MA374 Financial Engineering Lab

Assignment - 1

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Question - 1

Value of parameters given:-

$S_0=100, K=100, T=1, M=100,, r=8\%, \sigma=20\%$. I used these values to calculate the option prices and then carried out sensitivity analysis by varying one paraeter and then two at a time. The u and d values used are :-

$\Delta = T/M$

Set1 - $u=e^{\sigma\sqrt{(\Delta)}}, d=e^{-\sigma\sqrt{(\Delta)}}$

Set2 - $u=e^{\sigma\sqrt{(\Delta)}+(r-\sigma^2/2)t}, d=u=e^{-\sigma\sqrt{(\Delta)}+(r-\sigma^2/2)t}$

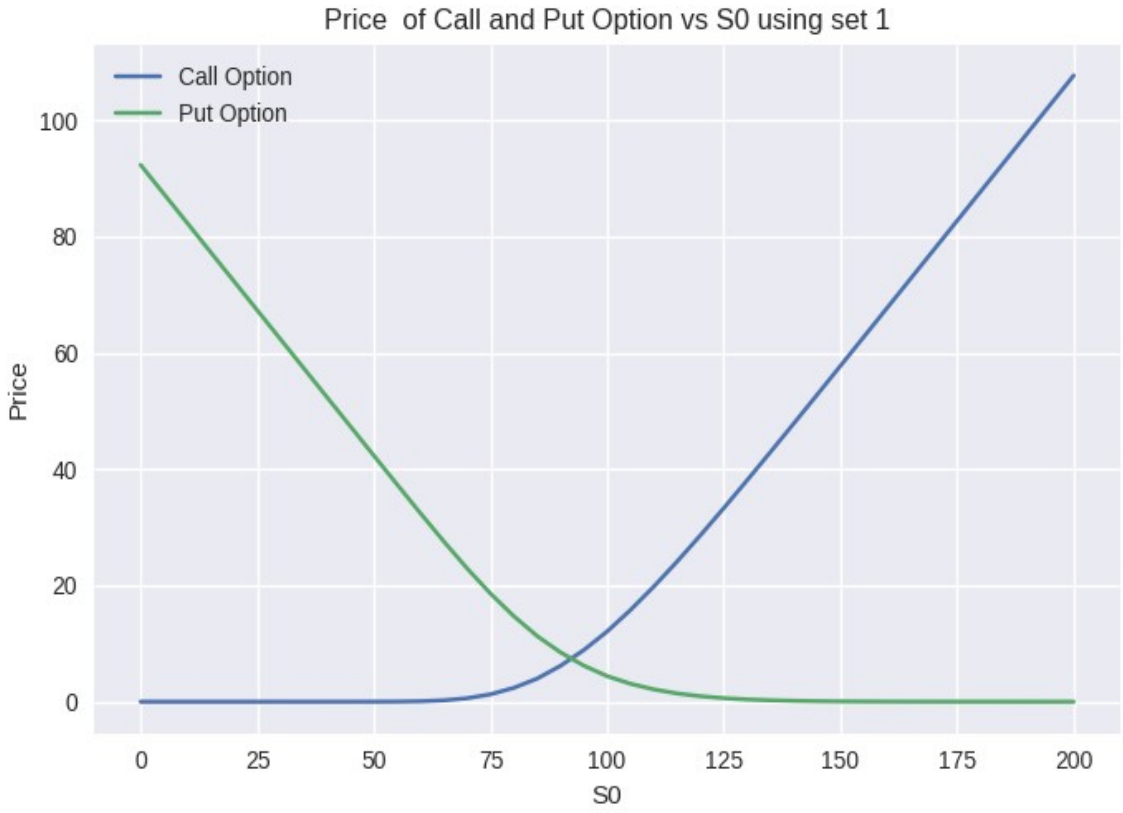
After that I calculated the inital stock price and then used the binomial model in order to calculate the inital call and put values. I calculated the value of p and q for the risk neutral model using the formulae $\hat{p}=e^{r\Delta}-d/u-d$ and q = 1-p. The value of option at final state was calculated as $(S(T)-K)^+, (K-S(T))^+$. Now for the intermediate time steps we just use the formulae:-

