

3-3.3-2

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Question: Construct a triangle with sides 5cm , 6cm and 7cm

Solution: Let the vertices of triangle be **A**, **B** and **C** and lengths of the sides opposing them be denoted by $a = 5\text{cm}$, $b = 6\text{cm}$ and $c = 7\text{cm}$ respectively.

By Cosine rule in $\triangle ABC$,

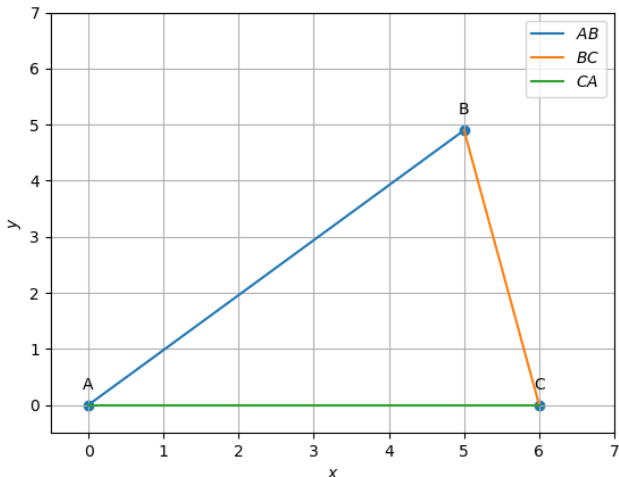
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{60}{84}$$

Let $\mathbf{A} = \mathbf{0}$ and $\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}$. Then $\mathbf{B} = c \begin{pmatrix} \cos A \\ \sin A \end{pmatrix}$

Substituting values we get, $\mathbf{A} = \mathbf{0}$, $\mathbf{B} = \begin{pmatrix} 5 \\ \sqrt{24} \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$



Code for this plot can be found at:

Codes/main.py
Codes/main.c

Alternate Solution: Let the vertices of triangle be \mathbf{A} , \mathbf{B} and \mathbf{C} and lengths of the sides opposing them be denoted by $a = 5\text{cm}$, $b = 6\text{cm}$ and $c = 7\text{cm}$ respectively.

Let $\mathbf{A} = \mathbf{0}$ and $\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}$, then \mathbf{B} satisfies

$$\|\mathbf{x} - \mathbf{A}\| = c$$

$$\|\mathbf{x} - \mathbf{C}\| = a$$

Since a point that lies on 2 circles lies on their radical axis, \mathbf{B} lies on the line

$$\begin{aligned} \|\mathbf{x}\|^2 - 2\mathbf{C}^T\mathbf{x} + b^2 - a^2 &= \|\mathbf{x}\|^2 - 2\mathbf{A}^T\mathbf{x} - c^2 \\ (\mathbf{C} - \mathbf{A})^T\mathbf{x} &= \frac{b^2 + c^2 - a^2}{2} \end{aligned}$$

Let $\mathbf{n} = (\mathbf{C} - \mathbf{A})$ and $d = \frac{b^2 + c^2 - a^2}{2}$

Then parametric form of line is $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

Where $\mathbf{h} = \frac{\mathbf{n}d}{\|\mathbf{n}\|^2}$ and \mathbf{m} be perpendicular to \mathbf{n} i.e $\mathbf{m} = \begin{pmatrix} -n_2 \\ n_1 \end{pmatrix}$

Substituting \mathbf{x} in equation of circle

$$\begin{aligned} (\mathbf{h} + k\mathbf{m} - \mathbf{A})^T(\mathbf{h} + k\mathbf{m} - \mathbf{A}) &= c^2 \\ ((\mathbf{h} - \mathbf{A}) + k\mathbf{m})^T((\mathbf{h} - \mathbf{A}) + k\mathbf{m}) &= c^2 \\ (\mathbf{h} - \mathbf{A})^T(\mathbf{h} - \mathbf{A}) + 2k\mathbf{m}^T(\mathbf{h} - \mathbf{A}) + k^2\mathbf{m}\mathbf{m}^T &= c^2 \end{aligned}$$

Since $\mathbf{m}^T\mathbf{h} = 0$

$$(\mathbf{h} - \mathbf{A})^T(\mathbf{h} - \mathbf{A}) - 2k\mathbf{m}^T\mathbf{A} + k^2\mathbf{m}\mathbf{m}^T = c^2$$

Using quadratic formula for k ,

$$\begin{aligned} k &= \frac{2\mathbf{m}^T\mathbf{A} \pm \sqrt{4(\mathbf{m}^T\mathbf{A})^2 - 4(\mathbf{m}\mathbf{m}^T)((\mathbf{h} - \mathbf{A})^T(\mathbf{h} - \mathbf{A}) - c^2)}}{2\mathbf{m}\mathbf{m}^T} \\ k &= \frac{\mathbf{m}^T\mathbf{A} \pm \sqrt{(\mathbf{m}^T\mathbf{A})^2 - (\mathbf{m}\mathbf{m}^T)((\mathbf{h} - \mathbf{A})^T(\mathbf{h} - \mathbf{A}) - c^2)}}{\mathbf{m}\mathbf{m}^T} \end{aligned}$$

Since we have assumed $\mathbf{A} = \mathbf{0}$, We can simplify this to

$$k = \frac{\pm \sqrt{(c^2 - \mathbf{h}^T\mathbf{h})}}{\sqrt{\mathbf{m}\mathbf{m}^T}}$$

Then \mathbf{x} is

$$x = \mathbf{h} \pm \frac{\sqrt{c^2 - \|\mathbf{h}\|^2} \mathbf{m}}{\|\mathbf{m}\|}$$

Now substituting data,

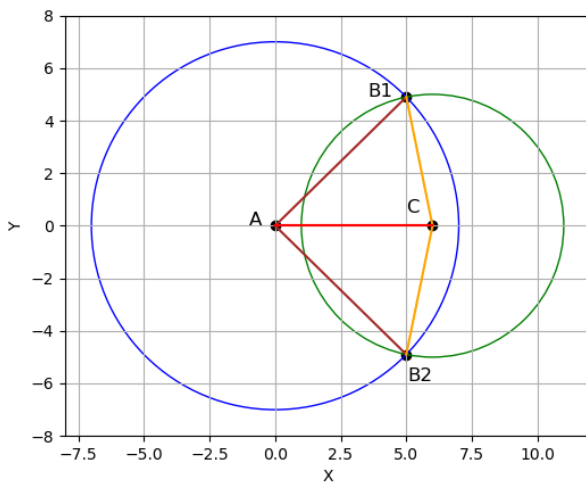
$$d = 30$$

$$\mathbf{h} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

$$k = \pm \frac{\sqrt{24}}{6}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ \pm \sqrt{24} \end{pmatrix}$$



Code for this plot can be found at:

Codes/alt.py
Codes/alt.c