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سیوا وناظر - 810899074 - تمرین سوم

$x_1 = 0 \rightarrow y_1 = -1 \qquad x_2 = \sqrt{2} \rightarrow y_2 = 1$ (1)

$\phi(x_1) = [1, \sqrt{2}(0), 0^2]^T = [1, 0, 0]^T$

$\phi(x_2) = [1, 2\sqrt{2}, \sqrt{2}^2]^T = [1, 2\sqrt{2}, 2]^T$

$y_1(w^T \phi(x_1) + w_0) \geq 1 : -1(w^T [1,0,0] + w_0) \geq 1$

$y_2(w^T \phi(x_2) + w_0) \geq 1 : 1(w^T [1, 2\sqrt{2}, 2] + w_0) \geq 1$

$w_0 + w_1 \leq -1$

$\Rightarrow -w_0 - w_1 \geq 1 \qquad \Rightarrow w_1 + 2\sqrt{2} w_2 + 2w_3 + w_0 \geq 1$

$\min \|w\|^2 = w_1^2 + w_2^2 + w_3^2$

$(w_1 + 2\sqrt{2} w_2 + 2w_3 + w_0) - (w_1 + w_0) \geq 2$

$\rightarrow 2\sqrt{2} w_2 + 2w_3 \geq 2 \rightarrow \sqrt{2} w_2 + w_3 \geq 1$

$\sqrt{2} w_2 + w_3 = 1$

$w_2 = \frac{1}{\sqrt{2}} \qquad w_3 = 0$

$w_1 + w_0 \leq -1 \rightarrow w_1 = -1 \quad w_0 = 0$

$w = [-1, \frac{1}{\sqrt{2}}, 0]^T$ الف

$\text{margin} = \frac{2}{\|w\|} \rightarrow$ ب)

$\|w\| = \sqrt{1 + \frac{1}{2} + 0} = \sqrt{3/2} = \sqrt{3/2}$

$\text{margin} = \frac{2\sqrt{2}}{\sqrt{3}}$

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$w = [-1, \frac{1}{\sqrt{2}}, 0]^T$ $\quad -1([-1, \frac{1}{\sqrt{2}}, 0][1,0,0] + w_0) = 1$ (2

$\Rightarrow -1(-1 + w_0) = 1 \rightsquigarrow 1 - w_0 = 1 \rightarrow w_0 = 0$

$\phi(n_2) = [1, \sqrt{2}, 2]$ $\quad w = 1([-1, \frac{1}{\sqrt{2}}, 0][1, \sqrt{2}, 2] + w_0) = 1$

$1(-1 + 1 + w_0) = 1 \rightsquigarrow 1(0 + w_0) = 1 \quad w_0 = 1$

$w = [-1, \frac{1}{\sqrt{2}}, 0]^T$ , $w_0 = 1$

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$$w = \sum_{i,\alpha_i>0} \alpha_i y_i x_i \qquad (2)$$

$$\Rightarrow 0{,}1414\begin{pmatrix}4\\2{,}9\end{pmatrix} - 0{,}1018\begin{pmatrix}2{,}5\\1\end{pmatrix} + 0{,}1018\begin{pmatrix}3{,}5\\4\end{pmatrix} - 0{,}1414\begin{pmatrix}2\\2{,}1\end{pmatrix}$$

$$\Rightarrow 0{,}1414\begin{pmatrix}2\\0{,}8\end{pmatrix} + 0{,}1018\begin{pmatrix}1\\3\end{pmatrix} \Rightarrow \begin{pmatrix}0{,}828 - 0{,}1018\\0{,}3312 - 0{,}054\end{pmatrix} = \begin{pmatrix}0{,}81\\0{,}2772\end{pmatrix}$$

$$b_i = y_i - w^T x_i \qquad b_1 = 1 - (0{,}81\ 0{,}2772)\begin{pmatrix}4\\2{,}9\end{pmatrix} = -3{,}04388$$

$$b_4 = -1 - (0{,}81\ 0{,}2772)\begin{pmatrix}2{,}5\\1\end{pmatrix} = -3{,}3022$$

$$b_7 = 1 - (0{,}81\ 0{,}2772)\begin{pmatrix}3{,}5\\4\end{pmatrix} = -2{,}9438$$

$$b_9 = -1 - (0{,}81\ 0{,}2772)\begin{pmatrix}2\\2{,}1\end{pmatrix} = -3{,}20212$$