$$V(T_i)=e^{-r\Delta}(\hat{p}*V(T_iH)+\hat{q}*V(T_iT))$$

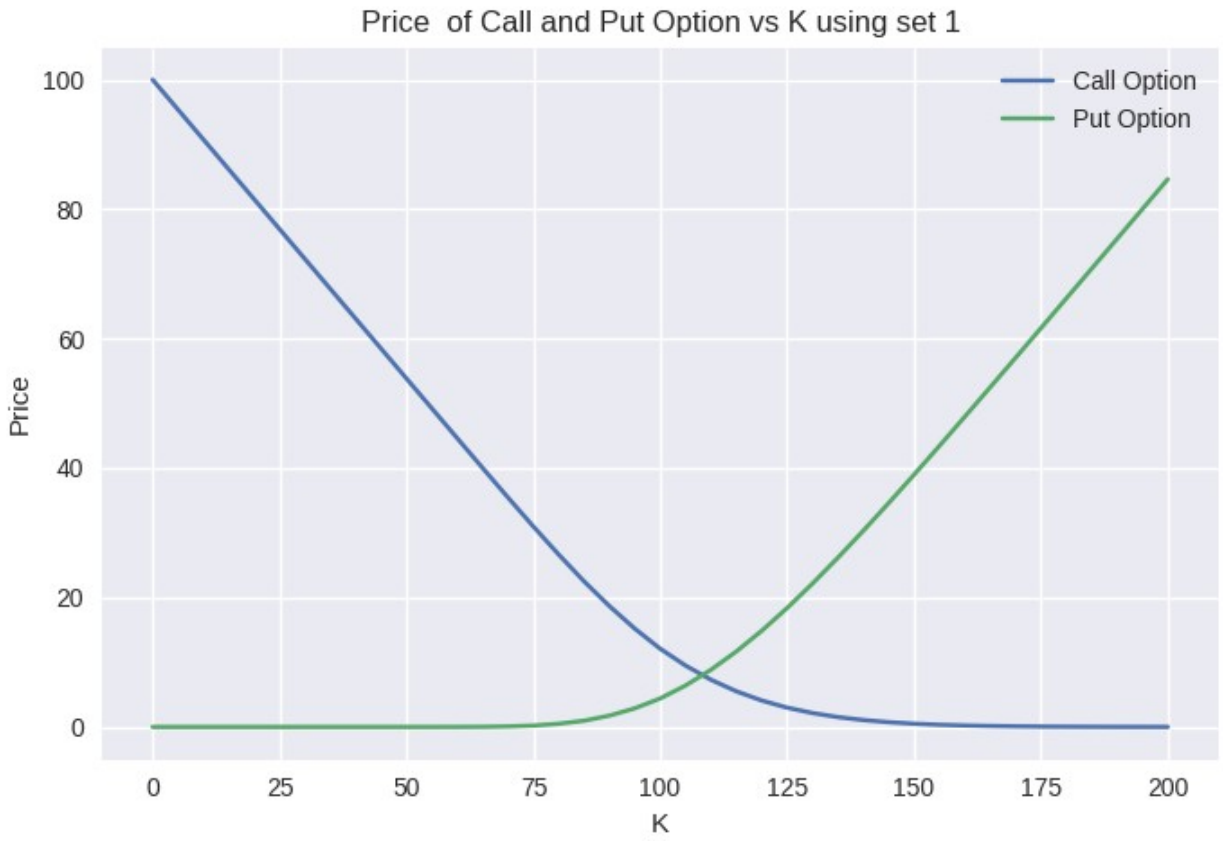
Here $V(T_iH), V(T_iT)$ are the option prices in one step future with up and down state respectively. In this way we backtrack and calculate the price of the option at time 0.

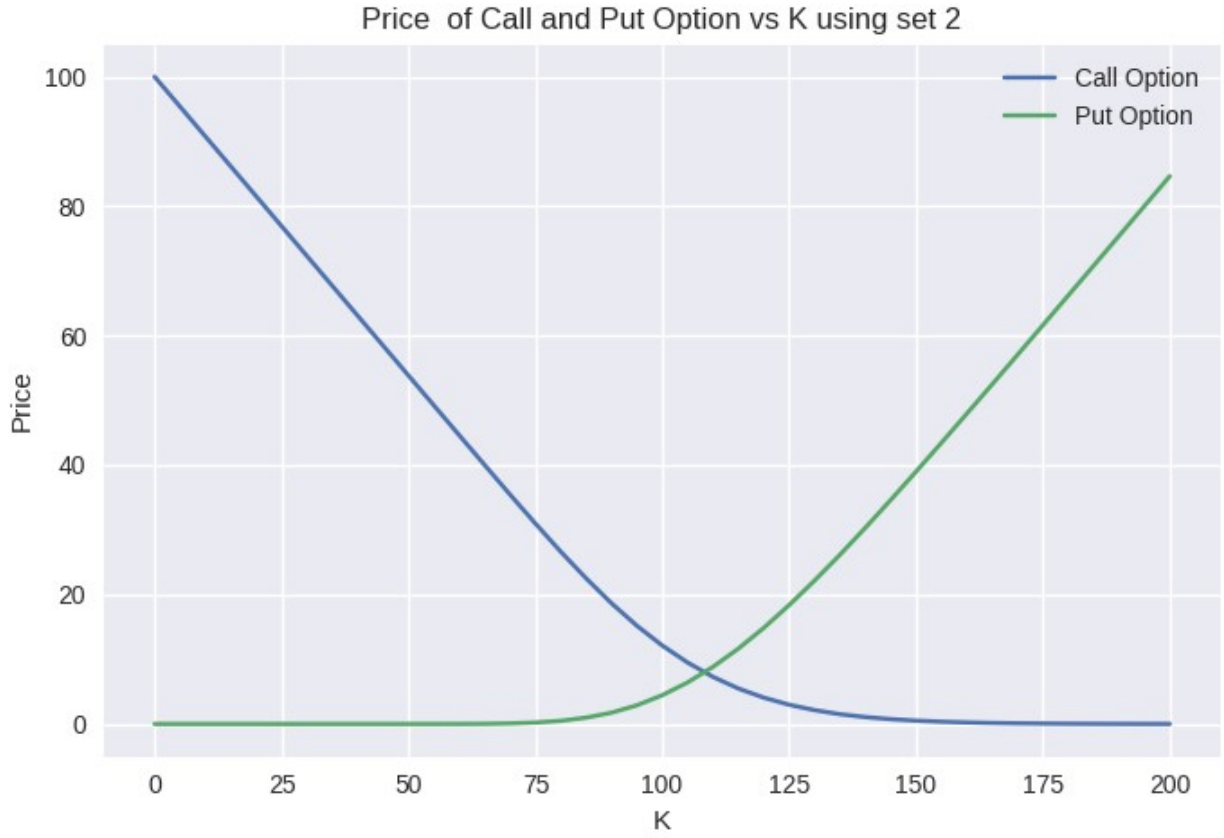
Set Number	Type of Option	Price
1	European Call	12.085380013710145
1	European Put	4.397014652374132
2	Eurpoean Call	12.123047074012453
2	European Put	4.434681712676473

