→ Bezier curves have global control ; B-spline curves have local control ↳ (shape based on control pt. ↳ degree of polynomial) ↳ degree doesnot depend upon on no. of control pts. ↳ It depends upon order of polynomial

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B- Spline Curve.

— Mainly there are two types of B-Spline Curves.

↳ Uniform Non-rational B-spline,

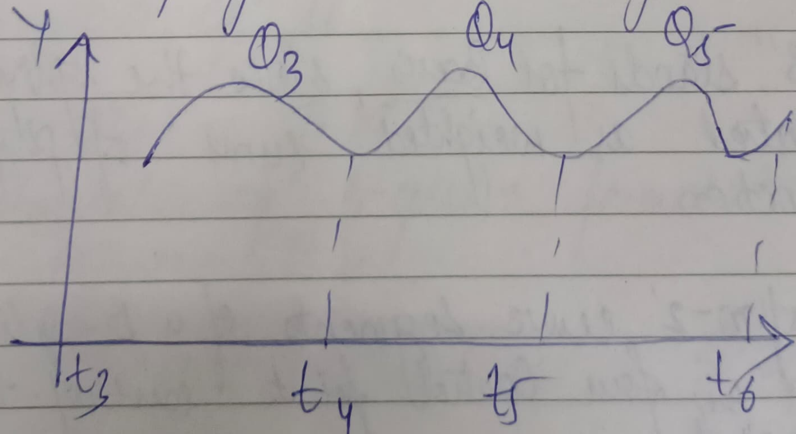
↳ Non-Uniform, Non-rational B-Spline

→ Uniform Non-rational B-Spline Curve :-

- B-Spline consists of a curve segments whose polynomial Coefficients depend on just a few control points. This is called local control because this control point affect only a small part of a curve.

- The Curve need not pass through the control points.

- Cubic B-splines approximate a series of $m+1$ control points $P_0, P_1, \ldots, P_m$, $m \geq 3$, with a curve consisting of $(m-2)$ cubic polynomial curve segments $Q_3, Q_4 \ldots Q_m$



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- The parameter range of Q_i is defined as $t_i \leq t \leq t_{i+1}$ for $3 \leq i \leq m$

- For each $i \geq 4$, there is a joint point or knot b/w Q_{i-1} and Q_i at the parameter value t_i known as knot value.

- The initial and final points are t_3 and t_{m+1}.

- So there are $(m+1)$ control points $(P_0, P_1 \ldots P_m)$ and $(m-2)$ segments $(Q_3, Q_4 \ldots Q_m)$ and $(m-1)$ knots $(t_3, t_4 \ldots t_m, t_{m+1})$

- The term Uniform means the knots are spaced at equal intervals of parameter 't'. We can assume that $t_3 = 0$ and $t_{i+1} - t_i = 1$.

- In a rational cubic polynomial curve, $x(t), y(t)$ & $z(t)$ are defined as ratio of two cubic ~~polynom~~ polynomials

- The "B" stands for Basis, since the spline can be represented as weighted sums of polynomial basis function

- Each of 'm-2' curve segments of a B-spline curve is defined by four control points out of the $(m+1)$ control points.

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- The curve segment Q_i is defined by points P_{i-3}, P_{i-2}, P_{i-1} and P_i. Thus, Geometry vector for segment Q_i is

$$G_{Bs_i} = \begin{bmatrix} P_{i-3} \\ P_{i-2} \\ P_{i-1} \\ P_i \end{bmatrix} \quad 3 \le i \le m.$$

Segment	Control Points	Parameter
Q_3	$P_0 \; P_1 \; P_2 \; P_3$	$t_3 = 0$; $t_4 = 1$
Q_4	$P_1 \; P_2 \; P_3 \; P_4$.	$t_4 = 1$; $t_5 = 2$
Q_5	$P_2 \; P_3 \; P_4 \; P_5$ –	$t_5 = 2$; $t_6 = 3$
⋮		
Q_m.	$P_{m-3} \; P_{m-2} \; P_{m-1} \; P_m$.	$t_m = m-3$, $t_{m+1} = m-2$

NOTE :- To satisfy the convex hull property, the blending func^n should be non-negative and sum = 1.

