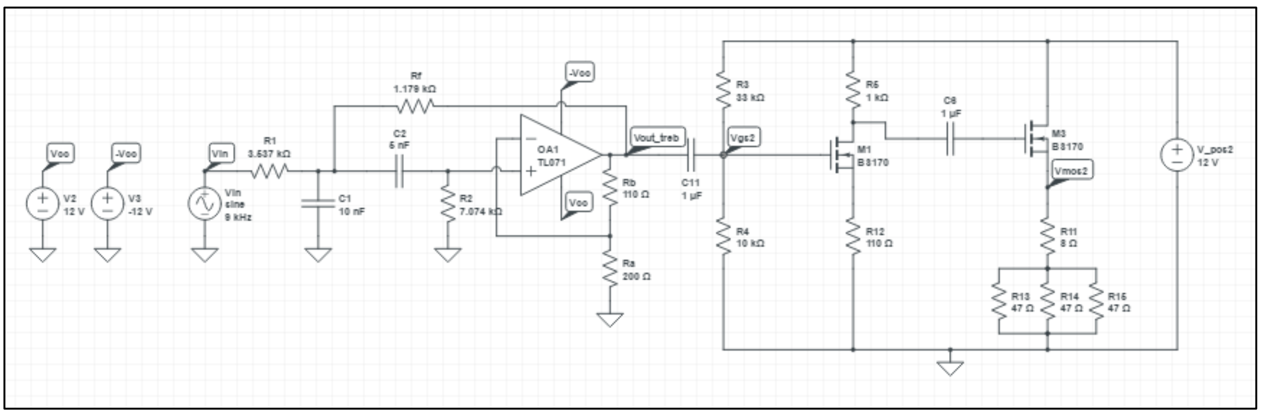


Figure 13.37: *Schematic of Amplifier Component Connected to the Treble Filter Component*



At the rightmost side of either schematic, we see the location for the single output DC power supply to power the amplifier component of the circuit. In order for us to measure this power, we used an oscilloscope to acquire the AC voltage waveform and, using Ohm's Law, deduced the power to be simply $P_{8\Omega} = \frac{V_{8\Omega}{}^2}{8\Omega}$. It was also instructive for us to measure the current waveform across the 8Ω load so that we had an idea as to how much current the current-gain MOSFET was able to supply. To verify that these AC voltage and current waveforms were accurate and expected, we compared them to the results of our prelab simulations.

Section 14: Experimental Procedure

Testing our amplifiers required a considerable amount of debugging. For the most part, we used the oscilloscope to get the waveforms that were useful for troubleshooting. In particular, we rigorously tested V_{GS} in each amplifier component to ensure that it was the bias point that we desired, which necessitated getting the voltage waveform at V_{GS} via the oscilloscope. At this point, we also employed the DMM simply to cross-check the oscilloscope to make sure that the DC offset was consistent. Then, after setting each filter component to its center frequency, we sought to maximize the gain which was the voltage across the 8Ω load. Hence, another round of oscilloscope waveforms was in order as we continued to measure the voltage waveforms of Vmos1 and Vmos2 as shown in ***Figures 13.36*** and ***13.37*** above. Indeed, we were looking for whether there was a noticeable gain from the input voltage of $300V_{RMS}$, implying an increase in the amplitude of the exiting signal, for both the bass and treble speakers.

Section 15: Experimental Results

In this section, we simply show the frequency responses of the voltage waveforms across the 8Ω speaker. As noted previously, we obtained these graphs through the use of an oscilloscope.

Figure 13.38: *Graph of Voltage across 8Ω Load at Center Frequency of Bass Filter*

