Internship Problems

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1 Introduction

Let us consider a cooperative non-orthogonal multiple access (NOMA) system, as in [1], with a base-station (BS) acting as a source (S), two relays R_1 and R_2 for virtual full-duplex (VFD) relaying, and a user D as the destination. The entire transmission process is carried out in three different time phases. In Phase 1, S superimposes signals $x_{r_1}(t_1)$ and $x_d(t_1)$ (i.e. signals of R_1 and D) with α and β fractions of total transmit power P_s respectively ($\beta = 1 - \alpha$), and broadcasts it to R_1 and R_2. The same is then decoded by both the relays.

In Phase 2, R_1 relays $x_d(t_1)$ to D with power P_{r1}. Concurrently, S also superimposes signals $x_{r_2}(t_2)$ and $x_d(t_2)$ (i.e. signals of R_2 and D) and transmits it to R_2. It is to note here that during this phase, R_2 receives a interfering signal from R_1. However, R_2 removes the IRI from R_1 through previous phase decoded signal $x_d(t_1)$. R_2 determines its own signal $x_{r_2}(t_2)$ after detecting and removing $x_d(t_2)$ through SIC. We consider a practical scenario of residual IRI and imperfect-SIC, with ζ_{ij} denoting level of residual interference at receiver j such that $\zeta_{sr_1} = 0$ implies a perfect-SIC.

The VFD-NOMA transmission process completes in Phase 3 by R_2 relaying the information $x_d(t_2)$ with power P_{r2} to D.

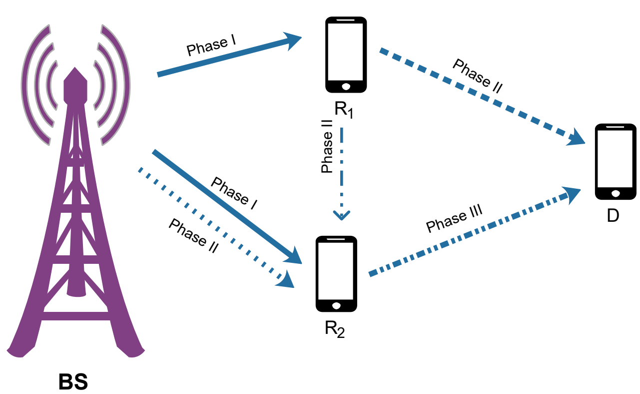

Figure 1: Two user based VFD-NOMA system model.

1.1 Outage probability of D

Outage probability (OP) is an important performance metric which determines the link failure probability. In particular, an outage event at D is defined as D failing to decode the signal transmitted by either R_1 or R_2.

1.1.1 Perfect-SIC and Perfect-IRI

Considering perfect-SIC and perfect-IRI, OP of D can be expressed as follows,

$$\begin{aligned} P_{out,p} = 1 - &\prod_{i=\{sr_1, sr_2\}} \prod_{j=\{r_1 d,\, r_2 d\}} \frac{1}{\Gamma(m_i)} \frac{1}{\Gamma(m_j)} \\ &\times \Gamma\Big(m_i, \frac{\mu_i\,(2^{\,3T_d} - 1)}{(1-\alpha\, 2^{\,3T_d})}\Big) \Gamma\Big(m_j, \mu_j\,(2^{\,3T_d} - 1)\Big), \end{aligned} \tag{1}$$

where $\mu_{sr_1} = \frac{m_{sr_1}}{\rho_s\, \bar{g}_{sr_1}}$, $\mu_{sr_2} = \frac{m_{sr_2}}{\rho_s\, \bar{g}_{sr_2}}$, $\mu_{r_1 d} = \frac{m_{r_1 d}}{\rho_{r1}\, \bar{g}_{r_1 d}}$, $\mu_{r_2 d} = \frac{m_{r_2 d}}{\rho_{r2}\, \bar{g}_{r_2 d}}$ and T_d is target rate of the destination user. m_{ij} is fading severity of Nakagami-m faded channel h_{ij}, where $i \in \{s, r_1, r_2\}$, $j \in \{r_1, r_2, d\}$. Further, $g_{ij} = |h_{ij}|^2$ and $\bar{g}_{ij} = \mathbb{E}[g_{ij}] = (\frac{D_{sd}}{D_{ij}})^{\eta}$, where D_{sd} is distance from S to D, D_{ij} is distance between node i and j, and η is path-loss exponent. ρ_s is transmit SNR, $\rho_s = \frac{P_s}{\sigma_N^2}$, $\rho_{r1} = \frac{P_{r1}}{\sigma_N^2}$, $\rho_{r2} = \frac{P_{r2}}{\sigma_N^2}$ and σ_N^2 is additive white Gaussian noise (AWGN) power. Also, Γ and Υ are the upper and lower gamma functions respectively, and $\mathbb{E}\{\cdot\}$ is the expected value.

