1

Mains - 14.A+B

ai24btech11030 - Shiven Bajpai

Section - E

1) Prove that $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$. (2002 - 5 Marks)

Section - F

1) Match The Following

(2005 - 6M)

Column I

Column II

a)
$$\sum_{i=1} \infty \tan^{-1} \left(\frac{1}{2i^2} \right) = t$$
, then $\tan t =$

- a) 1
- b) Sides a, b, c of a triangle ABC are in AP and b) $\frac{\sqrt{5}}{3}$ $\cos \theta_1 = \frac{a}{b+c}, \cos \theta_2 = \frac{b}{a+c}, \cos \theta_3 = \frac{c}{a+b}$ then $\tan^2\left(\frac{\theta_1}{2}\right) + \tan^2\left(\frac{\theta_3}{2}\right) =$ c) $\frac{2}{3}$
- c) A line is perpendicular to x + 2y + 2z = 0 and passes through (0,1,0). The perpendicular distance of this line from the origin is
- 2) Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(bxy) = \frac{\pi}{2}$. Match the statements in Column 1 with statements in Column II and indicate your answer by darkening the appropriate bubble in the 4x4 matrix given in the ORS.

a) If
$$a = 1$$
 and $b = 0$, then (x, y)

a) lies on the circle
$$x^2 + y^2 = 1$$

b) If
$$a = 1$$
 and $b = 1$, then (x, y)

b) lies on
$$(x^2 - 1)(y^2 - 1) = 0$$

c) If
$$a = 1$$
 and $b = 2$, then (x, y)

c) lies on
$$y = x$$

d) If
$$a = 2$$
 and $b = 2$, then (x, y)

d) lies on
$$(4x^2 - 1)(y^2 - 1) = 0$$

DIRECTIONS(Q.3): Following questions has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

3) a)
$$\left(\frac{1}{y^2} \left(\frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right)^{\frac{1}{2}}$$
 takes value

a)
$$\frac{1}{2} \sqrt{\frac{5}{3}}$$

- b) If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos \frac{x-y}{2}is$
 - c) $\frac{1}{2}$
- c) If $\cos\left(\frac{\pi}{4} x\right)\cos 2x + \sin x\sin 2x\sec x = \cos x\sin 2x\sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$ then possible value of $\sec x$ is
- d) If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x \neq 0$

Codes:

I - Integer Value Correct Type

1) The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^{i}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty} \left(\frac{-x}{2}\right)^{i} - \sum_{i=1}^{\infty} (-x)^{i}\right)$$

lying in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$ is. (Here, the inverse trignometric function $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2},\frac{pi}{2}\right]$ and $\left[0,\pi\right]$ respectively (JEE Adv. 2018)

2) The value of

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{10} + \frac{k\pi}{10}\sec\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals ____. (JEE Adv 2019)

SECTION B - JEE MAIN / AIEEE

- 1) $\cos^{-1}\left(\sqrt{\cos\alpha}\right) \tan^{-1}\left(\sqrt{\cos\alpha}\right)$, then $\sin x =$
 - a) $\tan^2\left(\frac{\alpha}{2}\right)$ c) $\tan \alpha$ b) $\cot^2\left(\frac{\alpha}{2}\right)$ d) $\cot\left(\frac{\alpha}{2}\right)$
- 2) The trignometric equation $sin^{-1}x = 2sin^{-1}a$ has a solution for [2003]

 - a) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) all real values of a b) $\frac{1}{2} < |\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| < \frac{1}{2}$
- 3) If $\cos^{-1} x \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 4xy \cos \alpha + y^2$ is equal to [2005]
 - a) $2\sin 2\alpha$

b) 4

- c) $4 \sin^2 \alpha$ d) $-4 \sin^2 \alpha$
- 4) If $\sin^{-1}\left(\frac{x}{5}\right) + \csc^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value
 - a) 4

b) 5

- 5) The value of $\cot \left(\csc^{-1} \frac{5}{3} + \tan^{-1} (2) 3 \right)$

- 6) If x,y,z are in AP and $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} 1z$ are also in A.P, then [JEE M 2013]

- a) x = y = zb) 2x = 3y = 6zc) 6x = 3y = 2zd) 6x = 4y = 3z
- 7) Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$, where |x| < 1 $\frac{1}{\sqrt{3}}$ Then a value of y is [JEE M 2015]
 - a) $\frac{3x-x^3}{1+3x}$ c) $\frac{3x-x^3}{1-3x}$ b) $\frac{3x+x^3}{1+3x}$ d) $\frac{3x+x^3}{1-3x}$
- 8) If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to [JEE M 2019 9 Jan M]