$$b = \text{avg}\{b_i\} \rightsquigarrow 3{,}123$$

$$h(x) = \begin{pmatrix}0{,}81\\0{,}2772\end{pmatrix}^T x - 3{,}123 = 0 \qquad \text{الف}$$

$$d = \frac{|wx_0 + b|}{\|w\|} \qquad wx_0 = 0{,}81 \times 1{,}9 + 0{,}2772 \times 1{,}9 \qquad \text{ب}$$

$$wx_0 = 2{,}06568$$

$$wx_0 + b = 2{,}06568 - 3{,}123 = -1{,}05732$$

$$\|w\| = \sqrt{0{,}81^2 + 0{,}2772^2} = 0{,}1856 \qquad d = \frac{1{,}05732}{0{,}1856} \simeq 1{,}235$$

$$\text{Margin} = \frac{1}{\|w\|} = \frac{1}{0{,}1856} \simeq 1{,}168$$

$1{,}235 > 1{,}168 \Rightarrow$ بیرون حاشیه است.

$$h(x) = (0,81)\underbrace{x_1}_{3} + (0,2772)\underbrace{x_2}_{3} - 3,123$$ (?

$$h(3,3) = 2,43 + 0,8316 - 3,123 = \underbrace{0,1386}_{\text{مثبت است}}$$

$$\Rightarrow y = +1$$

(3) الف) $z_i = \text{ReLU}(n w_i)$

$z_1 = \text{ReLU}(4) = 4 \quad z_2 = \text{ReLU}(4) = 4 \quad z_3 = \text{ReLU}(-4) = 0$

الف) $o = \sum_i z_i w'_i = 0.15 \times 4 + 4 + 2 \times 0 = 6$

$\hat{y} = \text{Sigmoid}(o) = \frac{1}{1+e^{-6}} = 0.9975$

ب) $SSE = (\hat{y} - y)^2 \qquad 1/2 (\hat{y} - y)$

$\rightarrow SSE = (0.9975 - 0)^2 = 0.9956$

ج) output layer

$L = 1/2 (\hat{y} - y) \rightarrow \frac{\delta L}{\delta \hat{y}} = (\hat{y} - y)$

sigmoid $\frac{d\hat{y}}{do} = \hat{y}(1 - \hat{y}) \qquad \frac{dL}{do} = \frac{\delta L}{\delta \hat{y}} \times \frac{\delta \hat{y}}{do} = \hat{y}(1-\hat{y})(\hat{y} - y)$

$= > 0.0025$

Hidden $\rightarrow \frac{\delta o}{\delta z_j} = w'_j - o \quad \frac{\delta L}{\delta z_j} = \frac{\delta L}{\delta o} \times \frac{\delta o}{\delta z_j}$

$\frac{\delta L}{\delta z_1} = 0.0025 \times 0.5 = 0.00125$

$$\frac{\delta L}{\delta z_2} = 0.00025 \times 1 = 0.00025$$

$$\frac{\delta L}{\delta z_3} = 0.00025 \times 2 = 0.0005$$

$$\Rightarrow \frac{\delta L}{\delta (w_1 x)} = 1 \times 0.000125 = 0.000125$$

$$\frac{\delta L}{\delta (w_2 x)} = 1 \times 0.00025 = 0.00025$$

$$\frac{\delta L}{\delta (w_3 x)} = 0 \times 0.0005 = 0$$

$$\frac{\delta L}{\delta w_j'} = \frac{\delta L}{\delta o} \times z_j$$

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$$\Rightarrow \frac{\delta L}{\delta w_1'} = 4 \times 0.1 \times 0.25 = 0.1$$

$$\frac{\delta L}{\delta w_2'} = 4 \times 0.1 \times 0.25 = 0.1$$

$$\frac{\delta L}{\delta w_3'} = 2 \times 0.1 \times 0.25 = 0 \rightarrow \nabla w' = \begin{pmatrix} 0.1 \\ 0.1 \\ 0 \end{pmatrix}$$

$$\frac{\delta L}{\delta w_{ij}} = \frac{\delta L}{\delta z_j} \times \frac{\delta z_j}{\delta w_{ij}} = \frac{\delta L}{\delta z_j} \times 4$$

$$\nabla w = \begin{pmatrix} 0.05 \\ 0.1 \\ 0 \end{pmatrix}$$