Mains - 14.A+B

ai24btech11030 - Shiven Bajpai

Section - E

1) Prove that $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\frac{x^2+1}{x^2+2}}$. (2002 - 5 Marks)

I - Integer Value Correct Type

1) The number of real solutions of the equation $\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(\frac{-x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$ lying in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$ is? (Here, the inverse trignometric function $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $[-\frac{\pi}{2},\frac{\pi}{2}]$ and $[0,\pi]$ respectively (JEE Adv. 2018)

2) The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{10}+\frac{k\pi}{10}\sec\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $[-\frac{\pi}{4},\frac{3\pi}{4}]$ equals (JEE Adv 2019)

Section B - JEE Main / AIEEE

1) $\cos^{-1}\left(\sqrt{\cos\alpha}\right) - \tan^{-1}\left(\sqrt{\cos\alpha}\right)$, then $\sin x =$ [2002]

   a) $\tan^2\left(\frac{\alpha}{2}\right)$
   b) $\cot^2\left(\frac{\alpha}{2}\right)$
   c) $\tan\alpha$
   d) $\cot\left(\frac{\alpha}{2}\right)$

2) The trignometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for [2003]

   a) $|\alpha| \geq \frac{1}{\sqrt{2}}$
   b) $\frac{1}{2} < |\alpha| < \frac{1}{\sqrt{2}}$
   c) all real values of a
   d) $|\alpha| < \frac{1}{2}$

3) If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to [2005]

   a) $2\sin 2\alpha$
   b) 4
   c) $4\sin^2\alpha$
   d) $-4\sin^2\alpha$

4) If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is [2007]

   a) 4
   b) 5
   c) 1
   d) 3

5) The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}(2)\,3\right)$

   a) $\frac{6}{17}$
   b) $\frac{3}{17}$
   c) $\frac{4}{17}$
   d) $\frac{5}{17}$

6) If x,y,z are in AP and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P, then [JEE M 2013]

   a) $x = y = z$
   b) $2x = 3y = 6z$
   c) $6x = 3y = 2z$
   d) $6x = 4y = 3z$

7) Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is [JEE M 2015]

   a) $\frac{3x-x^3}{1+3x}$
   b) $\frac{3x+x^3}{1+3x}$
   c) $\frac{3x-x^3}{1-3x}$
   d) $\frac{3x+x^3}{1-3x}$

8) If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to [JEE M 2019 - 9 Jan M]

   a) $\frac{\sqrt{145}}{12}$
   b) $\frac{\sqrt{145}}{10}$
   c) $\frac{\sqrt{146}}{12}$
   d) $\frac{\sqrt{145}}{11}$

Section - F

1) Match The Following (2005 - 6M)

Column I	Column II
a) $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then tan t =	a) 1
b) Sides a, b, c of a triangle ABC are in AP and $\cos\theta_1 = \frac{a}{b+c}, \cos\theta_2 = \frac{b}{a+c}, \cos\theta_3 = \frac{c}{a+b}$ then $\tan^2\left(\frac{\theta_1}{2}\right) + \tan^2\left(\frac{\theta_3}{2}\right) =$	b) $\frac{\sqrt{5}}{3}$ c) $\frac{2}{3}$
c) A line is perpendicular to $x + 2y + 2z = 0$ and passes through (0,1,0). The perpendicular distance of this line from the origin is	

2) Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(bxy) = \frac{\pi}{2}$.
Match the statements in Column 1 with statements in Column II and indicate your answer by darkening the appropriate bubble in the 4x4 matrix given in the ORS.

a) If $a = 1$ and $b = 0$, then (x, y)	a) lies on the circle $x^2 + y^2 = 1$
b) If $a = 1$ and $b = 1$, then (x, y)	b) lies on $(x^2 - 1)(y^2 - 1) = 0$
c) If $a = 1$ and $b = 2$, then (x, y)	c) lies on $y = x$
d) If $a = 2$ and $b = 2$, then (x, y)	d) lies on $(4x^2 - 1)(y^2 - 1) = 0$

DIRECTIONS(Q.3): *Following questions has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

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3) a) $\left(\frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1} y\right)+y\sin\left(\tan^{-1} y\right)}{\cot\left(\sin^{-1} y\right)+\tan\left(\sin^{-1} y\right)}\right)^2 + y^4\right)^{\frac{1}{2}}$ takes value	a) $\frac{1}{2}\sqrt{\frac{5}{3}}$
b) If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\frac{x-y}{2} is$	b) $\sqrt{2}$ c) $\frac{1}{2}$
c) If $\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$ then possible value of $\sec x$ is	d) 1
d) If $\cot\left(\sin^{-1}\sqrt{1 - x^2}\right) = \sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x \neq 0$	

Codes:

	P	**Q**	**R**	**S**
(a)	4	3	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	3	4	1	2