The parametric eq^n is

$$Q(t) = T \cdot M \cdot G.$$

The ~~parametric eq^n~~ B-spline formulation for curve segment 'i' is

$$Q_i(t) = T_i \, M_{Bs} \, G_{Bs_i} \quad t_i \le t < t_{i+1}$$

where

$$T_i = \left[(t-t_i)^3 \;\; (t-t_i)^2 \;\; (t-t_i) \;\; 1 \right].$$

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The B-Spline basis matrix (M_{BS}) $= \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$

The B-spline blending func^n

$$B_{BS} = T_i \cdot M_{BS}$$

The blending func^n for each curve segment are exactly same, because for each segment i the values of $t - t_i$ range from ~~0 to~~ 0 at $t = t_i$ and 1 at $t = t_{i+1}$

If we replace $t - t_i$ by t and the interval $[t_i \; t_{i+1}]$ by $[0 \; 1]$.

$$B_{BS} = T \cdot M_{BS} = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

$$= \left[\frac{-t^3+3t^2-3t+1}{6}, \frac{3t^3-6t^2+4}{6}, \frac{-3t^3+3t^2+3t+1}{6}, \frac{t^3}{6} \right]$$

$$Q(t) = B_{BS} \cdot G_{BS}$$

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Properties of B-Spline Curves

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(1) Q_i and Q_{i+1} are C^0, C^1 & C^2 continous where they join.

(2) The sum of the B-spline basis funcⁿ for any parameter value is 1.

(3) The maximum order of the curve is equal to the no. of vertices of defining polygon.

(4) The degree of B-spline polynomial is independent on the no. of vertices of defining polygon.

(5) B-spline allows the local control over the curve surface because each vertex affects the shape of a curve only over a range of parameter values.

~~PROJECTION~~

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UNIT- 7.

Topics to be Covered :-

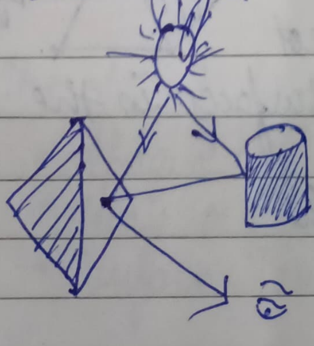
Transparency, Shadows, Constant - Intensity Shading, Gouraud Shading, Phong Shading, Wireframe-Visibility Methods, Recursive Ray, Tracing Algorithm, Radiosity Model

Illumination Model and Surface Rendering Model:-

- An Illumination Model / Lighting Model / Shading Model is used to calculate the intensity of light that we should see at a given point on the surface of an object.

- A Surface Rendering Algorithm uses the intensity calculation from an illumination model to determine the light intensity for all projected pixel positions.

- Illumination is a technique to intensify (brightness will be added) an object or point by applying certain model in order to get a realistic image.



Light Sources :-

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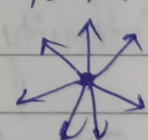
- Light source is the source which emits light.
- Total reflected light =

Light directly from light source
Light Emitting source

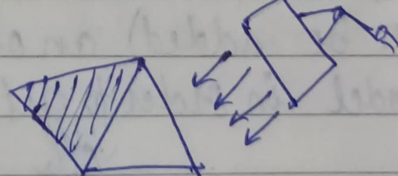
+ Light from Reflecting Surfaces
Light Reflecting sources.

- A Surface i.e. not directly exposed to light may still be visible if nearby objects are illuminated.

- Point Source :-
  - ✓ Rays from the source follows radially diverging paths from the source.
  - ✓ Dimension is small in comparison to the size of objects in the scene.
  - ✓ eg- sun.



- Distributed light source :-



✓ The Area of the source is not small compared to the surfaces in the scene.

✓ eg - A long flourescent light

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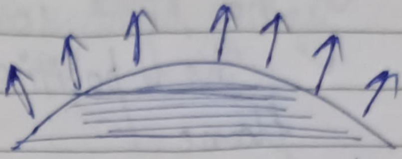
– Diffuse Reflection



✓ Surfaces that are rough or grainy, tend to scatter the reflected light in all directions. This scattered light is called diffuse reflection.

