GATE - Maths - 2008 - 1-17

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1) Consider the subspace $W = \{[a_{ij}] : a_{ij} = 0 \text{ if } i \text{ is even}\}$ of all 10x10 real matrices. Then the dimension of W is

a) 25 b) 50 c) 75 d) 100

2) Let S be the open unit disk and $f : S \to C$ be a real-valued analytic function with $f(0) = 1$. Then the set $\{z \in S : f(z) \neq 1\}$ is

a) empty
b) nonempty finite
c) countably infinite
d) uncountable

3) Let $E = \{(x, y) \in R^2 : 0 \leq x \leq 1, 0 \leq y \leq x\}$. Then $\int_E \int (x + y)\, dx\, dy$ is equal to

a) -1 b) 0 c) 112 d) 1

4) For $(x, y) \in R^2$, let $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{if } (x, y) = (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ Then

a) f_x and f_y exist at (0,0), and f is continuous at (0,0)
b) f_x and f_y exist at (0,0), and f is discontinuous at (0,0)
c) f_x and f_y do not exist at (0,0), and f is continuous at (0,0)
d) f_x and f_y do not exist at (0,0), and f is discontinuous at (0,0)

5) Let y be a solution of $y' = e^{-y^2} - 1$ on $[0, 1]$ which satisfies $y(0) = 0$. Then

a) $y(x) > 0$ for $x > 0$
b) y changes sign in $[0, 1]$
c) $y(x) < 0$ for $x > 0$
d) $y = 0$ for $x > 0$

6) For the equation $x(x - 1)y'' + sin(x)y' + 2x(x - 1)y = 0$, consider the following statements

P: $x = 0$ is a regular singular point.
Q: $x = 1$ is a regular singular point.
Then

a) both P and Q are true
b) P is true but Q is false
c) P is false but Q is true
d) both P and Q are false

7) Let $G = R\backslash\{0\}$ and $H = \{-1, 1\}$ be groups under multiplication. Then the map $\phi : G \to H$ defined by $\phi(x) = \frac{x}{|x|}$ is

a) not a homomorphism

b) a one-one homomorphism, which is not onto
c) an onto homomorphism, which is not one-one
d) an isomorphism

8) The number of maximal ideals in $\mathbb{Z}_{27}$ is

a) 0 b) 1 c) 2 d) 3

9) For $1 \leq p \leq \infty$, let $\| \|_p$ denote the p-norm on $\mathbb{R}^2$. If $\| \|_p$ satisfies the parallelogram law, then p is equal to

a) 1 b) 2 c) 3 d) ∞

10) Consider the initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$. The aim is to compute the value of $y_1 = y(x_1)$, where $x_1 = x_0 + h(h > 0)$. At $x = x_1$, if the value of y_1 is equated to the corresponding value of the straight line passing through (x_0, y_0) and having the slope equal to the slope of the curve $y(x)$ at $x = x_0$, then the method is called

a) Euler's method
b) Improved Euler's method
c) Backward Euler's method
d) Taylor series method of order 2

11) The solution of $xu_x + yu_y = 0$ is of the form

a) $f(y/x)$ b) $f(x + y)$ c) $f(x - y)$ d) $f(x)$

12) If the partial differential equation $(x-1)^2u_{xx} - (y-2)^2u_{yy}, +2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 \backslash S$, then S is

a) $\{(x, y) \in \mathbb{R} : x = 1 \text{ or } y = 2\}$
b) $\{(x, y) \in \mathbb{R} : x = 1\}$
c) $\{(x, y) \in \mathbb{R}^2 : x = 1 \text{ and } y = 2\}$
d) $\{(x, y) \in \mathbb{R}^2 : y = 2\}$

13) Let E be a connected subset of $\mathbb{R}$ with at least two elements. Then the number of elements in E is

a) exactly two
b) countably infinite
c) more than two but finite
d) uncountable

14) Let X be a non-empty set. Let ϵ_1, and ϵ_2, be two topologies on X such that ϵ_1, is strictly contained in ϵ_2. If $I : (X, \epsilon_1) \to (X, \epsilon_2)$ is the identity map, then
a) both I and I^{-1} are continuous
b) I is continuous but I^{-1} is not continuous
c) both I and I^{-1} are not continuous
d) I is not continuous but I^{-1} is continuous

15) Let $X_1, X_2, \ldots X_{10}$ be a random sample from $N(80, 3^2)$ distribution. Define

$$S = \sum_{i=1}^{10} U_i \text{ and } T = \sum_{i=1}^{10} \left(U_i - \frac{S}{10} \right)^2$$

where $U_i = \frac{X_i - 80}{3}, i = 1, 2, \ldots, 10$. Then the value of $E(ST)$ is equal to

a) 0 b) 1 c) 10 d) $\frac{80}{3}$

16) Two (distinguishable) fair coins are tossed simultaneously. Given that ONE of them lands up head, the probability of the OTHER to land up tail is equal to

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$

17) Let $c_{ij} \geq 2$ be the cost of the $(i, j)^{th}$ cell of an assignment problem. If a new cost matrix is generated by the elements $c^*_{ij} = \frac{1}{2} c_{ij} + 1$, then

a) optimal assignment plan remains unchanged and cost of assignment decreases
b) optimal assignment plan changes and cost of assignment decreases
c) optimal assignment plan remains unchanged and cost of assignment increases
d) optimal assignment plan changes and cost of assignment increases