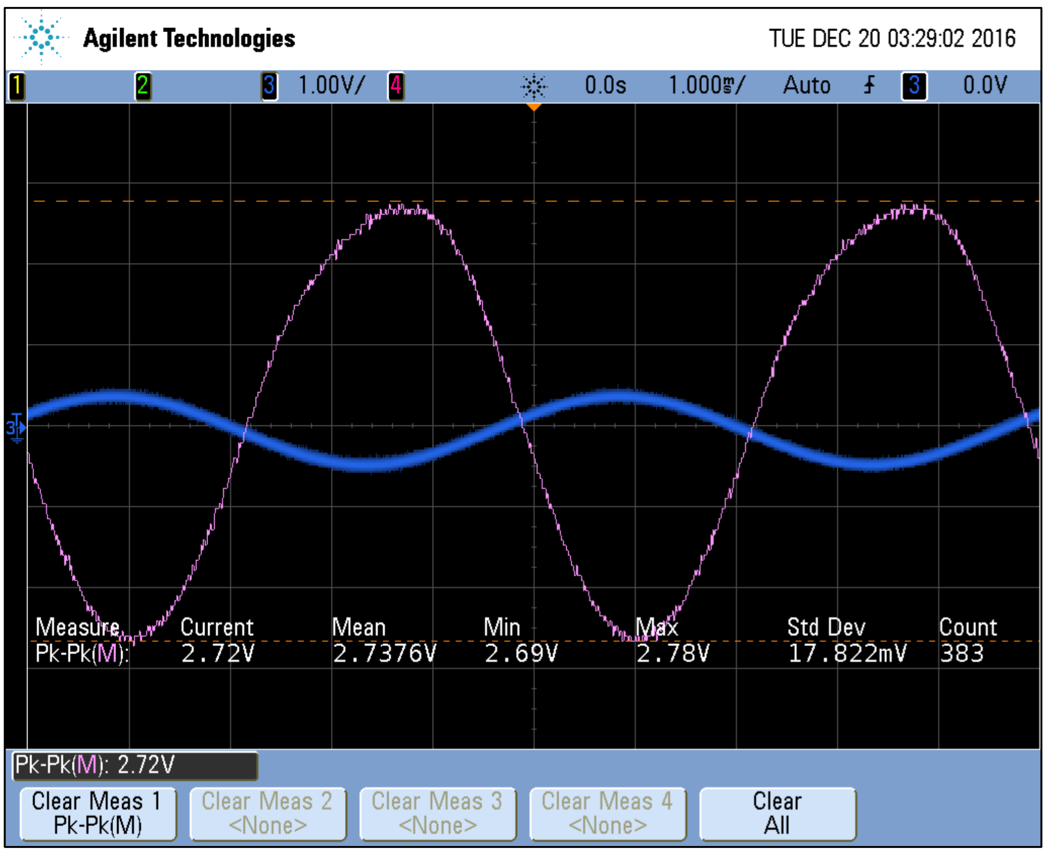
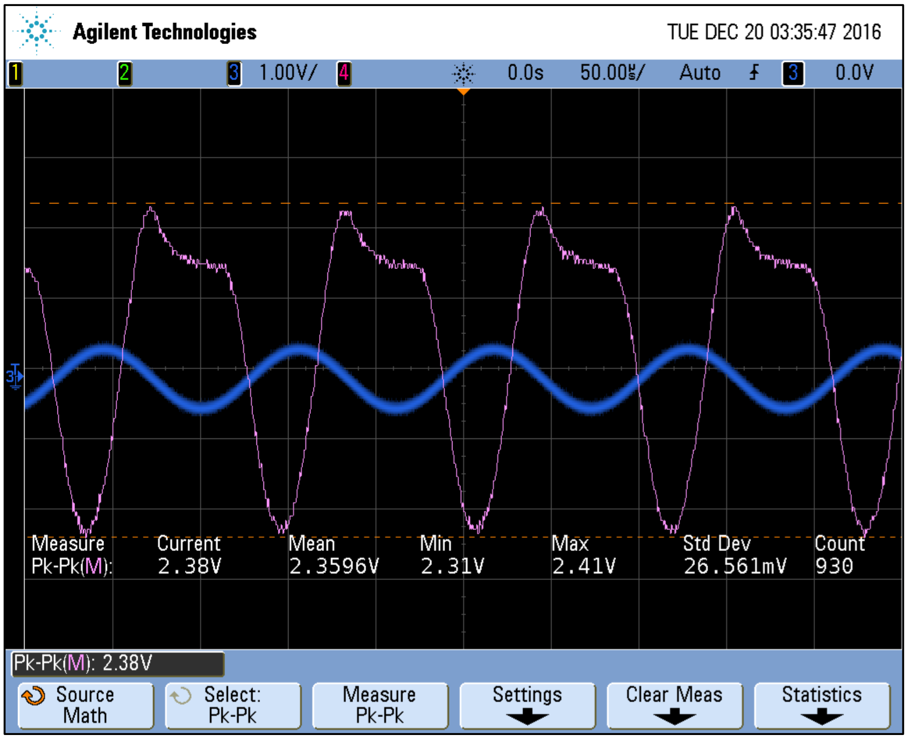


Figure 13.39: *Graph of Voltage across 8Ω Load at Center Frequency of Treble Filter*



Section 16: Error Analysis

For the amplifier component, our most common source of error was overheating the rightmost MOSFET. Initially, we constantly replaced it in the circuit, but we eventually came up with the solution of a heat sink which worked well for the most part. Furthermore, we inserted three parallel 47Ω resistors in series with the speaker to reduce the current through the loop. Nevertheless, the former frying of the transistor could have adversely affected the other components of the circuit without us realizing, such as burning through the resistors and capacitors. As evident in ***Figure 13.38*** and ***Figure 13.39***, the graph of the voltage across the speaker at the center frequency of the bass filter is in nearly perfect sinusoidal form, whereas at the center frequency of the treble filter, the curve is not so clean. We realized that when we challenged the treble filter, the DC bias pushed the signal past the saturation point, resulting in clipping. This was in part due to the nonzero gain of the treble filter, but fortunately, this clipping was not apparent when we played our music through the iPhone 6s. Furthermore, the sound was relatively quieter compared to some others' final projects, although the signal was certainly amplified. To achieve a louder amplification, we could have implemented a preamp at the beginning of the amplifier component to amplify the voltage without changing the current. This would result in greater power across the speaker and a louder output. Nonetheless, the signal was indeed amplified and of high quality with barely if any noise.

Section 17: Conclusion

Overall, we were very successful in implementing the amplifier component, as we outputted a clear and strong signal. In retrospect, this signal could have been louder with the addition of a preamp; however, for the ends of this project, it was more than sufficient. Furthermore, we adeptly ensured that there was minimal noise. The main adverse outcome was the mild clipping of signal at the center frequency of the treble filter due to the gain in the treble filter. Looking back, we realize that perfecting the filter is crucial to the rest of the project as its slight imperfections are reflected even in other components. Ultimately however, the amplifier component combined well with the filter to yield in high quality, robust sound.

