3-3.3-2

ai24btech11030 - Shiven Bajpai

Question: Construct a triangle with sides $5cm$, $6cm$ and $7cm$

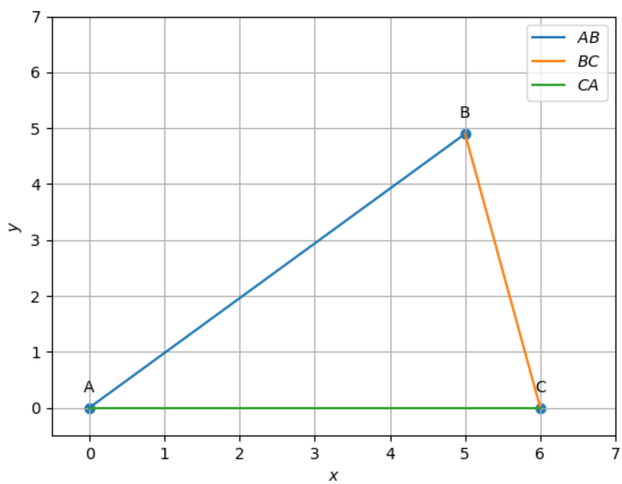
Solution: Let the vertices of triangle be **A**, **B** and **C** and lengths of the sides opposing them be denoted by $a = 5cm$, $b = 6cm$ and $c = 7cm$ respectively.

By Cosine rule in ΔABC,

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{60}{84}$$

Let $\mathbf{A} = \mathbf{0}$ and $\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}$. Then $\mathbf{B} = c\begin{pmatrix} \cos A \\ \sin A \end{pmatrix}$

Substituting values we get, $\mathbf{A} = \mathbf{0}, \mathbf{B} = \begin{pmatrix} 5 \\ \sqrt{24} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$



Code for this plot can be found at:

Codes/main.py
Codes/main.c

Alternate Solution: Let the vertices of triangle be $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ and lengths of the sides opposing them be denoted by $a = 5cm$, $b = 6cm$ and $c = 7cm$ respectively.

Let $\mathbf{A} = \mathbf{0}$ and $\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}$, then $\mathbf{B}$ satisfies

$$\|\mathbf{x} - \mathbf{A}\| = c$$
$$\|\mathbf{x} - \mathbf{C}\| = a$$

Since a point that lies on 2 circles lies on their radical axis, $\mathbf{B}$ lies on the line

$$\|\mathbf{x}\|^2 - 2\mathbf{C}^{\mathrm{T}}\mathbf{x} + b^2 - a^2 = \|\mathbf{x}\|^2 - 2\mathbf{A}^{\mathrm{T}}\mathbf{x} - c^2$$
$$(\mathbf{C} - \mathbf{A})^{\mathrm{T}}\mathbf{x} = \frac{b^2 + c^2 - a^2}{2}$$

Let $\mathbf{n} = (\mathbf{C} - \mathbf{A})$ and $d = \frac{b^2+c^2-a^2}{2}$

Let parametric form of line is $\mathbf{x} = \mathbf{h} + k\mathbf{m}$
Where $\mathbf{h} = \frac{\mathbf{n}d}{\|\mathbf{n}\|^2}$ and $\mathbf{m}$ be perpendicular to $\mathbf{n}$ i.e $\mathbf{m} = \begin{pmatrix} -\mathbf{n}_2 \\ \mathbf{n}_1 \end{pmatrix}$

Substituting $\mathbf{x}$ in equation of circle

$$(\mathbf{h} + k\mathbf{m} - \mathbf{A})^{\mathrm{T}}(\mathbf{h} + \mathbf{k}m - \mathbf{A}) = c^2$$
$$((\mathbf{h} - \mathbf{A}) + k\mathbf{m})^{\mathrm{T}}((\mathbf{h} - \mathbf{A}) + k\mathbf{m}) = c^2$$
$$(\mathbf{h} - \mathbf{A})^{\mathrm{T}}(\mathbf{h} - \mathbf{A}) + 2k\mathbf{m}^{\mathrm{T}}(\mathbf{h} - \mathbf{A}) + k^2\mathbf{m}^{\mathrm{T}}\mathbf{m} = c^2$$

Since $\mathbf{m}^T\mathbf{h} = 0$

$$(\mathbf{h} - \mathbf{A})^{\mathrm{T}}(\mathbf{h} - \mathbf{A}) - 2k\mathbf{m}^{\mathrm{T}}\mathbf{A} + k^2\mathbf{m}^{\mathrm{T}}\mathbf{m} = c^2$$

Using quadratic formula for k,

$$k = \frac{2\mathbf{m}^{\mathrm{T}}\mathbf{A} \pm \sqrt{4(\mathbf{m}^{\mathrm{T}}\mathbf{A})^2 - 4(\mathbf{m}^{\mathrm{T}}\mathbf{m})((\mathbf{h} - \mathbf{A})^{\mathrm{T}}(\mathbf{h} - \mathbf{A}) - c^2)}}{2\mathbf{m}^{\mathrm{T}}\mathbf{m}}$$

$$k = \frac{\mathbf{m}^{\mathrm{T}}\mathbf{A} \pm \sqrt{(\mathbf{m}^{\mathrm{T}}\mathbf{A})^2 - (\mathbf{m}^{\mathrm{T}}\mathbf{m})((\mathbf{h} - \mathbf{A})^{\mathrm{T}}(\mathbf{h} - \mathbf{A}) - c^2)}}{\mathbf{m}^{\mathrm{T}}\mathbf{m}}$$

Since we have assumed $\mathbf{A} = \mathbf{0}$, We can simplify this to

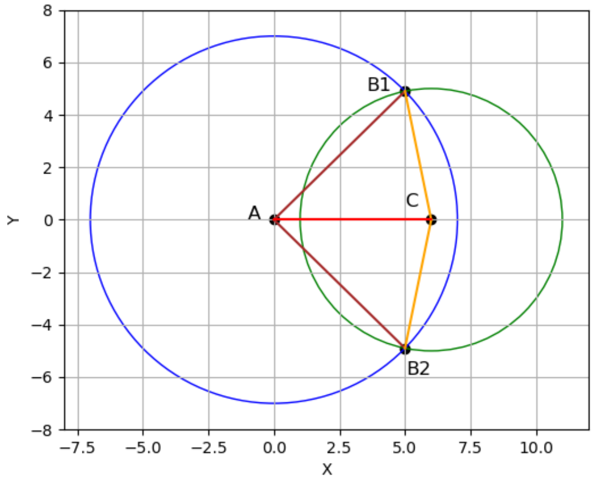
$$k = \frac{\pm\sqrt{(c^2 - \mathbf{h}^{\mathrm{T}}\mathbf{h})}}{\sqrt{\mathbf{m}^{\mathrm{T}}\mathbf{m}}}$$

Then $\mathbf{x}$ is

$$\mathbf{x} = \mathbf{h} \pm \frac{\sqrt{c^2 - ||\mathbf{h}||^2}\ \mathbf{m}}{||\mathbf{m}||}$$

Now substituting data,

$$d = 30$$
$$\mathbf{h} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$
$$\mathbf{m} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$
$$k = \pm\frac{\sqrt{24}}{6}$$
$$\mathbf{B} = \begin{pmatrix} 5 \\ \pm\sqrt{24} \end{pmatrix}$$



Code for this plot can be found at:

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Codes/alt.py
Codes/alt.c
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