1-1.9-4

ai24btech11030 - Shiven Bajpai

Question: If $\|\mathbf{a}\| = 4$ and $-3 \le \lambda \le 2$, then $\|\lambda\mathbf{a}\|$ lies in

1) $[0, 12]$
2) $[2, 3]$
3) $[8, 12]$
4) $[-12, 8]$

Solution: 1) $[0, 12]$

$$\begin{aligned}\|\mathbf{a}\| &= \sqrt{\mathbf{a}\mathbf{a}^{\mathrm{T}}} \\ \|\lambda\mathbf{a}\| &= \sqrt{(\lambda\mathbf{a})\,(\lambda\mathbf{a})^{\mathrm{T}}} \\ &= \sqrt{\lambda^2\mathbf{a}\mathbf{a}^{\mathrm{T}}} \\ &= |\lambda|\sqrt{\mathbf{a}\mathbf{a}^{\mathrm{T}}}\end{aligned}$$

$$\|\lambda\mathbf{a}\| = |\lambda|\,\|\mathbf{a}\|$$

Now, since

$$\begin{aligned}-3 \le \lambda &\le 2 \\ 0 \le |\lambda| &\le 3 \\ 0\|\mathbf{a}\| \le |\lambda|\,\|\mathbf{a}\| &\le 3\|\mathbf{a}\| \\ 0 \le \|\lambda\mathbf{a}\| &\le 12\end{aligned}$$

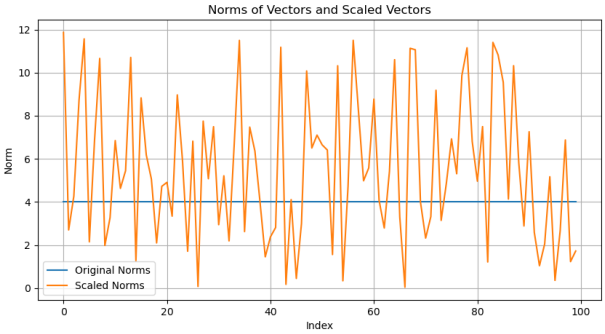

Fig. 4: Plot of norms of 100 random vectors, multiplied by random λ for each vector, One can see that norm of $\lambda\mathbf{a}$ is in range $[0, 12]$

Code for this plot can be found at:

Codes/main.py
Codes/main.c