Part III: Final Integration

Section 18: Breakdown of Specification

In this part of the report, we connect our power supply to both TL071 OpAmps and both MOSFET amplifiers, effectively powering these active components. Thus, instead of relying upon an external power supply, which is largely unaffected by the size of its load, we use our power supply and therefore must take into account its power budget. This power budget tells us whether it is possible to adequately support our filter and amplifier components as well as how much power will be delivered to the 8Ω speaker in each of the output channels. Furthermore, the AC/DC power supply serves as a useful replacement for the Triple Output DC Power Supply.

Section 19: Prelab Hand Calculations

The filters that connected to the power supply essentially represented the load. To calculate individual components of the power supply (built for the midterm project), we performed the following hand calculations.

First, we found the turn ratio of the transformer: $Turn\ Ratio = \frac{N_{Primary}}{N_{Secondary}} = 6.389$, and understanding that $\frac{V_{input}}{V_{output}} = \frac{N_{Primary}}{N_{Secondary}}$, we found that V_{output} comes out to be $18V_{RMS}$. For the rectifier sub-component, we implemented a full-wave rectifier whose bottom portion received -$9V_{RMS}$ or -12.73V ($-9 * \sqrt{2}$) and whose top portion received +$9V_{RMS}$ or 12.73V ($9 * \sqrt{2}$). Across each half of the rectifier, there is a characteristic voltage drop (V_f) across the diodes of 0.6V given in Ulaby's textbook *Circuits: Third Edition* for the diodes we are using in this laboratory; thus the net voltage drop across both rectifiers is 1.2 V. Next, we built a smoothing filter with the objective of maintain a ripple voltage within

2% of the 12.73V input given in the previous segment. The full-wave rectifier doubles the signal's frequency to 120Hz, so the period $T_{rect} = \frac{1}{120\ Hz} = 8.33\ ms$. To find the capacitor's charge and discharge time, we calculated: $\tau_{up} = \frac{T_{rect}}{12} = 0.69\ ms$ and $\tau_{down} = 12 * T_{rect} = 100ms$. The capacitance is given by $C = \frac{\tau_{up}}{2*R_D}$ where $R_D = 0.042\ \Omega$ and hence the capacitance is 8.267 mF, but by the process of experimentation, we arrived at a 100µF capacitance. Since $R_L = \frac{\tau_{down}}{C}$, the load resistance had to be *at least* 12.096Ω, but we ultimately used a much greater value for optimal results. Lastly, we built a voltage regulator by adding a zener diode, which ensured a maximum polarity of 12V across it. Unfortunately, due to the extremely heavy load, ultimately enough power was not delivered and less current was actually delivered than when simulated individually.

Section 20: Prelab Simulations

Figure 20.40: *Complete Schematic of Final Project*

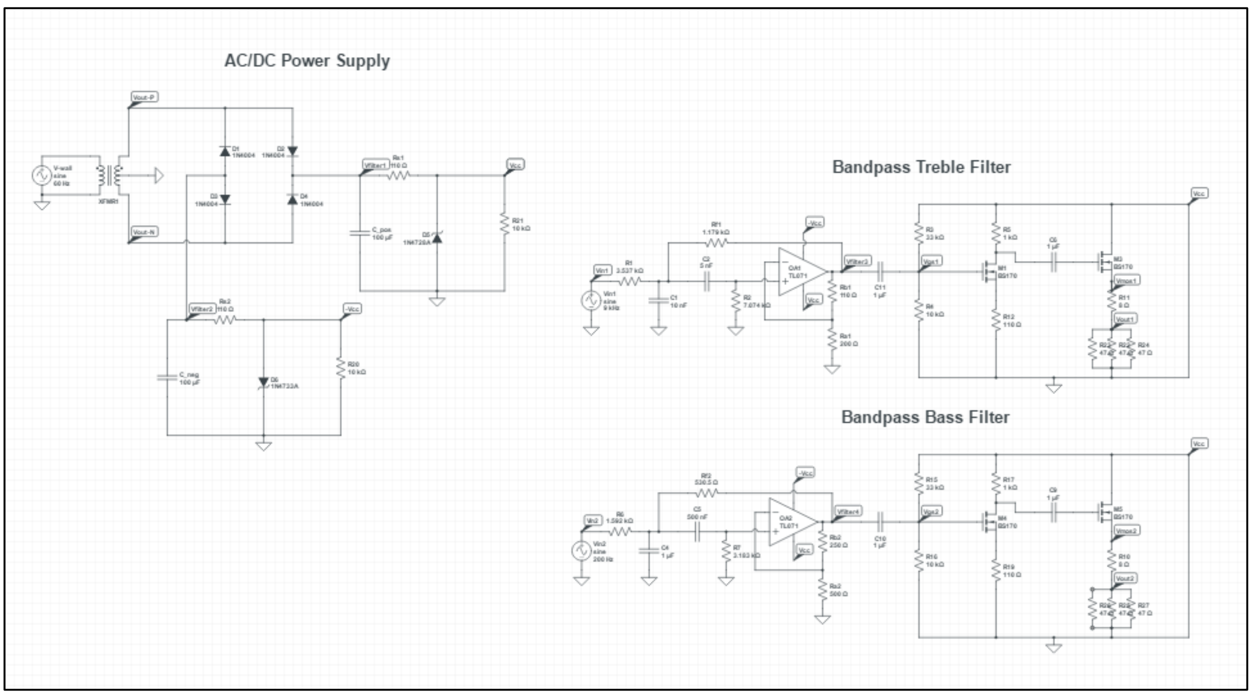


Figure 20.41: *Time-Domain Simulation of Load Output at Treble Filter Center Frequency*

