JEEM - 1Sep2021 - Shift1 - 1-15

ai24btech11030 - Shiven Bajpai

1) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Then $\lim_{x\to\frac{\pi}{4}} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2-\frac{\pi^2}{16}}$ is equal to: (1 Sep 2021 S1)

   a) $f(2)$ b) $2f(2)$ c) $2f(\sqrt{2})$ d) $4f(2)$

2) $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to: (1 Sep 2021 S1)

   a) $3\pi - 11$ b) $4\pi - 9$ c) $4\pi - 11$ d) $3\pi + 1$

3) Consider the system of linear equations:

$$-x + y + 2z = 0$$
$$3x - ay + 5z = 1$$
$$2x - 2y - az = 7$$

   Let S_1 be the set of all $a \in \mathbb{R}$ for which the system is inconsistent, and S_2 the set of all $a \in \mathbb{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then: (1 Sep 2021 S1)

   a) $n(S_1) = 2,\ n(S_2) = 2$
   b) $n(S_1) = 1,\ n(S_2) = 0$
   c) $n(S_1) = 2,\ n(S_2) = 0$
   d) $n(S_1) = 0,\ n(S_2) = 2$

4) Let the acute angle bisector of the planes $x-2y-2z+1 = 0$ and $2x-3y-6z+1 = 0$ be the plane P. Which of the following points lies on P? (1 Sep 2021 S1)

   a) $(3, 1, -\frac{1}{2})$
   b) $(-2, 0, -\frac{1}{2})$
   c) $(0, 2, -4)$
   d) $(4, 0, -2)$

5) Which of the following is equivalent to the Boolean expression $p \wedge \neg q$? (1 Sep 2021 S1)

   a) $\neg(q \to p)$ b) $\neg p \to \neg q$ c) $\neg(p \to \neg q)$ d) $\neg(p \to q)$

6) Two squares are chosen at random on a chessboard. The probability that they have a side in common is: (1 Sep 2021 S1)

a) $\frac{2}{7}$ b) $\frac{1}{18}$ c) $\frac{1}{7}$ d) $\frac{1}{9}$

7) If $y = y(x)$ is the solution of the differential equation $x^2dy + \left(y - \frac{1}{x}\right)dx = 0$; $x > 0$ and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to: (1 Sep 2021 S1)

a) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ b) $3 + \frac{1}{\sqrt{e}}$ c) $3 + e$ d) $3 - e$

8) If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1,\ x \in [0, \pi],$$

and S is the sum of all these solutions, then the ordered pair (n, S) is: (1 Sep 2021 S1)

a) $(3, \frac{13\pi}{9})$ b) $(2, \frac{2\pi}{3})$ c) $(2, \frac{8\pi}{9})$ d) $(3, \frac{5\pi}{3})$

9) The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements:

(S1) There exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) There exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

Then: (1 Sep 2021 S1)

a) Both (S1) and (S2) are true
b) (S1) is false and (S2) is true
c) Both (S1) and (S2) are false
d) (S1) is true and (S2) is false

10) Let

$$J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1}dx, \forall n > m \quad \text{and} \quad n, m \in \mathbb{N}.$$

Consider a matrix $A = [a_{ij}]_{3\times3}$ where

$$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & \text{if } i \leq j, \\ 0, & \text{if } i > j. \end{cases}$$

Then $\left|\text{adj}A^{-1}\right|$ is: (1 Sep 2021 S1)

a) $(15)^2 \times 2^{42}$ b) $(15)^2 \times 2^{34}$ c) $(105)^2 \times 2^{38}$ d) $(105)^2 \times 2^{36}$

11) The area enclosed by the curves $y = |\cos x - \sin x|$ and $y = \sin x + \cos x$, and the lines $x = 0$ and $x = \frac{\pi}{2}$ is: (1 Sep 2021 S1)

a) $2\sqrt{2} - 2$ b) $2 + 2\sqrt{2}$ c) $4 - 2\sqrt{2}$ d) $2 + 4\sqrt{2}$

12) The distance of the line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point $(2, -1, 6)$ is: (1 Sep 2021 S1)

a) $\sqrt{26}$ b) $2\sqrt{5}$ c) $2\sqrt{6}$ d) $4\sqrt{2}$

13) Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to: (1 Sep 2021 S1)

a) $\frac{75}{8}$ b) $\frac{125}{16}$ c) $\frac{25}{2}$ d) $\frac{15}{2}$

14) The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of the equation, is: (1 Sep 2021 S1)

a) 6 b) 2 c) 4 d) 8

15) Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + \cdots + (n-1) \cdot 1$, for $n \geq 4$. The sum

$$\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$$

is equal to: (1 Sep 2021 S1)

a) $\frac{e-1}{3}$ b) $\frac{e-2}{6}$ c) $\frac{e}{3}$ d) $\frac{e}{6}$