AI1110 Assignment1

Gadekar Shivendraraje Baban(CS22BTECH11022)

12.13.6.3 Question

Suppose that 5 % of men and 0.25 % of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Answer

let us define two random variables.
First we'll define X as random variable whose value is *1 if selected person is a male and 0 if selected person is a female.*
Then, we'll define Y as a random variable whose value is *1 if selected person has grey hair and 0 if selected person does not have grey hair.*

$$X = \begin{cases} 1 & \textit{person selected is a male.} \\ 0 & \textit{person selected is a female.} \end{cases} \tag{1}$$

$$Y = \begin{cases} 1 & \textit{person has grey hair.} \\ 0 & \textit{person doesn't have grey hair.} \end{cases} \tag{2}$$

Bayes' theorem will be useful while solving this question.

We want to find the conditional probability that person is a male given that the person has grey hair, i.e., $\Pr(X = 1/Y = 1)$.

By Bayes' theorem, we have:

$$\Pr(X = 1/Y = 1) = \Pr(Y = 1/X = 1) \cdot \Pr(X = 1) \,/\, \Pr(Y = 1) \tag{3}$$

We know that

$$\Pr(X = 1) = \Pr(X = 0) = 0.5. \tag{4}$$

since there are an equal number of men and women.
We also know that

$$\Pr(Y = 1/X = 1) = 0.05 \tag{5}$$

since 5% of men have grey hair.
To find $\Pr(Y = 1)$, we need to use the *law of total probability*. We can partition the sample space into two events: having grey hair and not having grey hair. Then:

$$\Pr(Y = 1) = \Pr(Y = 1/X = 1) \cdot \Pr(X = 1) + \Pr(Y = 1/X = 0) \tag{6}$$

We know that

$$\Pr(Y = 1/X = 0) = 0.0025 \tag{7}$$

since 0.25% of women have grey hair. We also know that

$$\Pr(X = 0) = 0.5 \tag{8}$$

since there are an equal number of men and women. Therefore:

$$\Pr(Y = 1) = 0.05 \cdot 0.5 + 0.0025 \cdot 0.5 = 0.02625 \tag{9}$$

Putting these values into *Bayes' theorem* to get:

$$\Pr(X = 1/Y = 1) = 0.05 \cdot 0.5/0.02625 = 0.9524 \tag{10}$$

Therefore, the probability that a grey-haired person selected at random is male is approximately 0.9524, or 95.24 %.