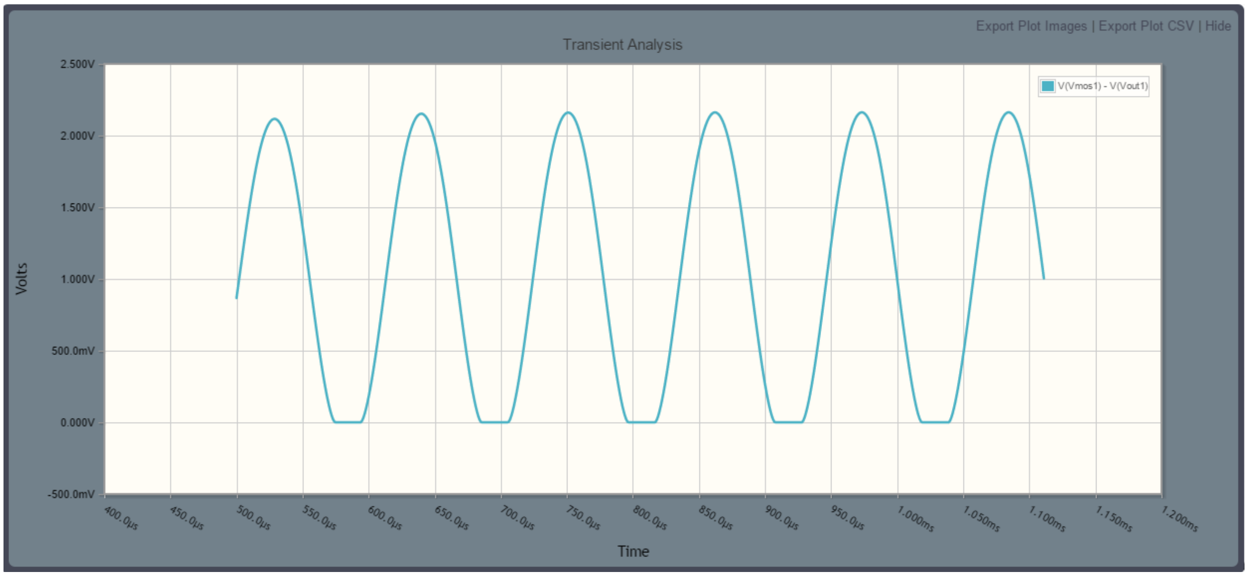
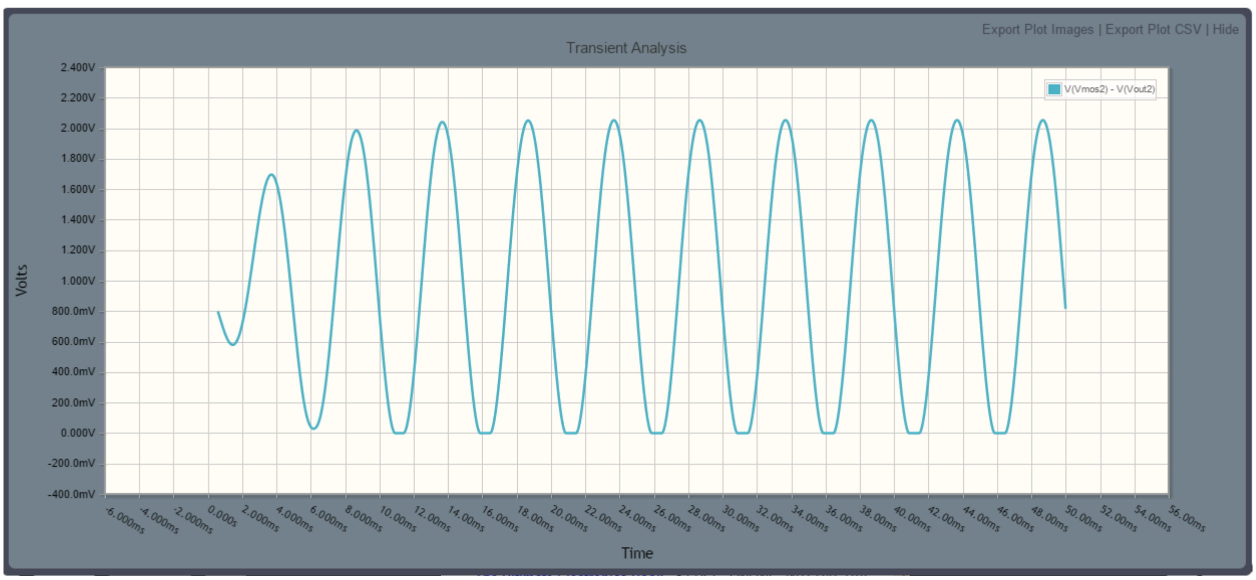


Figure 20.42: *Time-Domain Simulation of Load Output at Bass Filter Center Frequency*



Section 21: Experimental Setup

Here, the only change to our experimental setup is the implementation of our own AC/DC power supply instead of using the Triple Output DC Power Supply provided by the lab. As such, the lab equipment will be the same as before – documented in **Table 5.1** and **13.2** – but for the sake of completion, we shall also list the new equipment associated with the AC/DC power supply.

