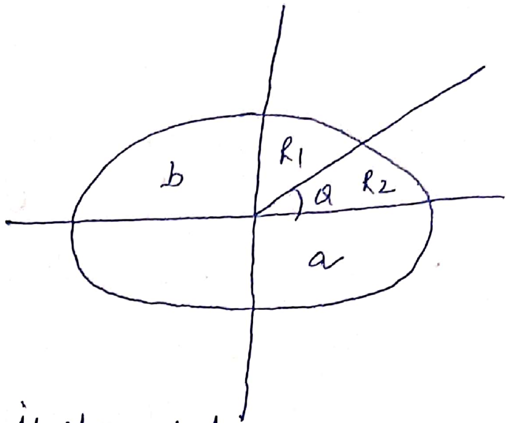
By Dr. Tanu

* Mid-point ellipse Drawing Algorithm -

Region 1, Region 2



Major Axis = 2a

Minor Axis = 2b

Major axis = $\left\{ \begin{array}{l} a = r_x \\ b = r_y \end{array} \right\}$ (is elongated form of Circle)

eqⁿ of ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f(x,y) = x^2 b^2 + y^2 a^2 = a^2 b^2$$

$$= x^2 b^2 + y^2 a^2 - a^2 b^2 = 0$$

Now replace with given (r_x and r_y)

$$\boxed{x^2 r_y^2 + r_x^2 y^2 - r_x^2 r_y^2 = 0} \quad -(1)$$

If we put any point in eqⁿ (1)

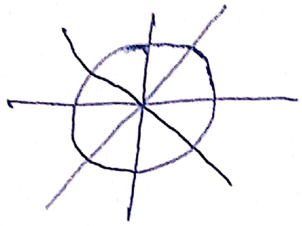
[= 0] point lies on ellipse

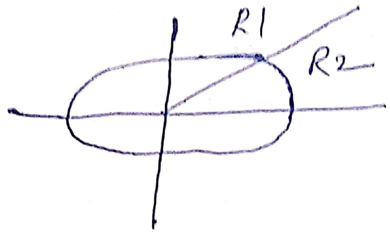
[< 0] point lies inside the ellipse

[> 0] outside the ellipse

Difference b/w Circle and ellipse :-

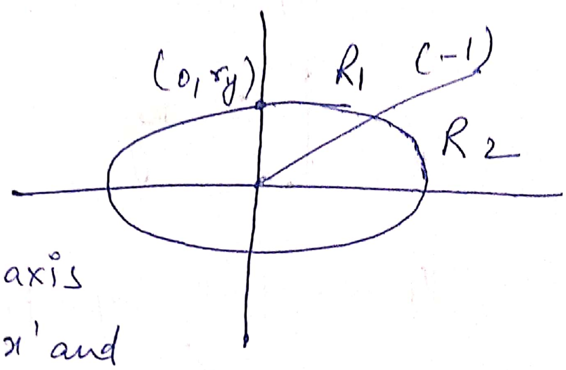
Circle	Ellipse
→ 8 ways symmetry	→ 4 ways symmetry
→ calculate 1 octand and rest of them is calculated	→ calculate atleast 2 region to complete the calculation





Quadrant -1 :- Region 1

* Start pt. : $(0, r_y)$
* Slope of curve : (<-1)
* Take unit step in x-axis till the boundary b/w 'x' and 'y' is not reached. $(x \rightarrow x+1)$



* We have to check the Y axis

$$\left\{ \begin{array}{c} y_1 \rightarrow y_0 + 1 \\ \text{or} \\ y_1 \rightarrow y_0 \end{array} \right\} \text{check at every iteration}$$

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Qudrant 2 : Region 2

* Slope of curve [≥ -1]

* Take unit step is y-axis direction till end of the quardrant. ($y_1 = y_0 - 1$).

FORMULA :-

Region :-

$$P_K = r_y^2 + \frac{1}{4} r_x^2 - r_x^2 r_y$$

Case 1 : if $P_K < 0$ then

$$x_{K+1} = x_K + 1$$

$$y_{K+1} = y_K$$

$$P_{K+1} = P_K + r_y^2 (1 + 2(x_K + 1))$$

if $P_K \geq 0$, then

$$x_{K+1} = x_K + 1$$

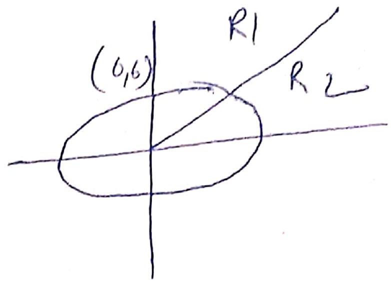
$$y_{K+1} = y_K - 1$$

$$P_{K+1} = P_K + r_y^2 (1 + 2(x_K + 1)) + 2 r_x^2 (1 - y_K)$$

Numerical :- Center $(0,0)$

$r_x = 8$

$r_y = 6$



Region 1 :-

Initial Point $(0, r_y) = (0,6)$

Iteration 0: $P_k = r_y^2 + \frac{1}{4} r_x^2 - r_x^2 r_y$

$= (6)^2 + \frac{1}{4}(8)^2 - (8)^2(6)$

$= 36 + \frac{64}{4} - 384$

$= -332$

Case 1 :- if $P_k < 0$

Iteration : 1 $x_{k+1} = x_k + 1 = 0+1 = 1$

$y_{k+1} = y_k = 6$

$P_{k+1} = P_k + r_y^2 (1 + 2(x_k+1))$

$= -332 + 6^2 (1 + 2(0+1))$

$= -332 + 36 \times 3$

$= -224.$

check for the $\boxed{2 r_y^2 x \geq 2 r_x^2 y}$

then change the region

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Region 1 :- Coordinates

k	P_k	x_{k+1}, y_{k+1}	$2r_y^2x$	$2r_x^2y$
0	-332	(0, 6)	0	768
1	-224	(1, 6)	72	768
2	-44	(2, 6)	144	768
3	208	(3, 6)	216	768
4	-108	(4, 5)	288	640
5	288	(5, 5)	360	640
6	244	(6, 4)	432	512
7	400	(7, 3)	$\boxed{504}$	$\boxed{384}$