✓ Color of an object is the color of the diffuse reflection of the incident light

eg:- 1) A blue object illuminated by a white light source reflects the blue component of the white light and absorbs all other components.

2) The same blue object ~~as~~ illuminated by red light appears black since all incident light is absorbed.

– Specular Reflection:- ✓ Light sources create highlights or bright spots called specular reflection.

✓ Mostly found on shiny surfaces.

Basic Illumination Models:-

– Lighting calculations are based on:-

✓ The optical properties of surfaces:-

- Opaque or transparent
- Shiny or Dull.
- Surface-Texture

✓ The relative position of the surface in a scene.

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- The light source specification: color, position.
- The position and orientation of the viewing plane.
- The background lighting conditions.

- Ambient light (background light)

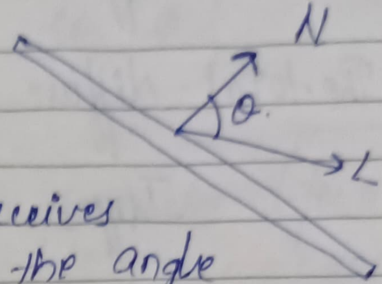
- The light i.e. the result from the light reflecting off other surfaces in the environment has no direction.
- Each light source has ambient light contribution (I_a).
- for a given surface, we can ~~apply~~ specify how much ambient light the surface can reflect using an ambient reflection coefficient, K_a ($0 < K_a < 1$).
- So the amount of light that the surface reflects is

$$\boxed{I_{amb} = K_a * I_a}$$

- Diffuse Light

- The illumination that a surface receives from a light source and reflects equally in all directions.
- This type of reflection is called Lambertian Reflection (Lambertian Surfaces).
- The brightness of the surface is independent of the

Observer position since the light is reflected in all directions equally.



- Lambert's Law :-

- How much light the surface receives from a light source depends on the angle b/w its normal and vector from the surface point d_a to the light

✓ **Lambert's Law :-** The radiant energy, I_d from a small surface d_a for a given light source is :-

$$\boxed{I_d = I_L * \cos\theta}$$

Where,

$I_L \rightarrow$ the intensity of the light source.

$\theta \rightarrow$ Angle b/w the surface Normal 'N' and light vector 'L'.

- The Diffuse Component :-

✓ **Surface Material property :-** Assuming that the surface can reflect K_d ($0 < K_d < 1$, the diffuse reflection coefficient), the amount of diffuse light :-

$$\boxed{I_{diff} = K_d * I_L * \cos\theta}$$

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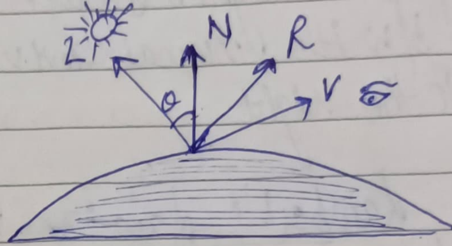
If N & L are normalized, $\cos\theta = N * L$.

$\Rightarrow I_{diff} = K_d * I_L * (N*L)$.

The total diffuse reflection = ambient + Diffuse

$I_{diff} = K_a * I_a + K_d * I_L * (N*L)$.



— Specular Light

✓ These are bright spots on Objects (such as polished metal, apple etc.)

✓ light reflected from the surfaces unequally to all directions

✓ The result of near total reflection of the incident light in a concentrated region around the specular reflection angle.

✓ The specular reflection angle equals the angle of the incident light, with two angles measured on opposite sides of the unit normal surface vector N.

✓ Here R: Unit vector in the direction of ideal specular reflection.

L: Unit vector directed toward the point light source.

V: Unit vector pointing to the viewer.

ϕ: Viewing angle relative to the specular reflection direction R.

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As per Phong specular-reflection model, the intensity of specular reflection is described as :-

$$\boxed{I_{spec} = W(\theta) \cdot I_l \cdot \cos^{n_s} \phi}$$

where,

$n_s \rightarrow$ specular reflection parameter (for a shiny surface $n_s = 100$ or more for a dull surface $n_s = 1$; for a perfect reflecter $n_s = \infty$).

