

1. Decision Variables = 25.

$$x_{ij} \quad \forall i \geq 1 \quad j \leq 5$$

Decision Variables \rightarrow Binary (0 or 1)

1 \rightarrow when person i is doing task j
otherwise 0.

Taking input cost matrix c as variable cost
 $\text{Cost}_{ij} \rightarrow$ Cost for person i doing task j .

OBJECTIVE FUNCTION

minimize: $\sum \sum \text{cost}(ij) \times x_{ij}$ (for $i \geq 1 \text{ \& } j \leq 5$)

CONSTRAINTS

$$\sum x_{ij} = 1 \quad \forall i \in \{1, 2, 3, 4, 5\} \text{ (j fixed)}$$

$$\sum x_{ij} = 1 \quad \forall j \in \{1, 2, 3, 4, 5\} \text{ (i fixed)}$$

Sum of rows & columns of decision variable matrix = 1.

OUTPUT MATRIX

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Interpretation: Person 4 is doing task 1

Person 3 is doing task 2

Person 1 is doing task 3

person 2 is doing task 4

person 5 is doing task 5

Accordingly, min^m. cost = $40 + 39 + 43 + 45 + 43$
 $= ₹ 210$

2. Decision Variables = 9 (integers)

OBJECTIVE FUNCTION:

Maximize: $\sum [Profit \times \{x_{ij}\}] \quad i \in \{1, 2, 3\} \quad j \in \{\text{Heavy Duty, Standard, Economy}\}$

CONSTRAINTS

Sum of total batteries & lead used in each plant has to be $<$ limits.

Total prodⁿ $<$ Total demand of same type.

Total Batteries:

$$P_i - \text{HeavyD} + P_i - \text{Standard} + P_i - \text{Economy} \leq AC_i$$

where $i \in \{1, 2, 3\}$ & $AC_1 = 550$, $AC_2 = 750$, $AC_3 = 225$

Total leads used:

$$\underbrace{\text{LeadR}_{-HD} \times P_i - HD}_{\text{Heavy Demand}} + \underbrace{\text{LeadR}_{-Stan} \times P_i - Stan}_{\text{Standard}} + \underbrace{\text{LeadR}_{-Exp} \times P_i - E}_{\text{Economy}} \leq \text{Max. Lead Prod}(i).$$

Total Demand:

$$P_i - \text{HeavyD} + P_i - \text{Standard} + P_i -$$

$$P_1 - \text{HeavyD} + P_2 - \text{HeavyD} + P_3 - \text{HeavyD} \leq 700$$

Similar for other 2 as well.

OUTPUT

$$\begin{bmatrix} 171 & 377 & 0 \\ 16 & 392 & 0 \\ 98 & 126 & 0 \end{bmatrix}$$

\Rightarrow COMPANY DOES NOT
PRODUCE ANY ECONOMY
BATTERY

INTERPRETATION

Plant 1 produces 171 HD, 377 Standard & 0 Economy Batteries.

Plant 2 produces 16 HD, 392 Standard & 0 Economy Batteries.

Plant 3 produces 98 HD, 126 Standard & 0 Economy Batteries.

b).

CONSTRAINT

In addition to 2a. above constraints:

$$\leq P_{\text{economy}} \geq 0.4 \times (\leq (P_{\text{economy}} + P_{\text{standard}} + P_{\text{HD}}))$$

OUTPUT

$$\begin{bmatrix} 476 & 0 & 0 \\ 0 & 0 & 450 \\ 199 & 0 & 0 \end{bmatrix}$$

\Rightarrow COMPANY DOES NOT
PRODUCE ANY STANDARD
BATTERY

INTERPRETATION

Plant 1 produces only 476 HD batteries.

Plant 2 ——— " ——— 450 Economy ———

Plant 3 ——— " ——— 199 HD batteries.