GATE - PH - 2015 - 53 - 65

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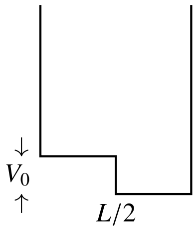
1) A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $v(0) = 10\,\mathrm{ms}^{-1}$. If the air exerts a frictional force of the form, $f = -kv$, then for $k = 0.05\,\mathrm{Nm}^{-1}\cdot\mathrm{s}$, the velocity (in ms^{-1}) at time $t = 0.2\,\mathrm{s}$ is ......(up to two decimal places). (use $g = 10\,\mathrm{ms}^{-2}$ and $e = 2.72$) (GATE PH 2015)
2) In the nuclear shell model, the potential is modeled as $V(r) = \frac{1}{2}m\omega^2r^2 - \lambda\,\mathbf{L}\cdot\mathbf{S}, \lambda > 0$. The correct spin-parity and isospin assignments for the ground state of ^{13}C is (GATE PH 2015)

a) $\frac{1}{2}^-; -\frac{1}{2}$ b) $\frac{1}{2}^+; -\frac{1}{2}$ c) $\frac{3}{2}^+; -\frac{1}{2}$ d) $\frac{3}{2}^-; -\frac{1}{2}$

3) A particle is confined in a box of length L as shown below.



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is (GATE PH 2015)

a) $E = \frac{\hbar^2\pi^2}{2mL^2} + V_0$ b) $E = \frac{\hbar^2\pi^2}{2mL^2} - \frac{V_0}{2}$ c) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{4}$ d) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{2}$

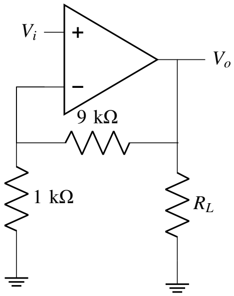
4) Suppose a linear harmonic oscillator of frequency ω and mass m is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_0\rangle + e^{\frac{\pi}{2}}|\psi_1\rangle\right) \text{ at } t = 0$$

where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\psi|x|\psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is .... (GATE PH 2015)
5) A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z axis. Their velocities are given by (GATE PH 2015)

a) $\mathbf{v}_1 = \mathbf{v}_2 = (0.8c)\hat{z}$
b) $\mathbf{v}_1 = -\mathbf{v}_2 = (0.8c)\hat{z}$
c) $\mathbf{v}_1 = -\mathbf{v}_2 = (0.6c)\hat{z}$
d) $\mathbf{v}_1 = (0.6c)\hat{z}; \quad \mathbf{v}_2 = (-0.8c)\hat{z}$

6) In the given circuit, if the open loop gain $A = 10^5$, the feedback configuration and the closed loop gain A_f are



(GATE PH 2015)

a) series-shunt, $A_f = 9$
b) series-series, $A_f = 10$
c) series-shunt, $A_f = 10$
d) shunt-shunt, $A_f = 10$

7) A function $y(z)$ satisfies the ordinary differential equation

$$y'' + \frac{1}{z}y' - \frac{m^2}{z^2}y = 0,$$

where $m = 0, 1, 2, 3, \dots$. Consider the four statements P, Q, R, S as given below.
P: z^m and z^{-m} are linearly independent solutions for all values of m
Q: z^m and z^{-m} are linearly independent solutions for all values of $m > 0$
R: $\ln z$ and 1 are linearly independent solutions for $m = 0$
S: z^m and $\ln z$ are linearly independent solutions for all values of m
The correct option for the combination of valid statements is (GATE PH 2015)

a) P, R and S only b) P and R only c) Q and R only d) R and S only

8) The entropy of a gas containing N particles enclosed in a volume V is given by $S = Nk_B \ln\left(\frac{aVE^{\frac{3}{2}}}{N^{\frac{5}{2}}}\right)$, where E is the total energy, a is a constant, and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by (GATE PH 2015)

a) $\mu = -k_B T\left[\ln\left(\frac{aVE^{\frac{3}{2}}}{N^{\frac{5}{2}}}\right) - \frac{5}{2}\right]$
b) $\mu = -k_B T\left[\ln\left(\frac{aVE^{\frac{3}{2}}}{N^{\frac{5}{2}}}\right) - \frac{3}{2}\right]$
c) $\mu = -k_B T\left[\ln\left(\frac{aVE^{\frac{3}{2}}}{N^{\frac{3}{2}}}\right) - \frac{5}{2}\right]$
d) $\mu = -k_B T\left[\ln\left(\frac{aVE^{\frac{3}{2}}}{N^{\frac{3}{2}}}\right) - \frac{3}{2}\right]$

9) Let the Hamiltonian for two spin-$\frac{1}{2}$ particles of equal masses m, momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, and positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ be

$$H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2(r_1^2 + r_2^2) + k\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$

where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1\,\text{eV}$ and $k = 0.2\,\text{eV}$. If the ground state has net spin zero, then the energy (in eV) is .... (GATE PH 2015)

10) The average energy U of a one dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature T is given by (GATE PH 2015)

a) $U = \frac{1}{2}\hbar\omega \coth\left(\frac{1}{2}\beta\hbar\omega\right)$

b) $U = \frac{1}{2}\hbar\omega \sinh\left(\frac{1}{2}\beta\hbar\omega\right)$

c) $U = \frac{1}{2}\hbar\omega \tanh\left(\frac{1}{2}\beta\hbar\omega\right)$

d) $U = \frac{1}{2}\hbar\omega \cosh\left(\frac{1}{2}\beta\hbar\omega\right)$

11) A monochromatic plane wave (wavelength = 600 nm) $E_0 \exp[i(kz - \omega t)]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1 \exp\,[i\,(\boldsymbol{k}_1 \cdot \boldsymbol{r} - \omega t)]$ in the first order of diffraction. Here $E_1 < E_0$ and $\boldsymbol{k}_1 = |\boldsymbol{k}_1|\left[\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}\right]$. The period (in μm) of the diffraction grating is ......(upto one decimal place) (GATE PH 2015)

12) The Heaviside function is defined as $H(t) = \begin{cases} +1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$ and its Fourier transform is given by $-\frac{2i}{\omega}$. The Fourier transform of $\frac{1}{2}\left[H(t+1/2) - H(t-1/2)\right]$ is (GATE PH 2015)

a) $\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}$

b) $\frac{\cos\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}$

c) $\sin\left(\frac{\omega}{2}\right)$

d) 0

13) The atomic masses of ${}^{152}_{63}\mathrm{Eu}$, ${}^{152}_{62}\mathrm{Sm}$, ${}^{1}_{1}\mathrm{H}$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (amu), respectively. Using the above information, the Q-value of the reaction ${}^{152}_{63}\mathrm{Eu} + n \rightarrow {}^{152}_{62}\mathrm{Sm} + p$ is ......$\times 10^{-3}$ amu (up to three decimal places) (GATE PH 2015)