$>$

Steps :-

$[P_k < 0]$ Iteration 2 :- $x_{k+1} = 1+1 = 2$

$y_{k+1} = 6$

$P_{k+1} = P_k + r_y^2(1 + 2(x_k+1))$

$= -224 + 6^2(1 + 2(1+1))$

$= -224 + 180$

$= -44$

Iteration 3 : $[P_K < 0]$

$x_{k+1} = x_k + 1 = 2+1 = 3$

$y_{k+1} = y_y = 6$

$P_{k+1} = P_k + r^2 y \left(1+2(x_k+1)\right)$

$= -44 + 6^2 \left(1+2(2+1)\right)$

$= -44 + 36 \times 7$

$= 208$

Iteration 4 : $[P_K > 0]$

$x_{k+1} = x_k + 1 = 3+1 = 4$

$y_{k+1} = y_k - 1 = 6-1 = 5$

$P_{k+1} = P_k + r_y^2 \left(1+2(x_k+1)\right) + 2r^2 x (1-y_k)$

$= 208 + 6^2 \left(1+2(3+1)\right) + 2\times 8^2 (1-6)$

$= 208 + (36 \times 9) + 2 \times 64 \times -5$

$= 208 + 324 - 640$

$= -108$

Iteration 5 : $[P_K < 0]$

$x_{k+1} = x_k + 1 \Rightarrow 4+1 = 5$

$y_{k+1} = y_k = 5$

$P_{k+1} = P_k + r^2 y \left(1+2(x_k+1)\right)$

$= -108 + 6^2 \left(1+2(4+1)\right)$

$= -108 + 36 \times 11$

$= 288$

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Iteration 6: $P_K > 0$

$x_{K+1} = x_K + 1 = 5 + 1 = 6$

$x_{K+1} = y_K = 5 - 1 = 4$

$P_{K+1} = P_K + r_y^2 (1 + 2(x_K + 1)) + 2 r_x^2 (1 - y_K)$

$= 288 + 6^2 (1 + 2(6)) + 2 \times 8^2 (1 - 5)$

$= 288 + (36 + 13) + (-512)$

$= 756 - 512$

$= 244$

Iteration 7: $[P_K > 0]$

$x_{K+1} = x_K + 1 = 6 + 1 = 7$

$y_{K+1} = y_K = 4 - 1 = 3$

$P_{K+1} = P_K + r_y^2 (1 + 2(x_K + 1)) + 2 r_x^2 (1 - y_K)$

$= 244 + 6^2 (1 + 2(7)) + 2 \times 8^2 (1 - 4)$

$= 244 + 540 - 384$

$= 400$

Region 2

$$\boxed{P_K = r_y^2 \left(x_k + \frac{1}{2}\right)^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2}$$

if $(P_K < 0)$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k - 1$$

$$P_{k+1} = P_k + 2r_y^2 (x_k + 1)$$

if $(P_K \geq 0)$

$$x_{k+1} = x_k$$

$$y_{k+1} = y_k - 1$$

$$P_{k+1} = P_k + r_x^2 (1 - 2(y_k - 1))$$

Initial point :- $(7, 3)$

$$P_K = 6_y^2 \left(x_k + \frac{1}{2}\right)^2 + r_x^2 (y_k + 1)^2 - r_x^2 r_y^2$$

$$= 6^2 \left(7 + \frac{1}{2}\right)^2 + 8^2 (3-1)^2 - 8^2 \times 6^2$$

$$= 36 \times \frac{225}{4} + 64 \times 4 - 2304$$

$$= 2025 + 256 - 2304$$

$$= -23$$

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K	P_K	x_{K+1}	y_{K+1}
0		7	3
1	-23	8	2
2	361	8	1
3	297	8	0

Iteration 2 : $P_K < 0$.

$$x_{K+1} = x_K + 1 = 7 + 1 = 8$$

$$y_{K+1} = y_K - 1 = 3 - 2 = 2$$

$$P_{K+1} = P_K + 2r_y^2 (x_K + 1) + r_x^2 (1 - 2(y_K - 1))$$

$$= -23 + 2 \times (6)^2 (7+1) + 8^2 (1 - 2(3-1))$$

$$= -23 + 576 + 64 (1-4)$$

$$= -23 + 576 - 192$$

$$= 361$$

Iteration 3 : $P_K > 0$

$$x_{K+1} = x_K = 8$$

$$x_{K+1} = y_K - 1 = 2 - 1 = 1$$

$$P_{K+1} = P_K + r_x^2 (1 - 2(y_K - 1))$$

$$= 361 + 8^2 (1 - 2(1))$$

$$= 361 - 64$$

$$= 297$$

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<u>Iteration 4</u> :- $P_k > 0$

$$x_{k+1} = x_k = 8$$

$$y_{k+1} = y_k - 1 = 1 - 1 = 0$$

stop the simulation once its satisfy the $(x, y) = (8, 0)$