Plots
Variation with S_0

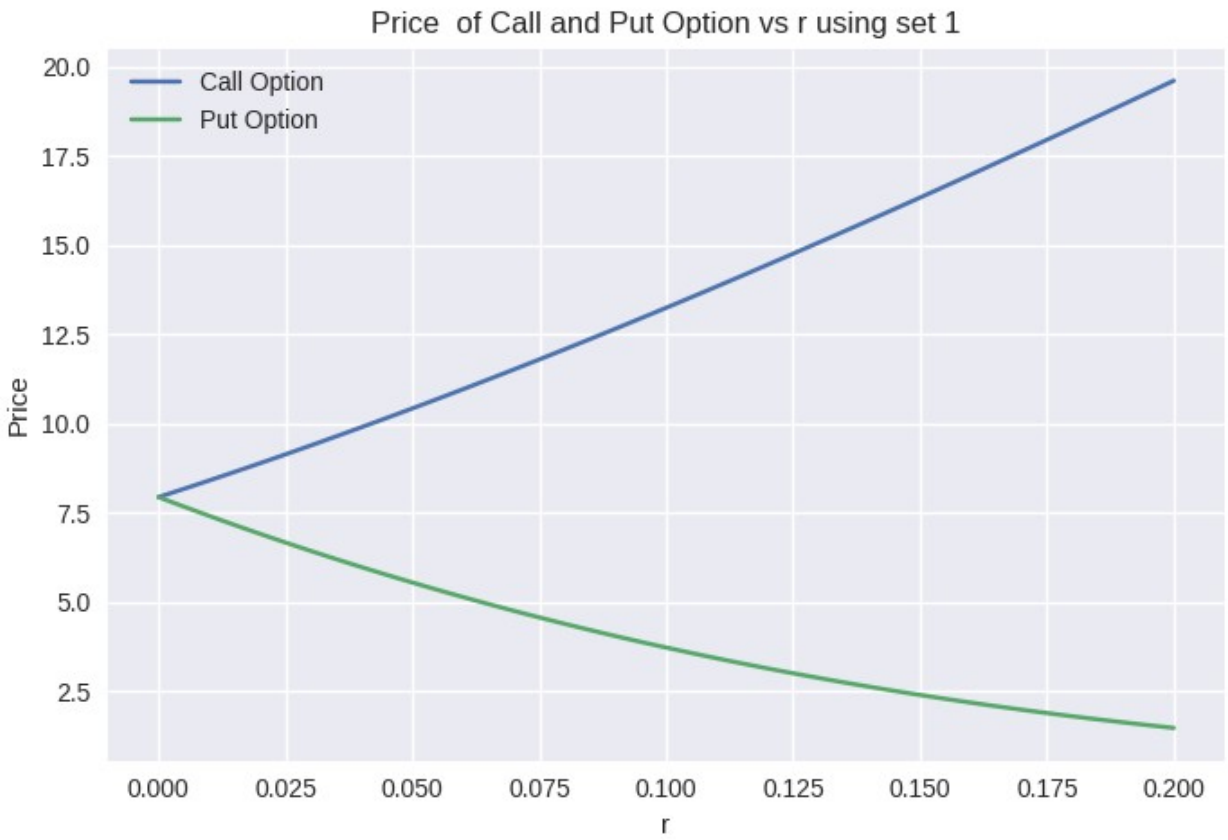


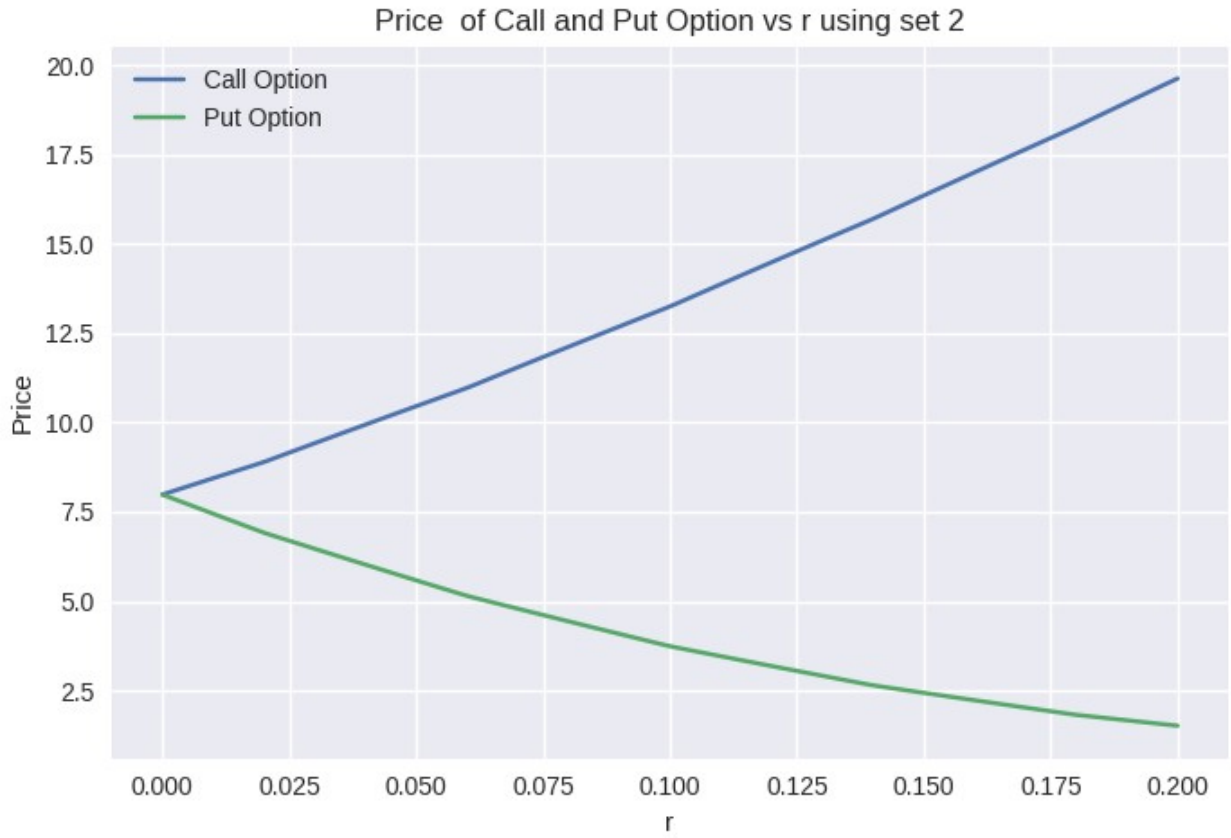
Variation with K



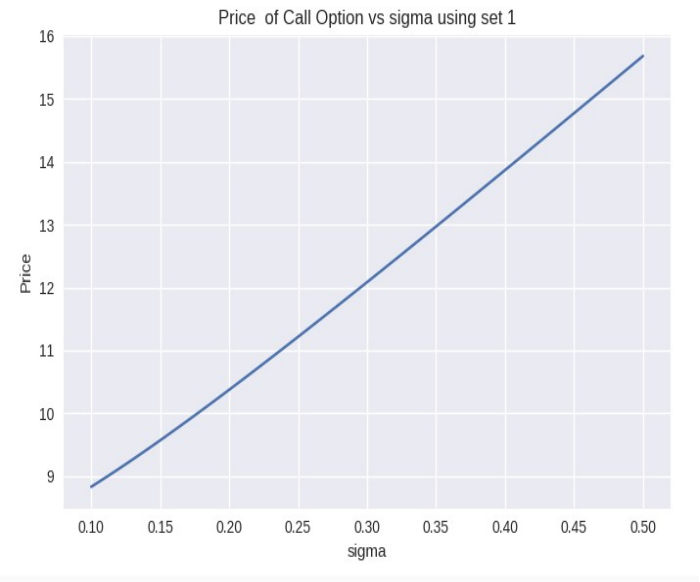


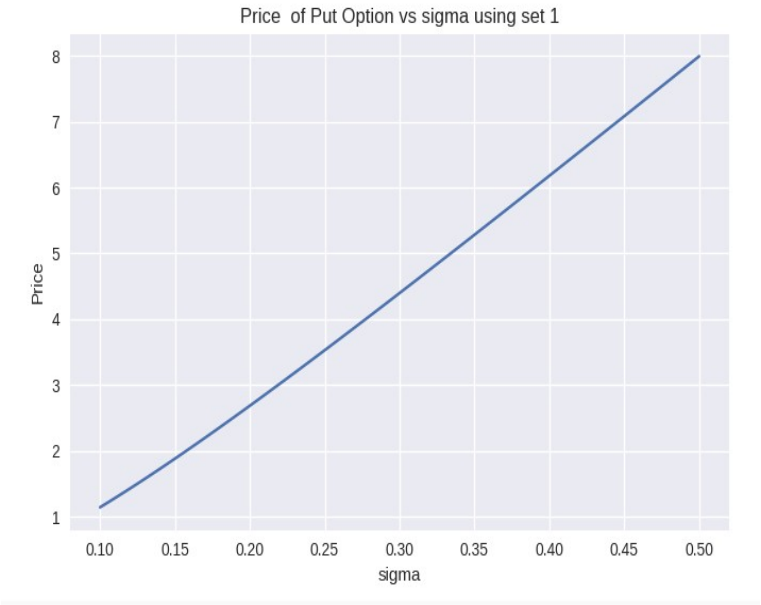