Table 21.3: *Lab Equipment of AC/DC Power Supply with Specifications*

Equipment	Manufacturer & Model Number (if given)
$115V_{RMS}$ - $18V_{RMS}$ Center-Tapped Transformer	N/A

Circuit Breaker	N/A
Zener Diodes	1N4742
General Purpose Rectifier Diodes	1N4004

In **Figure 20.40**, we can observe the interaction of all the subcomponents in the final project. We see that the AC/DC power supply is used to power the ±Vcc inputs of both TL071 OpAmps, but for the sake of better organization and easier debugging, we initially employed the Agilent Single Output power supply for each of the MOSFET amplifiers. Ultimately, we did use the AC/DC power supply to power both MOSFET amplifiers and have shown those results in later sections. To verify the output response of each of the 8Ω loads, we primarily used the oscilloscope to acquire each waveform. At one point, we did use the DMM to get the magnitude of the output voltage response and thus ensure consistency with our results from the oscilloscope. Moreover, both tools were very useful for each subcomponent. In particular, the DMM proved useful for obtaining the DC offset associated with each amplifier component; with that information, we could confirm the oscilloscope readings and simulation results.

Section 22: Experimental Procedure

In order to ensure that our power supply was working properly when interfaced with our filters and MOSFETs, we used the DMM to measure the output voltage of the power supply. It did indeed read approximately 12V (11.56V), so it was working well. Nevertheless, the power supply was not able to supply enough current when we connected it to the rest of the circuit, as the audio was noticeably quiet and distorted as compared to when we used the power supplies stationed in Detkin laboratory. The power supply was already assembled and ready to go from our work in the midterm project, so in the procedure, we had simply to test it to make sure that it supplied approximately 12V where necessary.

Section 23: Experimental Results

For comparison with our prelab simulations, here we show the oscilloscope readings of the load output at the center frequencies of the bass and treble filters, respectively. They are as follows:

Figure 23.43: *Time-Domain Graph of Load Output at Bass Filter Center Frequency*

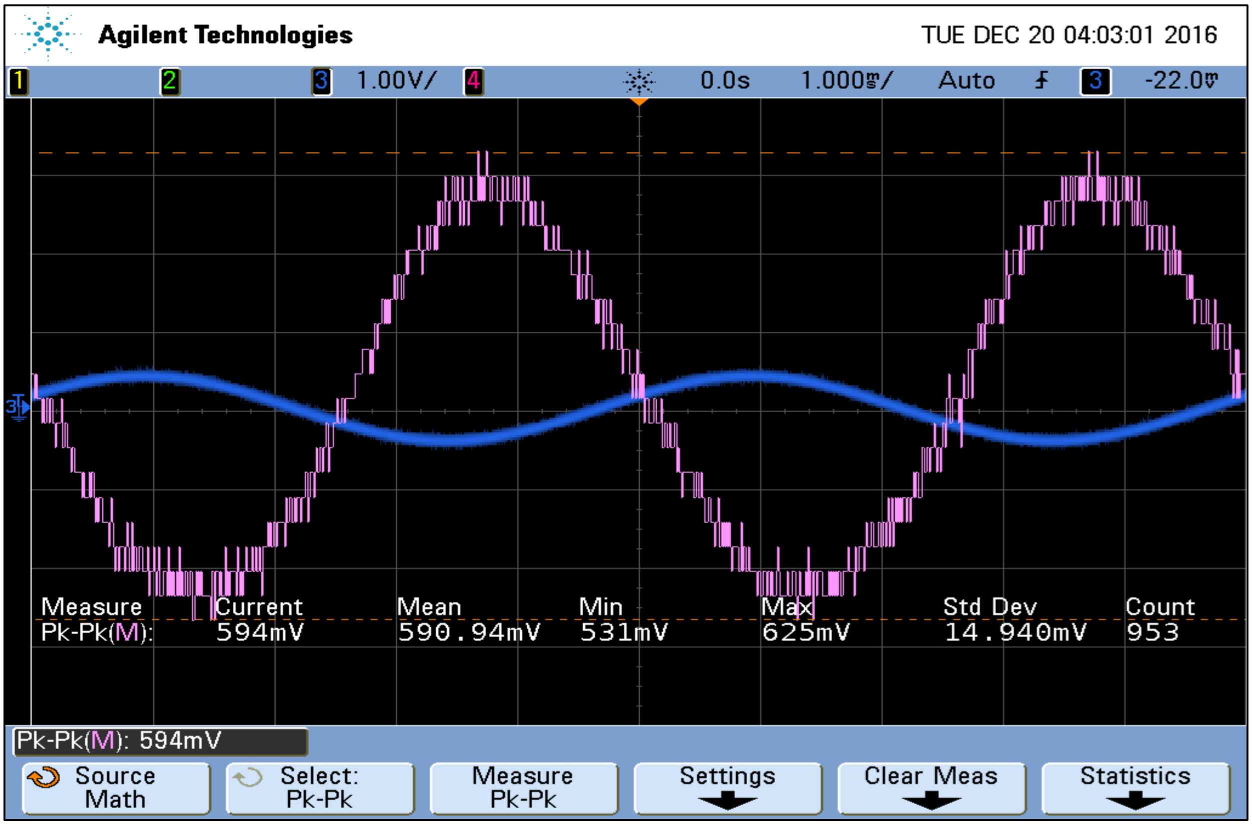
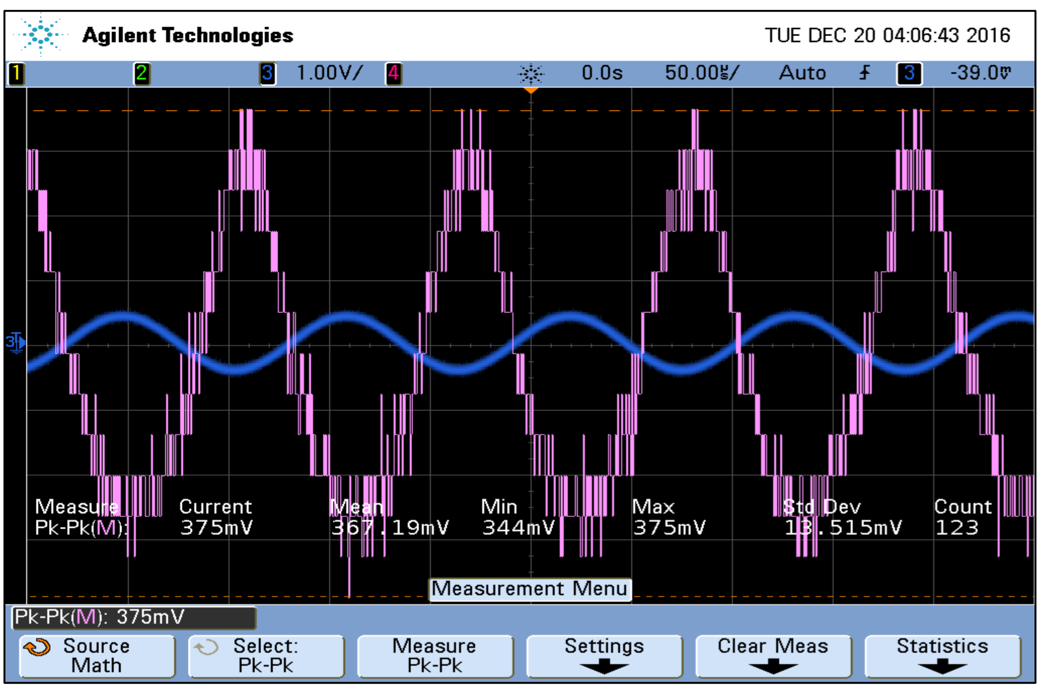