$W(\theta) =$ specular-reflection coefficient
At $\theta = 90°$, $W(\theta) = 1$, and all of the incident light is reflected..

I_l :- Intensity of light source.

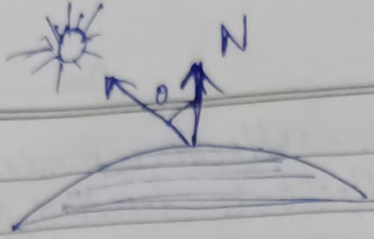
ϕ :- Viewing angle relative to the specular reflection direction R.

- Illumination Models calculate the intensity projected from a particular surface point in a specified viewing direction

- Types of illumination Model :-
  - Global illumination
  - Local illumination.

- Global illumination Model :- It takes into account the interaction of light from all the surfaces in the scene.

- Local illumination Model :- Only consider the light, the observer position and the object material properties.



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Illumination Vs Shading :-

- Illumination (lighting) model determine the color of a surface point by simulating some light attributes.

- Shading model applies the illumination models at a set of points and colours the whole image.

Shading Models for Polygons :-

- Constant Shading (flat shading) :- Compute illumination at any one point on the surface. Use face or one normal from a pair of edges. Good for far away light & viewer.

- Per Pixel Shading :- Compute illumination at every point on the surface.

- Interpolated Shading :- Compute illumination at vertices and interpolate color.

Polygon-Rendering Models :-

- The objects are usually polygon-mesh approximations of curved-surface objects.

- Each Polygon can be rendered with a single intensity or the intensity can be obtained at each point of the

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Surface using an interpolation scheme.

1) Constant - Intensity Shading / Flat Shading :-

- It is a fast and Simple method for rendering an object with polygon surfaces.

- A Single intensity is calculated for each polygon. All points over the surface of the polygon are then displayed with the same intensity value.

- Useful for quickly displaying the general appearance of a curved surface.

- Flat Shading of a polygon facets provides an accurate rendering for an object if all of the following assumptions are valid :-

↳ The object is a polyhedron and is not an approximation of an object with a curved surface.

↳ All light sources illuminating the object are sufficiently far from the surface so that N·L and the attenuation funcn are consto over the surface.

⇒ The viewing position is sufficiently far from the surface so that V·R is constant over the surface.

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Application :-

→ This algorithm is applied to the scene where both light source and viewer are far distant from the object.

→ To display fast moving object in a scene this algorithm is suitable.

Drawback :-

→ Algorithm fails to represent a scene where the intensity is varying uniformly. That means the intensity discontinuity can occur in flat shading. Gouraud shading overcomes this limitation.

Gouraud Shading :-

→ This is an intensity-interpolation scheme, developed by Gouraud.

→ It renders the polygon surfaces by linearly interpolating intensity values across the surface.

→ It eliminates the intensity discontinuities (which can occur in flat shading) as intensity values for each polygon are matched with values of adjacent polygons along the common edges

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→ It performs the following calculations :-

- Determine the average ~~and~~ unit normal vector at each polygon vertex.

- Apply an illumination model to each vertex to calculate the vertex intensity.

- Linearly interpolate the vertex intensities over the surface of the polygon.

Step 1.

- At each polygon vertex, we obtain a normal vector by averaging the surface normals of all polygons sharing that vertex.

- For any vertex position v, we obtain the unit vertex normal.

$$N_v = \frac{\sum_{k=1}^{n} N_k}{\left| \sum_{k=1}^{n} N_k \right|}$$

Step 2.

After finding the vertex normals at each vertex we can determine the intensity at the vertices from a lighting model.

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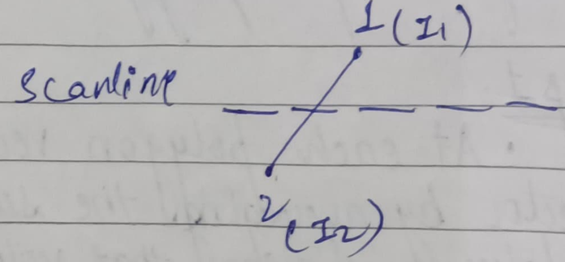
Step 3.

