

# Mains - 14.A+B

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## SECTION - E

- 1) Prove that  $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$ . (2002  
- 5 Marks)

## SECTION - F

1) Match The Following

(2005 - 6M)

**Column I****Column II**

- a)  $\sum_{i=1}^{\infty} \tan^{-1} \left( \frac{1}{2i^2} \right) = t$ , then  $\tan t =$
- b) Sides  $a, b, c$  of a triangle ABC are in AP and  $\cos \theta_1 = \frac{a}{b+c}, \cos \theta_2 = \frac{b}{a+c}, \cos \theta_3 = \frac{c}{a+b}$  then  $\tan^2 \left( \frac{\theta_1}{2} \right) + \tan^2 \left( \frac{\theta_3}{2} \right) =$
- c) A line is perpendicular to  $x + 2y + 2z = 0$  and passes through  $(0,1,0)$ . The perpendicular distance of this line from the origin is

- a) 1
- b)  $\frac{\sqrt{5}}{3}$
- c)  $\frac{2}{3}$

2) Let  $(x, y)$  be such that  $\sin^{-1}(ax) + \cos^{-1}(bxy) = \frac{\pi}{2}$ .

Match the statements in Column 1 with statements in Column II and indicate your answer by darkening the appropriate bubble in the 4x4 matrix given in the ORS.

- |   |                                       |
|---|---------------------------------------|
| a) If $a = 1$ and $b = 0$ , then $(x, y)$ | a) lies on the circle $x^2 + y^2 = 1$ |
| b) If $a = 1$ and $b = 1$ , then $(x, y)$ | b) lies on $(x^2 - 1)(y^2 - 1) = 0$   |
| c) If $a = 1$ and $b = 2$ , then $(x, y)$ | c) lies on $y = x$                    |
| d) If $a = 2$ and $b = 2$ , then $(x, y)$ | d) lies on $(4x^2 - 1)(y^2 - 1) = 0$  |

**DIRECTIONS(Q.3):** Following questions has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

- 3) a)  $\left( \frac{1}{y^2} \left( \frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)} \right)^2 + y^4 \right)^{\frac{1}{2}}$  takes value
- b) If  $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$  then possible value of  $\cos \frac{x-y}{2}$  is
- c) If  $\cos \left( \frac{\pi}{4} - x \right) \cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos \left( \frac{\pi}{4} + x \right) \cos 2x$  then possible value of  $\sec x$  is
- d) If  $\cot(\sin^{-1} \sqrt{1-x^2}) = \sin(\tan^{-1}(x\sqrt{6}))$ ,  $x \neq 0$

- a)  $\frac{1}{2} \sqrt{\frac{5}{3}}$
- b)  $\sqrt{2}$
- c)  $\frac{1}{2}$
- d) 1

**Codes:**

	P	Q	R	S
(a)	4	3	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	3	4	1	2

## I - INTEGER VALUE CORRECT TYPE

- 1) The number of real solutions of the equation  $\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty} \left(\frac{-x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i\right)$  lying in the interval  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  is? (Here, the inverse trigonometric function  $\sin^{-1} x$  and  $\cos^{-1} x$  assume values in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  and  $[0, \pi]$  respectively (JEE Adv. 2018))

- 2) The value of  $\sec^{-1}\left(\frac{1}{4} \sum_{k=0}^{10} \sec\left(\frac{7\pi}{10} + \frac{k\pi}{10} \sec \frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$  in the interval  $[-\frac{\pi}{4}, \frac{3\pi}{4}]$  equals (JEE Adv 2019)

## SECTION B - JEE MAIN / AIEEE

- 1)  $\cos^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha})$ , then  $\sin x =$  [2002]

- a)  $\tan^2\left(\frac{\alpha}{2}\right)$  c)  $\tan \alpha$   
b)  $\cot^2\left(\frac{\alpha}{2}\right)$  d)  $\cot\left(\frac{\alpha}{2}\right)$

- 2) The trigonometric equation  $\sin^{-1} x = 2 \sin^{-1} a$  has a solution for [2003]

- a)  $|\alpha| \geq \frac{1}{\sqrt{2}}$  c) all real values of  $a$   
b)  $\frac{1}{2} < |\alpha| < \frac{1}{\sqrt{2}}$  d)  $|\alpha| < \frac{1}{2}$

- 3) If  $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$ , then  $4x^2 - 4xy \cos \alpha + y^2$  is equal to [2005]

- a)  $2 \sin 2\alpha$  c)  $4 \sin^2 \alpha$   
b) 4 d)  $-4 \sin^2 \alpha$

- 4) If  $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ , then the value of  $x$  is [2007]

- a) 4 c) 1  
b) 5 d) 3

- 5) The value of  $\cot\left(\operatorname{cosec}^{-1} \frac{5}{3} + \tan^{-1}(2)3\right)$

- a)  $\frac{6}{17}$  c)  $\frac{4}{17}$   
b)  $\frac{3}{17}$  d)  $\frac{5}{17}$

- 6) If  $x, y, z$  are in AP and  $\tan^{-1} x, \tan^{-1} y$  and  $\tan^{-1} z$  are also in A.P, then [JEE M 2013]

- a)  $x = y = z$  c)  $6x = 3y = 2z$   
b)  $2x = 3y = 6z$  d)  $6x = 4y = 3z$

- 7) Let  $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ , where  $|x| < \frac{1}{\sqrt{3}}$ . Then a value of  $y$  is [JEE M 2015]

- a)  $\frac{3x-x^3}{1+3x}$  c)  $\frac{3x-x^3}{1-3x}$   
b)  $\frac{3x+x^3}{1+3x}$  d)  $\frac{3x+x^3}{1-3x}$

- 8) If  $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$  ( $x > \frac{3}{4}$ ), then  $x$  is equal to [JEE M 2019 - 9 Jan M]

- a)  $\frac{\sqrt{145}}{12}$  c)  $\frac{\sqrt{146}}{12}$   
b)  $\frac{\sqrt{145}}{10}$  d)  $\frac{\sqrt{145}}{11}$