Variation with r

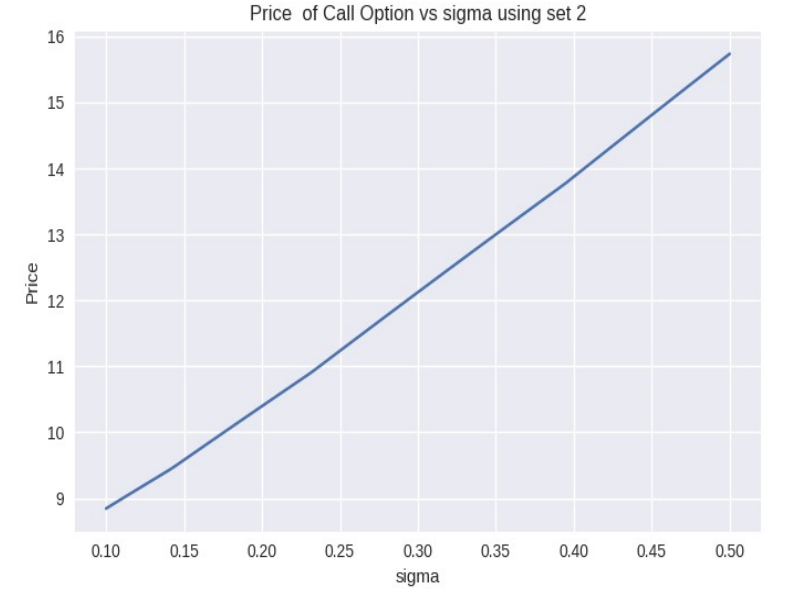




Variation with σ

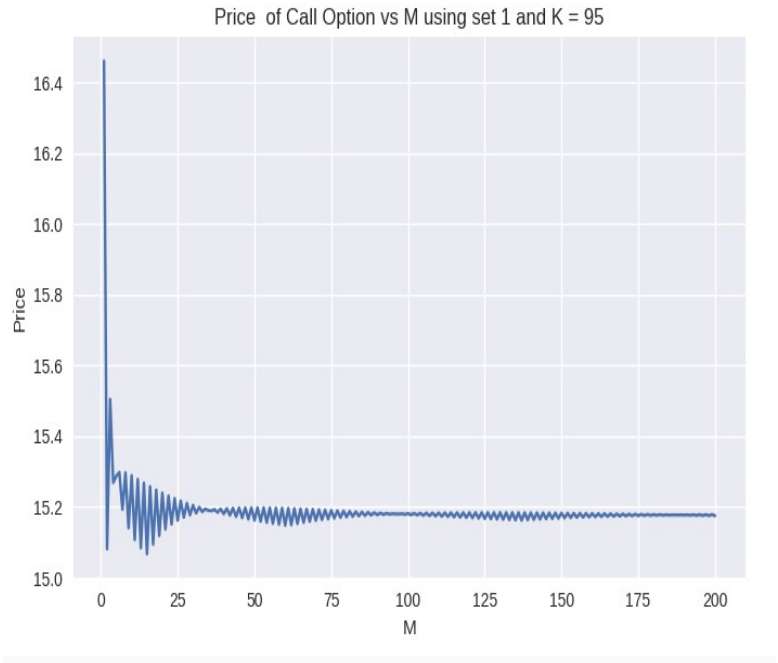


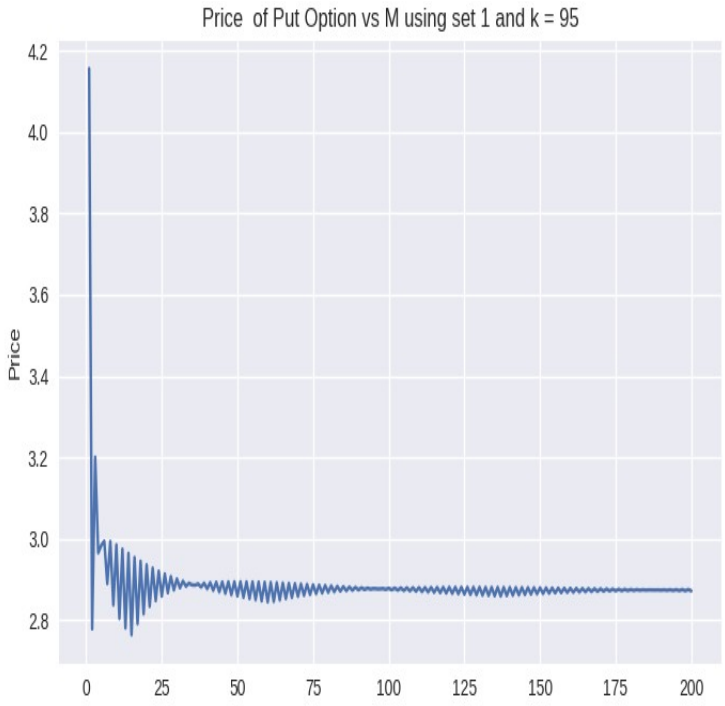


