

1-1.6-25

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Question: Three points $P(h, k)$, $Q(x_1, y_1)$ and $R(x_2, y_2)$ lie on a line. Show that

$$\frac{h - x_1}{y_2 - y_1} = \frac{k - y_1}{x_2 - x_1}$$

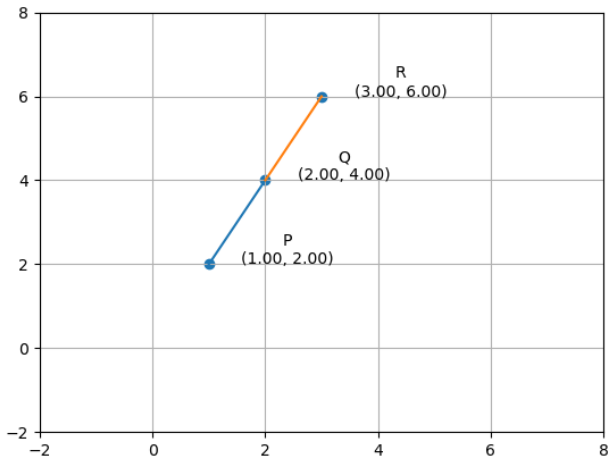
Solution: Since the points P, Q, R are collinear, The matrix $\begin{pmatrix} P - Q & R - Q \end{pmatrix}$ must have rank 1.

$$\begin{pmatrix} P - Q & R - Q \end{pmatrix}^T = \begin{pmatrix} h - x_1 & x_2 - x_1 \\ y - y_1 & y_2 - y_1 \end{pmatrix}$$

$$\begin{aligned} \text{Using } R_2 &\leftarrow R_2 - \frac{k - y_1}{h - x_1} R_1 \\ &= \begin{pmatrix} h - x_1 & x_2 - x_1 \\ 0 & y_2 - y_1 - \frac{(k - y_1)(x_2 - x_1)}{h - x_1} \end{pmatrix} \end{aligned}$$

Since matrix has rank 1, we can say

$$\begin{aligned} y_2 - y_1 - \frac{(k - y_1)(x_2 - x_1)}{h - x_1} &= 0 \\ \frac{h - x_1}{y_2 - y_1} &= \frac{k - y_1}{x_2 - x_1} \end{aligned}$$



```
:!python3 main.py
Taking point P to be: [[1.]
[2.]]
Taking point Q to be: [[2.]
[4.]]
Taking point R to be: [[3.]
[6.]]
Calculated LHS to be: 2.0
Calculated RHS to be: 2.0
```

Code for this plot can be found at:

Codes/main.py
Codes/main.c