

Mains - 14.A+B

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SECTION - E

- 1) Prove that $\cos \tan^{-1} x \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$. (2002 - 5 Marks)

SECTION - F

- 1) Match The Following (2005 - 6M)

Column I

Column II

- a) $\sum_{i=1}^{\infty} \tan^{-1} \left(\frac{1}{2i^2} \right) = t$, then $\tan t =$ a) 1
 b) Sides a, b, c of a triangle ABC are in AP and $\cos \theta_1 = \frac{a}{b+c}$, $\cos \theta_2 = \frac{b}{a+c}$, $\cos \theta_3 = \frac{c}{a+b}$ then $\tan^2 \left(\frac{\theta_1}{2} \right) + \tan^2 \left(\frac{\theta_2}{2} \right) =$ b) $\frac{\sqrt{5}}{3}$
 c) A line is perpendicular to $x+2y+2z=0$ and passes through $(0, 1, 0)$. The perpendicular distance of this line from the origin is c) $\frac{2}{3}$
- 2) Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(bxy) = \frac{\pi}{2}$.
 Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubble in the 4x4 matrix given in the ORS. (2007)

- | | |
|---|---------------------------------------|
| a) If $a = 1$ and $b = 0$, then (x, y) | a) lies on the circle $x^2 + y^2 = 1$ |
| b) If $a = 1$ and $b = 1$, then (x, y) | b) lies on $(x^2 - 1)(y^2 - 1) = 0$ |
| c) If $a = 1$ and $b = 2$, then (x, y) | c) lies on $y = x$ |
| d) If $a = 2$ and $b = 2$, then (x, y) | d) lies on $(4x^2 - 1)(y^2 - 1) = 0$ |

DIRECTIONS (Q.3): Following questions has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

- 3) Match List I with List II and select the correct answer using the code given below the lists: (JEE Adv. 2013)

List I

- a) $\left(\frac{1}{y^2} \left(\frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)} \right)^2 + y^4 \right)^{\frac{1}{2}}$
takes value
- b) If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos \frac{x-y}{2}$ is
- c) If $\cos\left(\frac{\pi}{4} - x\right) \cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right) \cos 2x$ then possible value of $\sec x$ is
- d) If $\cot\left(\sin^{-1} \sqrt{1-x^2}\right) = \sin\left(\tan^{-1}(x\sqrt{6})\right)$, $x \neq 0$

List II

- a) $\frac{1}{2} \sqrt{\frac{5}{3}}$
- b) $\sqrt{2}$
- c) $\frac{1}{2}$
- d) 1

Codes:

	P	Q	R	S
(a)	4	3	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	3	4	1	2

I - INTEGER VALUE CORRECT TYPE

- 1) The number of real solutions of the equation $\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty} \left(\frac{-x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i\right)$ lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is? (Here, the inverse trigonometric function $\sin^{-1} x$ and $\cos^{-1} x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively) (JEE Adv. 2018)
- 2) The value of $\sec^{-1}\left(\frac{1}{4} \sum_{k=0}^{10} \sec\left(\frac{7\pi}{10} + \frac{k\pi}{10} \sec \frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals (JEE Adv 2019)

SECTION B - JEE MAIN / AIEEE

- 1) $\cos^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha})$, then $\sin x =$ (2002)
- a) $\tan^2\left(\frac{\alpha}{2}\right)$ b) $\cot^2\left(\frac{\alpha}{2}\right)$ c) $\tan \alpha$ d) $\cot\left(\frac{\alpha}{2}\right)$
- 2) The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$ has a solution for (2003)
- a) $|\alpha| \geq \frac{1}{\sqrt{2}}$ c) all real values of a
- b) $\frac{1}{2} < |\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| < \frac{1}{2}$
- 3) If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to (2005)

a) $2 \sin 2\alpha$

b) 4

c) $4 \sin^2 \alpha$

d) $-4 \sin^2 \alpha$

4) If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is (2007)

a) 4

b) 5

c) 1

d) 3

5) The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$

a) $\frac{6}{17}$

b) $\frac{3}{17}$

c) $\frac{4}{17}$

d) $\frac{5}{17}$

6) If x, y, z are in AP and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in A.P, then (JEE M 2013)

a) $x = y = z$

b) $2x = 3y = 6z$

c) $6x = 3y = 2z$

d) $6x = 4y = 3z$

7) Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is (JEE M 2015)

a) $\frac{3x-x^3}{1+3x}$

b) $\frac{3x+x^3}{1+3x}$

c) $\frac{3x-x^3}{1-3x}$

d) $\frac{3x+x^3}{1-3x}$

8) If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$ ($x > \frac{3}{4}$), then x is equal to (JEE M 2019 - 9 Jan M)

a) $\frac{\sqrt{145}}{12}$

b) $\frac{\sqrt{145}}{10}$

c) $\frac{\sqrt{146}}{12}$

d) $\frac{\sqrt{145}}{11}$