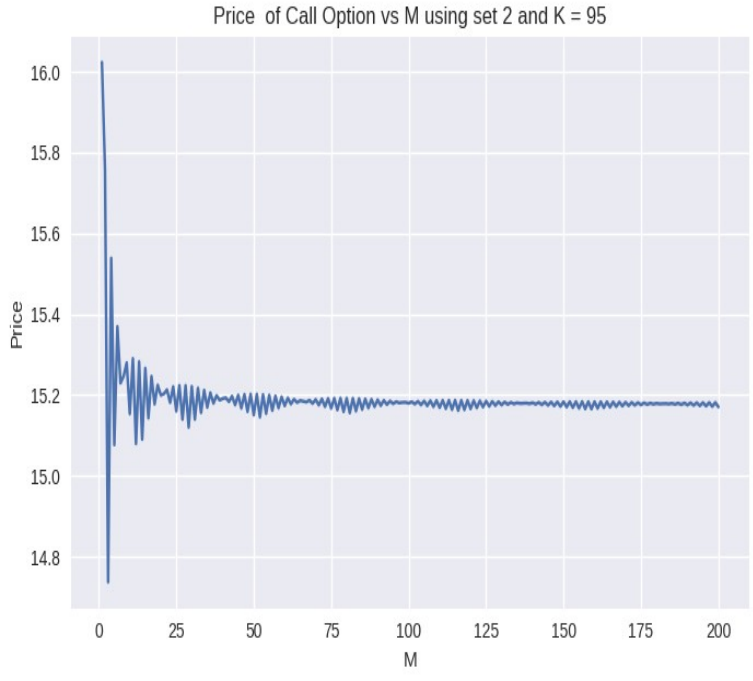


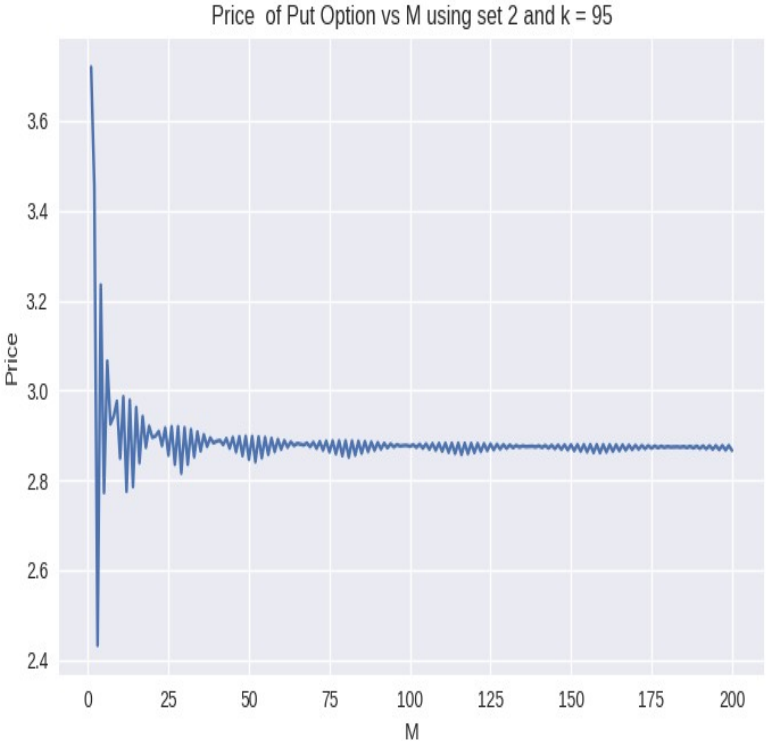
Variation with M

K=95

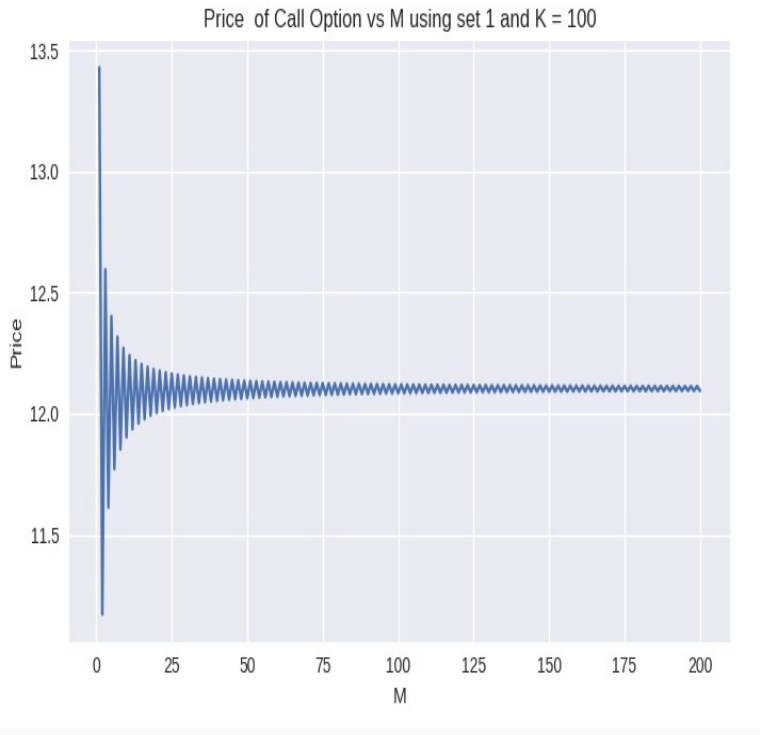


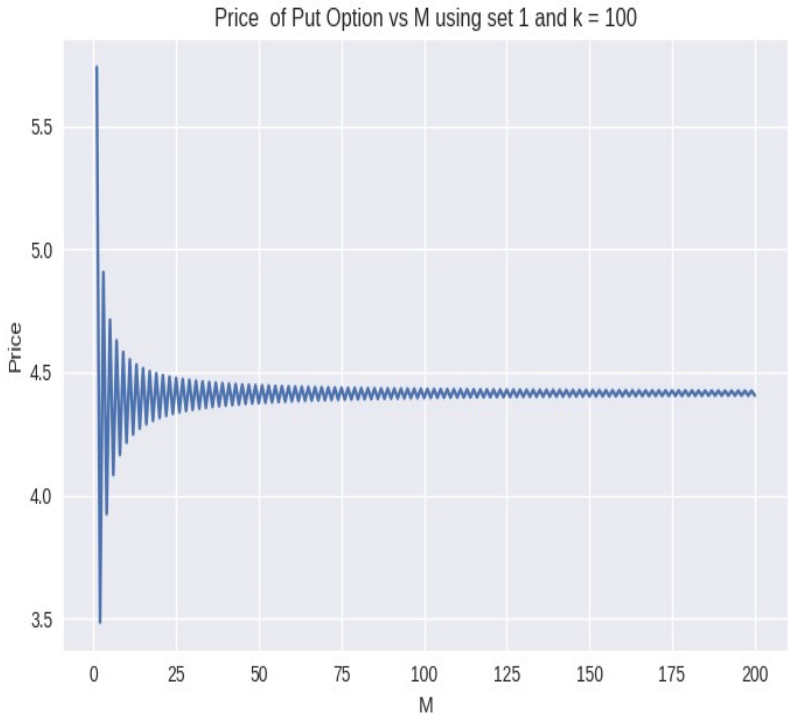