Figure 23.44: *Time-Domain Graph of Load Output at Treble Filter Center Frequency*



Section 24: Error Analysis

Our power supply was ultimately assembled for a smaller load than the combination of our filter and amplifier. Consequently, while the outputted music was somewhat audible when inputting signal via an iPhone 6s, there was a substantial amount of noise as the power supply was just not able to provide enough current to such a heavy load. Thus, if we were able to decrease the size of the load or add more passive components to the power supply circuitry such that it could deliver enough current, we would likely minimize this error and produce the same output as when we used the power supplies stationed in Detkin laboratory.

Section 25: Final Conclusion

Ultimately, our audio docking station was a success, as we aptly built original bass and treble filters and amplifiers accompanied by a power supply to yield in high quality, amplified music, given an input signal. A major takeaway from this project was understanding the extent to which different components of a circuit can affect each other: for example, the imperfections of our treble filter (5.2dB gain) were reflected in the voltage clipping across the speaker while we were completing the amplifier segment, and the power supply worked flawlessly without a load but was noticeably insufficient upon combining it with the rest of the audio docking station. Moving forward in our circuit building futures, we will make sure to understand individual component-to-component interactions at a deeper level. Nonetheless, we learned a great deal about the applications of filters and transistors through this final project, and accomplished creating our very own audio docking station.

