AI1110 Assignment1

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12.13.6.3 Question

Suppose that 5 % of men and 0.25 % of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Answer

Bayes' theorem will be useful while solving this question.

Let A be the event that the person is male.
Let B be the event that the person has grey hair.
We want to find the probability of A given B, i.e., $\Pr(A/B)$.

By Bayes' theorem, we have:

$$\Pr(A/B) = \Pr(B/A) \cdot \Pr(A) \,/\, \Pr(B) \tag{1}$$

We know that

$$\Pr(male) = \Pr(female) = 0.5. \tag{2}$$

since there are an equal number of men and women.
We also know that

$$\Pr(B/A) = 0.05 \tag{3}$$

since 5% of men have grey hair.
To find $\Pr(B)$, we need to use the *law of total probability*. We can partition the sample space into two events: having grey hair and not having grey hair. Then:

$$\Pr(B) = \Pr(B/A) \cdot \Pr(A) + \Pr(B/A') \cdot \Pr(A') \tag{4}$$

We know that

$$\Pr(B/A') = 0.0025 \tag{5}$$

since 0.25% of women have grey hair. We also know that

$$\Pr(A') = 0.5 \tag{6}$$

since there are an equal number of men and women. Therefore:

$$\Pr(B) = 0.05 \cdot 0.5 + 0.0025 \cdot 0.5 = 0.02625 \tag{7}$$

Putting these values into *Bayes' theorem* to get:

$$\Pr(A/B) = 0.05 \cdot 0.5/0.02625 = 0.9524 \tag{8}$$

Therefore, the probability that a grey-haired person selected at random is male is approximately 0.9524, or 95.24 %.