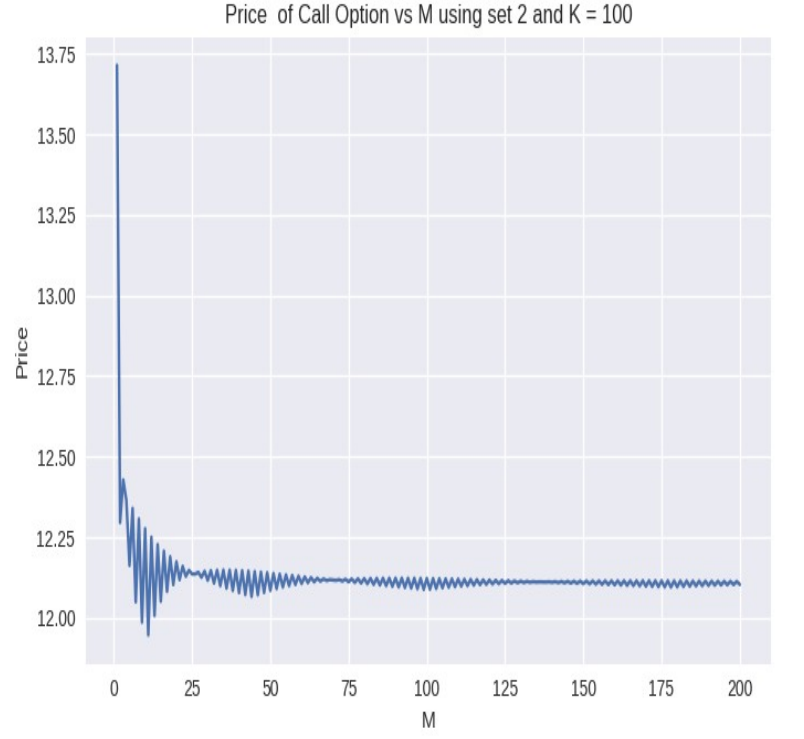


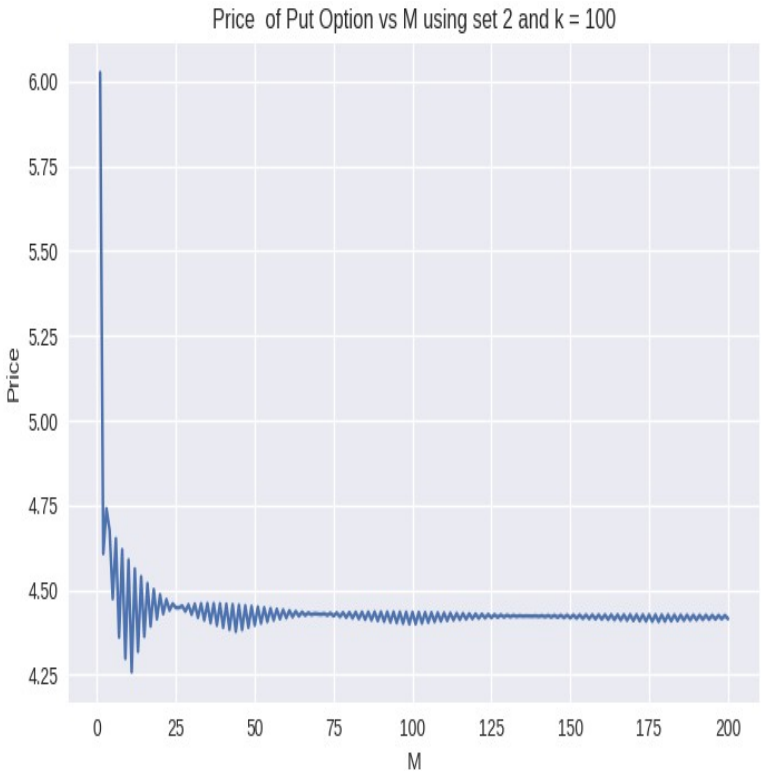


K=100

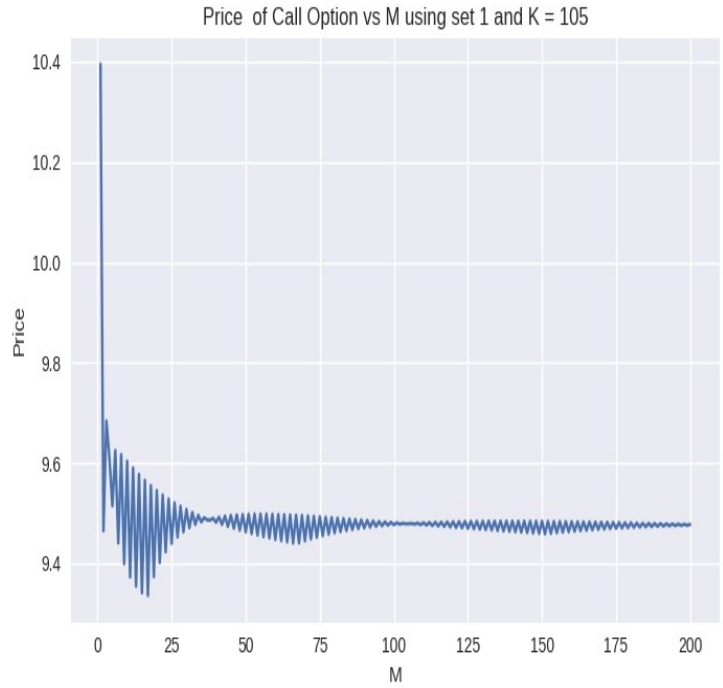

Price of Call Option vs M using set 1 and K = 100
Price
M
13.5
13.0
12.5
