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1) الف) poisson $= P(x=k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

اگر فرض کنیم متغیر x که تصادفی است، نشان دهنده تعداد وقوع یک رویداد در یک بازه زمانی مشخص است.

این عبارت دقیقاً با تابع چگالی احتمال نمایی : $f(x,\lambda) = \lambda e^{-\lambda x}$

توزیع نمایی برابر است. (با توزیع λ) ⟸ پواسون یک حالت خاص از نمایی می‌تواند باشد

ب) $P(y|x,\theta) = \dfrac{\lambda^y}{y!}e^{-\lambda} = \dfrac{e^{y\theta' x}e^{-e^{\theta' x}}}{y!}$

$$P(y_1 \dots y_m | x_1 \dots x_m; \theta) = \prod_i^m \frac{e^{y_i\theta' x_i} e^{-e^{\theta' x_i}}}{y_i!}$$

$$\xrightarrow{\ln} L(\theta|x,y) = \sum_i^m (y_i\theta' x_i - e^{\theta' x_i})$$

$$\rightarrow \frac{\delta L(\theta|x,y)}{\delta\theta} = 0 \rightarrow$$ گرادیانت دیسنت

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(2) (الف) $\phi = \frac{1}{m} \sum 1\{y^i = 1\} = \frac{2}{5} = \frac{4}{10} = 0{,}4$

$$\mu_0 = \frac{\sum 1\{y^i = 0\} x^i}{\sum 1\{y^i = 0\}} = 1 \qquad \mu_1 = \frac{\sum 1\{y^i = 1\} x^i}{\sum 1\{y^i = 1\}} = \frac{11}{2} = 5{,}5$$

$$\text{variance} = \frac{1}{m} \sum (x - \mu_{y^i})^2 = \frac{1}{5}(0{,}25 + 0{,}25 + 1 + 1) = 0{,}5$$

(ب) $P(x|y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_y)^2}{2\sigma^2}}$

$$P(y) = \phi^y (1-\phi)^{1-y}$$

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \left(e^{-\frac{(x-\mu_0)^2}{2\sigma^2}} (1-\phi) + e^{-\frac{(x-\mu_1)^2}{2\sigma^2}} \phi \right)$$

$$P(y|x) = \frac{1}{P(x)} \times P(x|y) P(y)$$

$$P(y|x) = \frac{e^{-\frac{(x-\mu_y)^2}{2\sigma^2}} \phi^y (1-\phi)^{1-y}}{e^{-\frac{(x-\mu_0)^2}{2\sigma^2}} (1-\phi) + e^{-\frac{(x-\mu_1)^2}{2\sigma^2}} \phi}$$

$$P(y|x) = \frac{e^{-\left(\frac{(x-\mu_y)^2}{1}\right)} (0{,}4)^y (0{,}6)^{(1-y)}}{0{,}6\, e^{(x-1)^2} + 0{,}4\, e^{-(x-5{,}5)^2}}$$

$$P(y|x=3{,}5) = \frac{e^{-(3{,}5-\mu_y)^2} (0{,}4)^y (0{,}6)^{1-y}}{0{,}6\, e^{-(2{,}5)^2} + 0{,}4\, e^{-9}}$$

$$P(y|x=3{,}5) = 828\, (0{,}4)^y (0{,}6)^{(1-y)} e^{-(3{,}5-\mu_y)^2}$$

PRESTIGE

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③ 1- calculate class priors: $P(Y=Y) = \frac{4}{9}$

$P(Y=N) = \frac{5}{9}$

$x = (T, F, 1, 0)$

- $P(a_i|Y)$: $P(a_1 = T|Y) = \frac{3}{4}$

$P(a_1 = F|Y) = \frac{1}{4}$ $\quad$ $P(a_1 = T|N) = \frac{1}{5}$

$P(a_1 = F|N) = \frac{4}{5}$

$P(a_2 = T|Y) = \frac{2}{4} = \frac{1}{2}$ $\quad$ $P(a_2 = F|Y) = \frac{2}{4} = \frac{1}{2}$

$P(a_2 = T|N) = \frac{3}{5}$ $\quad$ $P(a_2 = F|N) = \frac{2}{5}$

for $a_3 \rightarrow$ Gaussian naive Bayes

$P(a_3 = 1 | Y = N) \Rightarrow$

$$\text{Mean} = \frac{8+7+4+5+1}{5} = \frac{25}{5} = 5$$

$$\text{variance} = \frac{3^2 + 2^2 + 1 + 0 + 4^2}{4} = \frac{9+4+1+16}{4} = \frac{30}{4} = 7{,}5$$

$P(a_3 = 1 | Y = Y) \Rightarrow$

$$\text{Mean} = \frac{5+7+3+6}{4} = \frac{21}{4} = 5{,}25$$

$$\text{variance} = \frac{0{,}0625 + 3{,}0625 + 5{,}0625 + 0{,}5625}{3} = \frac{8{,}75}{3} = 2{,}91$$

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$P(a_3=1 \mid Y=N) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$\Rightarrow \frac{1}{\underbrace{\sqrt{2\times 3{,}14\times 7{,}5}}_{0{,}145710}} \times \underbrace{e^{\left(\frac{-(1-5)^2}{2\times 7{,}5}\right)}}_{0{,}344153} = \underline{0{,}05014}$

$P(a_3=1 \mid Y=Y)$

$\Rightarrow \frac{1}{\underbrace{\sqrt{2\times 3{,}14\times 2{,}91}}_{0{,}233923}} \times \underbrace{e^{\left(\frac{-(1-\overset{25}{5{,}1})^2}{2\times 2{,}91}\right)}}_{0{,}06398} \Rightarrow 0{,}01496$

$P(Y=N \mid a) = \frac{\cancel{8}}{9} \times \frac{1}{\cancel{8}} \times \frac{2}{5} \times 0{,}05014 = 0{,}002228$

$P(Y=Y \mid a) = \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{9}_3} \times \frac{1}{2} \times 0{,}01496 = 0{,}00\cancel{7}893$

با توجه به احتمالات به دست آمده ← احتمال کلاس Y بیشتر از N به دست آمد.