

1. Write a program to determine the initial price of an American call and an American put option in the binomial model with the following data:

$$S(0) = 100; K = 100; T = 1; M = 100; r = 8\%; \sigma = 20\%.$$

Use the following set of u and d for your program:

$$u = e^{\sigma\sqrt{\Delta t} + (r - \frac{1}{2}\sigma^2)\Delta t}; d = e^{-\sigma\sqrt{\Delta t} + (r - \frac{1}{2}\sigma^2)\Delta t}.$$

Here $\Delta t = \frac{T}{M}$, with M being the number of subintervals in the time interval $[0, T]$. Use the continuous compounding convention in your calculations (i.e., both in \tilde{p} and in the pricing formula).

Now, plot the initial prices of both call and put options by varying one of the parameters at a time (as given below) while keeping the other parameters fixed (as given above) :

- (a) $S(0)$.
 - (b) K .
 - (c) r .
 - (d) σ .
 - (e) M (Do this for three values of K , $K = 95, 100, 105$).
2. Write a program to determine the initial price of the *lookback* (European) option in the binomial model, using the basic binomial algorithm (used in earlier lab assignments), with the following data:

$$S(0) = 100; T = 1; r = 8\%; \sigma = 20\%.$$

The payoff of the *lookback* option is given by

$$V = \max_{0 \leq i \leq M} S(i) - S(M),$$

where $S(i) = S(i\Delta t)$ with $\Delta t = \frac{T}{M}$ (M being the number of subintervals of the time interval $[0, T]$). Use the continuous compounding convention in your calculations (i.e., both in \tilde{p} and in the pricing formula). Use the following values of u and d for your program:

$$u = e^{\sigma\sqrt{\Delta t} + (r - \frac{1}{2}\sigma^2)\Delta t}; \quad d = e^{-\sigma\sqrt{\Delta t} + (r - \frac{1}{2}\sigma^2)\Delta t}$$

- (a) Obtain the initial price of the option for $M = 5, 10, 25, 50$.
 - (b) How do the values of options at time $t = 0$ compare for the above values of M that you have taken ?
 - (c) Tabulate the values of the options at all intermediate time points for $M = 5$.
3. Repeat Problem 2 using a (Markov based) computationally efficient binomial algorithm. Make a comparative analysis of the two algorithms, like computational time, the value of M it can handle, etc.
 4. As in Problem 3, use a (Markov based) computationally efficient binomial algorithm to price an European call option. Make a comparative analysis of the two algorithms, like computational time, the value of M it can handle, etc.