Appendix A: Hand Calculations of Filter Component

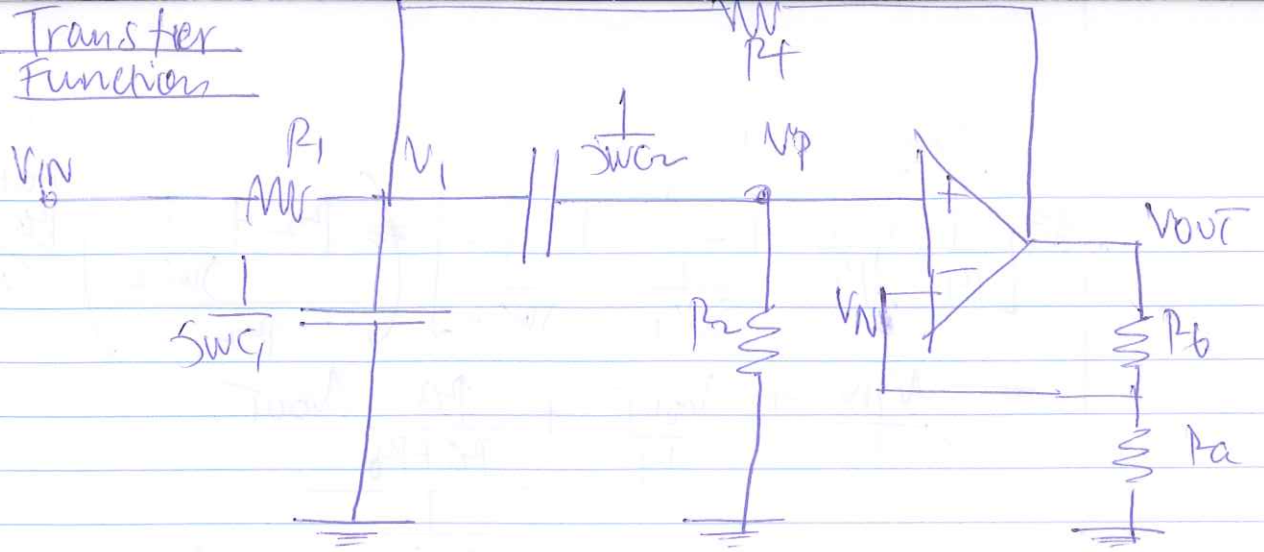
Transfer Function

Order of Filters

Cutoff & Center Frequencies, Bandwidth, and Selectivity

The Appendix A has been merged as a PDF.

Transfer Functions



$$V_N = \frac{R_a}{R_a + R_b} V_{OUT}$$ (Voltage Division)

$$V_p = V_N = \frac{R_a}{R_a + R_b} V_{OUT} \quad (1)$$

$$\cancel{V_1}\; V_p = \frac{R_2}{R_2 + \frac{1}{j\omega C_2}} V_1$$ (Voltage Division)

$$\therefore V_1 = \frac{R_2 + \frac{1}{j\omega C_2}}{R_2} V_p = \frac{R_2 + \frac{1}{j\omega C_2}}{R_2} \frac{R_a}{R_a + R_b} V_{OUT} \quad (2)$$

At V_1 by KCL,

$$\frac{V_1 - V_{IN}}{R_1} + \frac{V_1}{\frac{1}{j\omega C_1}} + \frac{V_1 - V_P}{\frac{1}{j\omega C_2}} + \frac{V_1 - V_{OUT}}{R_f} = 0$$

$$\cancel{V_1}\; V_1\left[\frac{1}{R_1} + \frac{1}{\frac{1}{j\omega C_1}} + \frac{1}{\frac{1}{j\omega C_2}} + \frac{1}{R_f}\right] = \frac{V_{IN}}{R_1} + \frac{V_P}{\frac{1}{j\omega C_2}} + \frac{V_{OUT}}{R_f}$$

Using ① and ②,

$$\left[\frac{1}{R_1}+\frac{1}{R_f}+\frac{1}{\frac{1}{j\omega C_1}}+\frac{1}{\frac{1}{j\omega C_2}}\right]\left(\frac{R_2+\frac{1}{j\omega C_2}}{R_2}\right)\frac{\frac{R_a}{R_a+R_b}}{\cancel{\frac{R_a+R_b}{R_a}}}V_{OUT}$$

$$=\frac{V_{IN}}{R_1}+\frac{V_{OUT}}{R_f}+\frac{\frac{R_a}{R_a+R_b}}{\frac{1}{j\omega C_2}}V_{OUT}$$

$$\therefore V_{OUT}\left[\left(\frac{1}{R_1}+\frac{1}{R_f}+j\omega C_1+j\omega C_2\right)\left(1+\frac{1}{j\omega R_2C_2}\right)\left(\frac{\frac{R_a}{R_a+R_b}}{\cancel{\frac{R_a+R_b}{R_a}}}\right)-\frac{1}{R_f}-\left(\frac{R_a}{R_a+R_b}\right)j\omega C_2\right]=\frac{V_{IN}}{R_1}$$

let $\frac{R_a}{R_a+R_b}=x$

$$\Rightarrow V_{OUT}\left[\left(\frac{1}{R_1}+\frac{1}{R_f}+j\omega C_1+j\omega C_2\right)\left(1+\frac{1}{j\omega R_2C_2}\right)x-\frac{1}{R_f}-xj\omega C_2\right]$$

$$=\frac{V_{IN}}{R_1}$$

$$\Rightarrow V_{OUT}\left[\frac{1x}{R_1}+\frac{1x}{R_f}+j\omega C_1x+\cancel{j\omega C_2x}+\frac{1x}{j\omega R_1R_2C_2}+\frac{1x}{j\omega R_fR_2C_2}+\frac{C_1x}{R_2C_2}+\frac{1x}{R_2}-\frac{1}{R_f}-\cancel{xj\omega C_2}\right]=\frac{V_{IN}}{R_1}$$