12.0
11.5
0
25
50
75
100
125
150
175
200

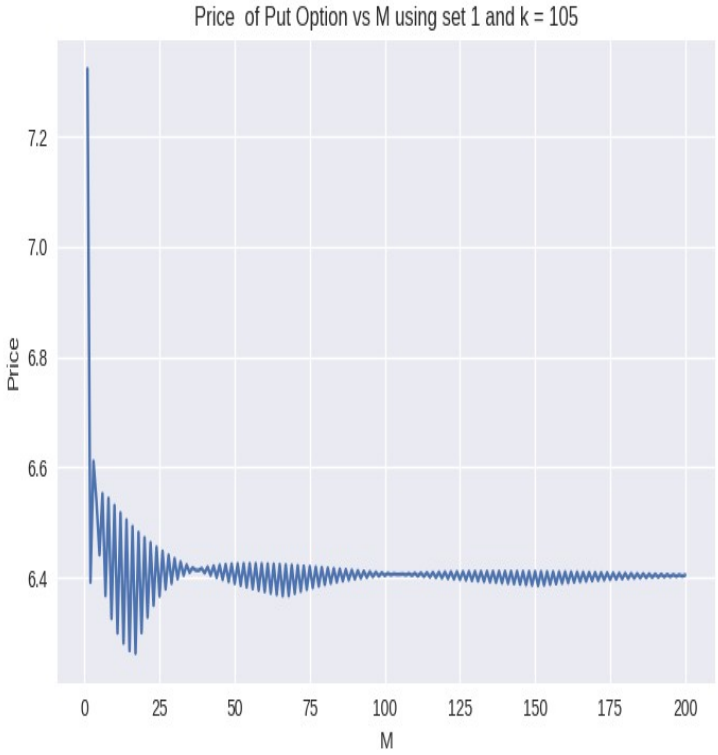

Price of Put Option vs M using set 1 and k = 100
Price
M
5.5
5.0
4.5
4.0
3.5
0
25
50
75
100
125
150
175
200