1.1.2 Imperfect-SIC and Residual-IRI

OP of user D with imperfect-SIC and residual-IRI can be expressed as,

$$P_{out,ip} = 1 - \prod_{i=\{1,\,2\}} \frac{\Gamma(m_{sr_i},\, \mu_{sr_i} T_1)\, {T_4}^{m_{r_1 r_2}}\, {T_{51}}^{n}\, \mathbb{K}_1(k,n)}{\Gamma(m_{sr_i})\, \Gamma(m_{r_1 r_2})\, e^{T_{51}}\, (T_{51} + T_4)^{k + m_{r_1 r_2}}} \prod_{j=\{1,\,2\}} \frac{\Gamma(m_{r_j d}, \mu_{r_j d}\, u_d)}{\Gamma(m_{r_j d})}, \tag{2}$$

where $T_4 = \frac{\mu_{r_1 r_2}}{\zeta_{r_1 r_2}}$, $T_{51} = \mu_{sr_2} T_1$, $T_1 = \frac{u_d}{(\beta - u_d \alpha)}$. $u_d = 2^{3T_d} - 1$, is the target SINR of the destination user, $\mathbb{K}_1(p,q) = \sum_{q=0}^{m_{sr_2}-1} \mathbb{K}_2(p,q)$ and $\mathbb{K}_2(p,q) = \sum_{p=0}^{q} \frac{\Gamma(p + m_{r_1 r_2})}{p!\,(q-p)!}$.

2 Problem 1 - Power optimization in VFD-NOMA with imperfect-SIC and residual-IRI

As evident from (2), given relays R_1 and R_2, in order to reduce the IRI and thereby minimize OP, optimization of power transmitted by relay R_1, P_{r1} is essentially required (as it interferes with signal received at R_2).

2.1 Problem definition

$$(\mathbb{P}2): \min_{P_r} \quad P_{out,ip}, \quad \text{subject to}$$

$$C1: P_{min} \leq P_{r1} \leq P_{max},$$

where constraint $C1$ indicates the limit on transmit power of R_2 with minimum and maximum values being P_{min} and P_{max}, respectively.

2.2 Example

Let us consider typical values of the system parameters as $\rho_s = 25$ dB, $P_{r2} = 0.5$, $P_{min} = 0.1$, $P_{max} = 1$, $\eta = 3$, $\alpha = 0.3$, $m_{ij} \triangleq m = 1$, and $\zeta_{ij} = 0.3$, where $i \in \{s, r_1, r_2\}$ and $j \in \{r_1, r_2\}$. The values of $\{D_{sr_1}, D_{sr_2}, D_{r_1r_2}\}$ are taken as $\{3\,D_{sd}/7, D_{sd}/7, 3\,D_{sd}/7\}$, with $D_{r_1d} = 1 - D_{sr_1}$ and $D_{r_2d} = 1 - D_{sr_2}$.

For the aforementioned values, minimum OP achieved is $P_{out,ip} = 0.020$ with optimal P_r^* value of 0.2870.

3 Problem 2 - Relay selection in VFD-NOMA with perfect-SIC and perfect-IRI

Given that the first relay R_1 is fixed, selection of relay R_2 from a given set of K relays ($\{U_1, U_2, ...U_K\}$) is required to minimize the overall OP of destination user.

3.1 Problem definition

$$(\mathbb{P}1): \min_{\boldsymbol{X}} \quad P_{out,p}, \quad \text{subject to}$$

$$C2: x_k \in \{0, 1\}, \qquad C3: \sum_{k=0}^{K} x_k = 1,$$

where $\boldsymbol{X}$ is a binary vector of order $1 \times K$ whose element x_k takes value 1 if relay U_k is selected as the R_2 and 0 if not. Thus, constraint $C2$ ensures that a particular relay is assigned a value either 0 or 1. Further, $C3$ limits the number of selected relays exactly to one.

3.2 Example

Similar to problem 1, the typical values of the system parameters can be considered as $\rho_s = 25$ dB, $P_{r1} = 0.5$, $P_{r2} = 0.5$, $\eta = 4$, $\alpha = 0.05$, $m = 3$. Also, $T_d = T_r = 0.1$, and $\{D_{sr_1}, D_{r_1r_2}\}$ are taken as $\{D_{sd}/2, D_{sd}/2\}$.

Further, let us consider $K = 5$ with distance of the relays from S as $\{D_{su_1}, D_{su_2}, D_{su_3}, D_{su_4}, D_{su_5}\} = \{0.1\,D_{sd}, 0.3\,D_{sd}, 0.5\,D_{sd}, 0.7\,D_{sd}, 0.9\,D_{sd}\}$. Now, on solving for the same, the minimum OP obtained is 0.0034 for relay U_3. Thereby, the optimal binary vector is $\boldsymbol{X}^* = [x_1\,x_2\,x_3\,x_4\,x_5] = [0\,0\,1\,0\,0]$.

References

[1] J. Jose, S. Parvez, and V. Bhatia, "VFD-NOMA Under Imperfect SIC and Residual Inter-Relay Interference Over Generalized Nakagami-m Fading Channels," *IEEE Commun. Lett.*, vol. 25, no. 2, pp. 646–650, Oct. 2020.