1-1.9-4

ai24btech11030 - Shiven Bajpai

Question: If $\|\mathbf{a}\| = 4$ and $-3 \le \lambda \le 2$, then $\|\lambda\mathbf{a}\|$ lies in

1) $[0, 12]$
2) $[2, 3]$
3) $[8, 12]$
4) $[-12, 8]$

Solution: 1) $[0, 12]$

$$\|\mathbf{a}\| = \sqrt{\mathbf{aa}^{\mathrm{T}}}$$

$$\|\lambda\mathbf{a}\| = \sqrt{(\lambda\mathbf{a})\,(\lambda\mathbf{a})^{\mathrm{T}}}$$

$$= \sqrt{\lambda^2\mathbf{aa}^{\mathrm{T}}}$$

$$= |\lambda|\,\sqrt{\mathbf{aa}^{\mathrm{T}}}$$

$$\|\lambda\mathbf{a}\| = |\lambda|\,\|\mathbf{a}\|$$

Now, since

$$-3 \le \lambda \le 2$$

$$0 \le |\lambda| \le 3$$

$$0\|\mathbf{a}\| \le |\lambda|\,\|\mathbf{a}\| \le 3\|\mathbf{a}\|$$
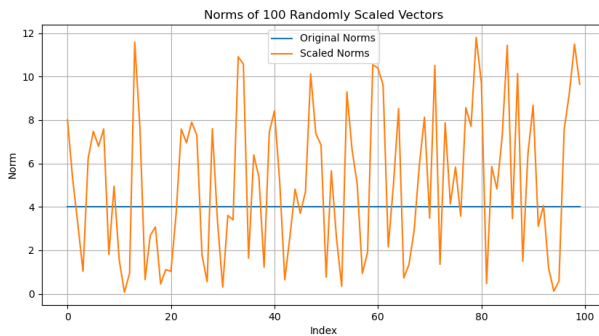
$$0 \le \|\lambda\mathbf{a}\| \le 12$$

Fig. 4: Norms of 100 randomly scaled vectors, clearly $\|\lambda \mathbf{a}\| \leq 12$

Code for this plot can be found at:

Codes/main.py
Codes/main.c