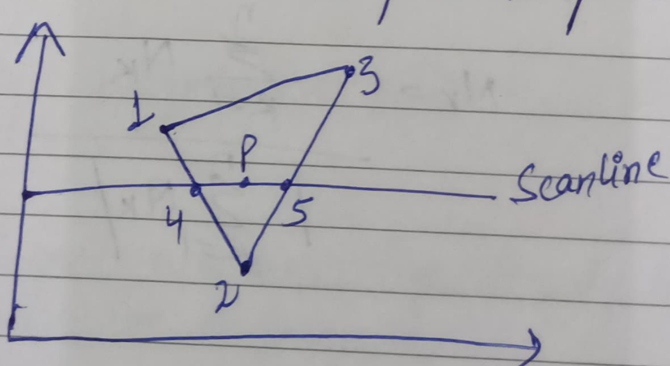
1. For each scanline, the intensity at the intersection of the scanline with a polygon edge is linearly interpolated from the intensities at the edge end points.

- It uses a fast method for obtaining the intensity at point 4 by interpolating b/w the intensities of point 1 (I_1) and point 2 (I_2) using only the vertical displacement.



Example :-

Assume that we know I_1, I_2, I_3 from step 2.



Intensity of point 4 is calculated as:-

$$I_4 = \frac{y_4 - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y_4}{y_1 - y_2} I_2$$

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Similarly,

$$I_5 = \frac{y_5 - y_3}{y_2 - y_3} I_2 + \frac{y_2 - y_5}{y_2 - y_3} I_3.$$

The intensity of an interior point P is calculated as:-

$$I_p = \frac{x_5 - x_p}{x_5 - x_4} I_4 + \frac{x_p - x_4}{x_5 - x_4} I_5$$

If the intensity at edge position (x, y) is interpolated as

$$I = \frac{y - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y}{y_1 - y_2} I_2$$

then we can obtain the intensity along this edge for the ~~next~~ next scanline, $(y-1)$ as

$$I' = I + \frac{I_2 - I_1}{y_1 - y_2}$$

Advantages

- It can remove the discontinuities associated with the constant shading model.

Disadvantages

Highlights on the surface are sometimes displayed with anomalous shapes, and the linear interpolation can cause bright or dark intensity streaks, called Mach ~~bon~~ bands to appear in the surface.

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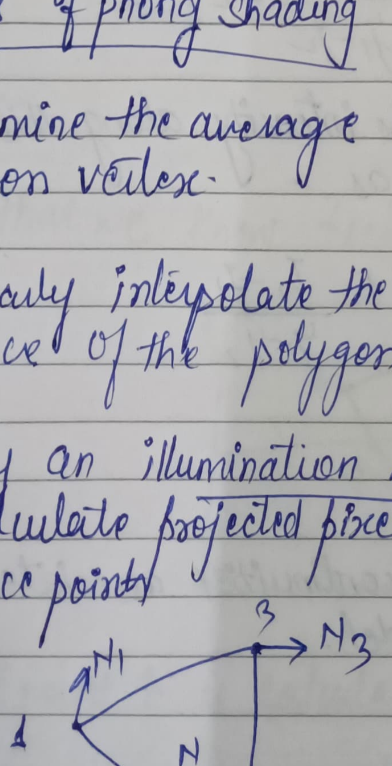
This is Overcome by Phong Shading.

Phong Shading / Normal-Vector Interpolating Shading

- This is a more accurate method for rendering polygon surface.
- This method first interpolate normal vectors and then apply the illumination model to each surface point

→ steps of phong shading

- Determine the average unit normal vector at each polygon vertex.
- Linearly interpolate the vertex normals over the surface of the polygon.
- Apply an illumination model along each scanline to calculate projected pixel intensities for the surface points



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The normal vector 'N' for the scan line intersection point along the edge b/w vertices 1 and 2 is obtained as:

$$N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2$$

- Incremental Methods are used to evaluate normals b/w scanlines and along each individual scanline.

- At each pixel position along a scanline, the illumination model is applied to determine the surface intensity at that point

Advantages

- Intensity Calculations are more accurate
- Suitable for shiny surfaces.

Disadvantages – Needs More Calculations