~~Dividing numerator and denominator by x in LHS,~~
Factoring out x,

$$x V_{OUT}\left[\frac{1}{R_1} + \frac{1}{R_f} + j\omega C_1 + \frac{1}{j\omega R_1 R_2 C_2} + \frac{1}{j\omega R_2 R_f C_2} + \frac{C_1}{R_2 C_2} + \frac{1}{R_2} - \frac{1}{x R_f}\right] = \frac{V_{IN}}{R_1}$$

$$H(\omega) = \frac{V_{OUT}}{V_{IN}} = \frac{\frac{1}{x R_1}}{\frac{1}{R_1} + \frac{1}{R_f} + j\omega C_1 + \frac{1}{j\omega R_1 R_2 C_2} + \frac{1}{j\omega R_2 R_f C_2} + \frac{C_1}{R_2 C_2} + \frac{1}{R_2} - \frac{1}{x R_f}}$$

$$= \frac{\frac{R_a + R_b}{R_a}\frac{1}{R_1 C_1}}{\frac{1}{R_1 C_1} + \frac{1}{R_f C_1} + j\omega + \frac{1}{j\omega R_1 R_2 C_1 C_2} + \frac{1}{j\omega R_2 R_f C_1 C_2} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} - \frac{R_a + R_b}{R_a R_f C_1}}$$

$$= \frac{\left(1 + \frac{R_b}{R_a}\right)\frac{1}{R_1 C_1}}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} - \frac{R_b}{R_a R_f C_1} + j\omega + \frac{1}{j\omega R_1 R_2 C_1 C_2} + \frac{1}{j\omega R_2 R_f C_1 C_2}}$$

$$= \frac{\left(1 + \frac{R_b}{R_a}\right) \frac{1}{R_1C_1} \quad j\omega R_1R_2R_fC_1C_2}{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_aR_fC_1}\right) j\omega R_1R_2R_fC_1C_2 - \omega^2 R_1R_2R_fC_1C_2}$$

$$+ \quad R_f + R_1$$

$$= \frac{\left(1 + \frac{R_b}{R_a}\right) \frac{j\omega}{R_1C_1}}{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_aR_fC_1}\right)\omega - \frac{\omega^2}{j} + \frac{R_f + R_1}{j R_1R_2R_fC_1C_2}}$$

$$= \frac{\left(1 + \frac{R_b}{R_a}\right) \frac{\omega}{R_1C_1}}{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_aR_fC_1}\right)\omega + j\left(\omega^2 - \frac{R_1 + R_f}{R_1R_2R_fC_1C_2}\right)}$$

Magnitude

$$M(\omega) = \frac{\left(1+\frac{R_b}{R_a}\right)\frac{\omega}{R_1C_1}}{\sqrt{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)^2\omega^2+\left(\omega^2-\frac{R_1+R_f}{R_1R_2R_fC_1C_2}\right)^2}}$$

Center Frequency

$$\omega_0 = \sqrt{\frac{R_1+R_f}{R_1R_2R_fC_1C_2}} \text{ rad/s}$$

Gain at ω_0

$$M(\omega_0) = \frac{\left(1+\frac{R_b}{R_a}\right)\frac{\omega_0}{R_1C_1}}{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)\omega_0}$$

$$= \frac{\left(1+\frac{R_b}{R_a}\right)\frac{1}{R_1C_1}}{\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}} \quad V/V$$

Corner Frequencies w_c

At w_c, $M(w_c) = \frac{1}{\sqrt{2}} M(w_0)$

$$\frac{\left(1+\frac{R_b}{R_a}\right)\frac{w_c}{R_1C_1}}{\sqrt{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)^2 w_c^2+\left(w_c^2-\frac{R_1+R_f}{R_1R_2R_fC_1C_2}\right)^2}} = \frac{1}{\sqrt{2}}\frac{\left(1+\frac{R_b}{R_a}\right)\frac{1}{R_1C_1}}{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)}$$

Cancelling same terms and squaring both sides,

$$\frac{w_c^2}{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)^2 w_c^2+\left(w_c^2-\frac{R_1+R_f}{R_1R_2R_fC_1C_2}\right)^2} = \frac{1}{2\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)^2}$$

(3) $$w_c^2\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}-\frac{R_b}{R_aR_fC_1}\right)^2 = \left(w_c^2-\frac{R_1+R_f}{R_1R_2R_fC_1C_2}\right)^2$$

Taking square root and keeping both sides +ve, in (3)

$$\omega_c^2 - \omega_c\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right) - \frac{R_1+R_f}{R_1R_2R_fC_1C_2} = 0$$

$$\omega_c = \frac{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right) \pm \sqrt{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}-\frac{R_b}{R_aR_fC_1}\right)^2 + \frac{4(R_1+R_f)}{R_1R_2R_fC_1C_2}}}{2}$$

Only term that is +ve is

$$(\omega_{c2})\quad \omega_c = \frac{1}{2}\left[\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right) + \sqrt{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right)^2 + 4\,\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}\right]$$

Now taking square root and making one side negative in (3),

$$\omega_c^2 + \omega_c\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{R_b}{R_aR_fC_1}\right) - \frac{R_1+R_f}{R_1R_2R_fC_1C_2} = 0$$

$$\omega c = \frac{-\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_G R_f C_1}\right) \pm \sqrt{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_G R_f C_1}\right)^2 + 4\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}}{2}$$

only positive term is

wc1 $$\omega c = \frac{-\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_G R_f C_1}\right) + \sqrt{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_G R_f C_1}\right)^2 + 4\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}}{2}$$

<u>Bandwidth</u> $= \omega c_2 - \omega c_1$

$$= \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_G R_f C_1}$$

$$Q = \frac{\omega_0}{B} = \frac{\sqrt{\frac{R_1+R_f}{R_1R_2R_fC_1C_2}}}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{R_b}{R_G R_f C_1}}$$