# Mains - 14.A+B

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#### Section - E

1) Prove that  $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$ . (2002) - 5 Marks)

#### I - Integer Value Correct Type

- 1) The number of real solutions of the equation  $\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i\right)$  $\cos^{-1}\left(\sum_{i=1}^{\infty}\left(\frac{-x}{2}\right)^i-\sum_{i=1}^{\infty}\left(-x\right)^i\right)$  lying in the interval  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  is? (Here, the inverse trignometric function  $\sin^{-1} x$  and  $\cos^{-1} x$  assume values in  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and  $[0, \pi]$  respectively (JEE Adv.
- The value  $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{10} + \frac{k\pi}{10}\sec\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$ 2) The in the interval  $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$  equals (JEE Adv 2019)

# SECTION B - JEE MAIN / AIEEE

- 1)  $\cos^{-1}(\sqrt{\cos \alpha}) \tan^{-1}(\sqrt{\cos \alpha})$ , then  $\sin x =$ (2002)
  - a)  $\tan^2\left(\frac{\alpha}{2}\right)$ b)  $\cot^2\left(\frac{\alpha}{2}\right)$

- c)  $\tan \alpha$ d)  $\cot \left(\frac{\alpha}{2}\right)$
- 2) The trignometric equation  $\sin^{-1} x = 2 \sin^{-1} a$ has a solution for (2003)

  - a)  $|\alpha| \ge \frac{1}{\sqrt{2}}$  c) all real values of a b)  $\frac{1}{2} < |\alpha| < \frac{1}{\sqrt{2}}$  d)  $|\alpha| < \frac{1}{2}$
- 3) If  $\cos^{-1} x \cos^{-1} \frac{y}{2} = \alpha$ , then  $4x^2 4xy \cos \alpha + y^2$ is equal to (2005)
  - a)  $2 \sin 2\alpha$
- c)  $4\sin^2\alpha$

b) 4

- d)  $-4\sin^2\alpha$
- 4) If  $\sin^{-1}\left(\frac{x}{5}\right) + \csc^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ , then the value

a) 4

c) 1

b) 5

- d) 3
- 5) The value of  $\cot\left(\csc^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$

- 6) If x,y,z are in AP and  $\tan^{-1} x$ ,  $\tan^{-1} y$  and  $\tan^{-1} z$ are also in A.P, then (JEE M 2013)
- a) x = y = zb) 2x = 3y = 6zc) 6x = 3y = 2zd) 6x = 4y = 3z
- 7) Let  $\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2}\right)$ , where |x| < 1 $\frac{1}{\sqrt{3}}$ . Then a value of y is
  - a)  $\frac{3x-x^3}{1+3x}$
- c)  $\frac{3x-x^3}{1-3x}$
- b)  $\frac{3x+x^3}{1+3x}$
- d)  $\frac{3x+x^3}{1+2x}$
- 8) If  $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$ , then x is equal to (JEE M 2019 9 Jan M)
  - a)  $\frac{\sqrt{145}}{12}$
- c)  $\frac{\sqrt{146}}{12}$
- b)  $\frac{\sqrt{145}}{10}$
- d)  $\frac{\sqrt{145}}{11}$

# Section - F

1) Match The Following

(2005 - 6M)

## Column I

### Column II

a) 
$$\sum_{i=1}^{\infty} \tan^{-1} \left( \frac{1}{2i^2} \right) = t$$
, then  $\tan t =$ 

- b) Sides a, b, c of a triangle ABC are in AP and b)  $\frac{\sqrt{5}}{3}$   $\cos \theta_1 = \frac{a}{b+c}, \cos \theta_2 = \frac{b}{a+c}, \cos \theta_3 = \frac{c}{a+b}$  then  $\tan^2\left(\frac{\theta_1}{2}\right) + \tan^2\left(\frac{\theta_3}{2}\right) =$  c)  $\frac{2}{3}$
- c) A line is perpendicular to x + 2y + 2z = 0 and passes through (0,1,0). The perpendicular distance of this line from the origin is
- 2) Let (x, y) be such that  $\sin^{-1}(ax) + \cos^{-1}(bxy) = \frac{\pi}{2}$ . Match the statements in Column 1 with statements in Column II and indicate your answer by darkening the appropriate bubble in the 4x4 matrix given in the ORS.
  - a) If a = 1 and b = 0, then (x, y)

a) lies on the circle  $x^2 + y^2 = 1$ 

b) If a = 1 and b = 1, then (x, y)

b) lies on  $(x^2 - 1)(y^2 - 1) = 0$ 

c) If a = 1 and b = 2, then (x, y)

c) lies on y = x

d) If a = 2 and b = 2, then (x, y)

d) lies on  $(4x^2 - 1)(y^2 - 1) = 0$ 

**DIRECTIONS(Q.3):** Following questions has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

3) a) 
$$\left(\frac{1}{y^2} \left(\frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right)^{\frac{1}{2}}$$
 takes value

- a)  $\frac{1}{2} \sqrt{\frac{5}{3}}$
- b) If  $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of  $\cos \frac{x-y}{2}is$
- c)  $\frac{1}{2}$
- c) If  $\cos\left(\frac{\pi}{4} x\right)\cos 2x + \sin x\sin 2x\sec x = \cos x\sin 2x\sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$  then possible value of  $\sec x$  is
- d) If  $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x \neq 0$

# **Codes:**

- (a) 4 3 1 2
- (*b*) 4 3 2 1
- (c) 3 4 2 1
- (*d*) 3 4 1 2