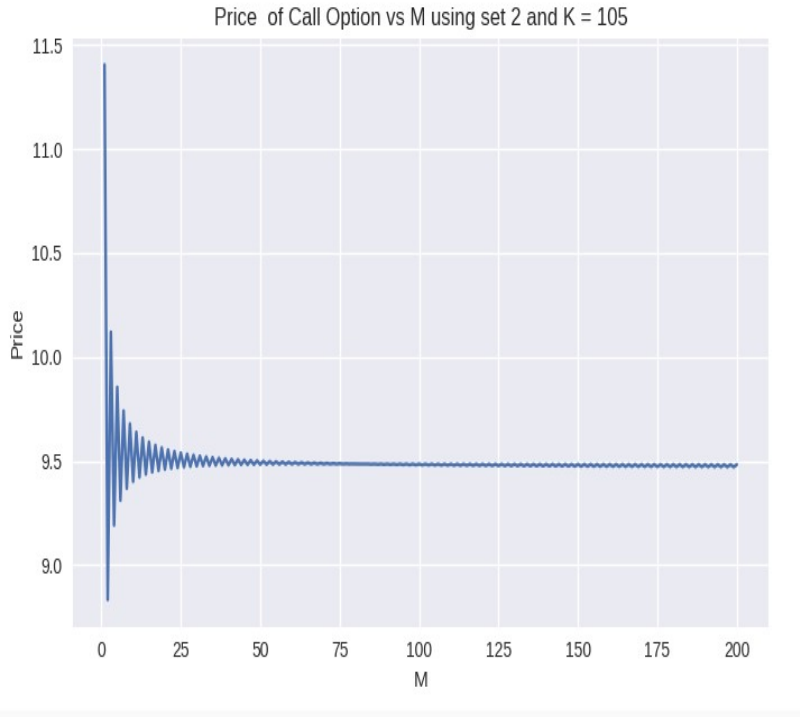

Price of Call Option vs M using set 2 and K = 100
Price
M
13.75
13.50
13.25
13.00
12.75
12.50
12.25
12.00
0
25
50
75
100
125
150
175
200

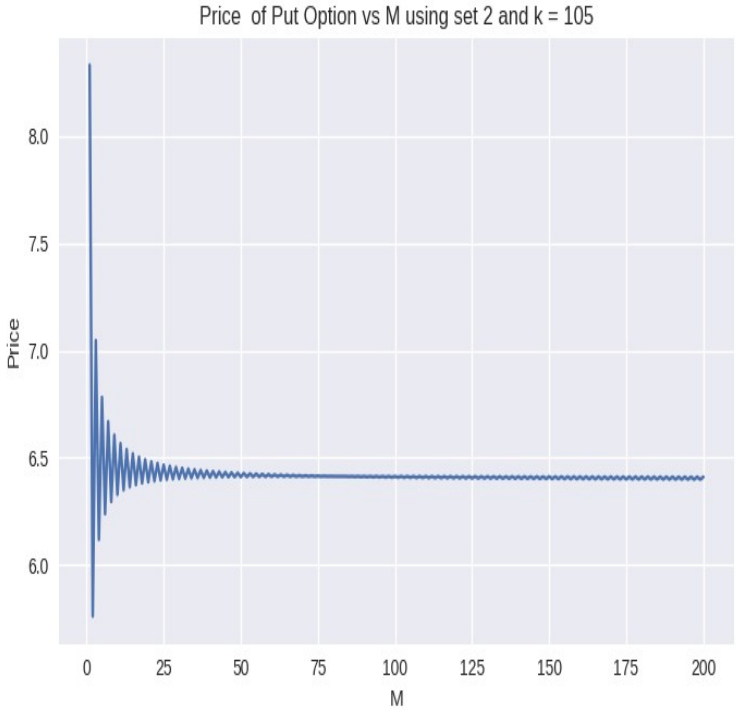

Price of Put Option vs M using set 2 and k = 100
Price
M
6.00
5.75
5.50
5.25
5.00
4.75
4.50
4.25
0
25
50
75
100
125
150
175
200

K=105


Price of Call Option vs M using set 1 and K = 105
Price
M
10.4
10.2
10.0
9.8
9.6
9.4
0
25
50
75
100
125
150
175
200

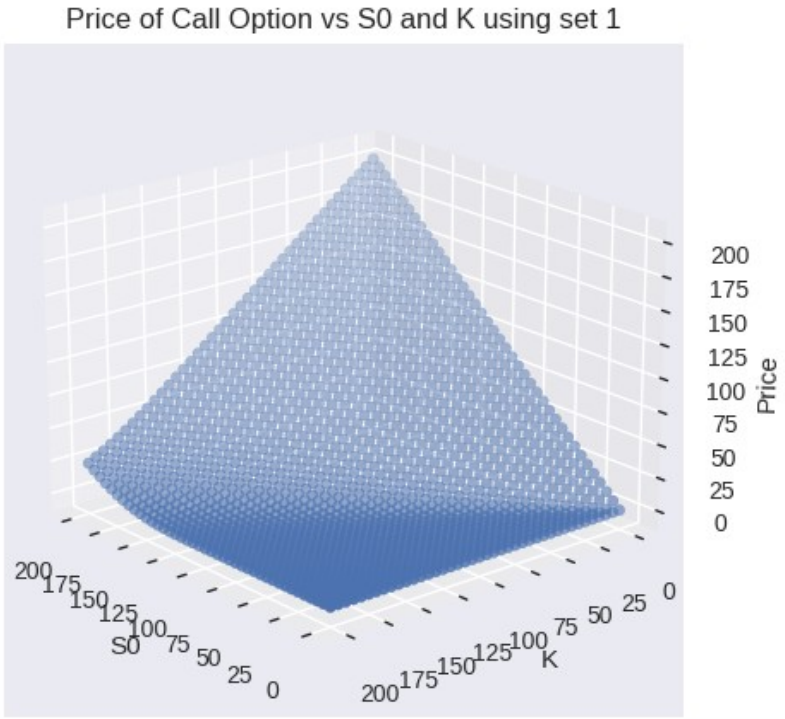

Price of Put Option vs M using set 1 and k = 105
Price
M
7.2
7.0
6.8
6.6
6.4
0
25
50
75
100
125
150
175
200

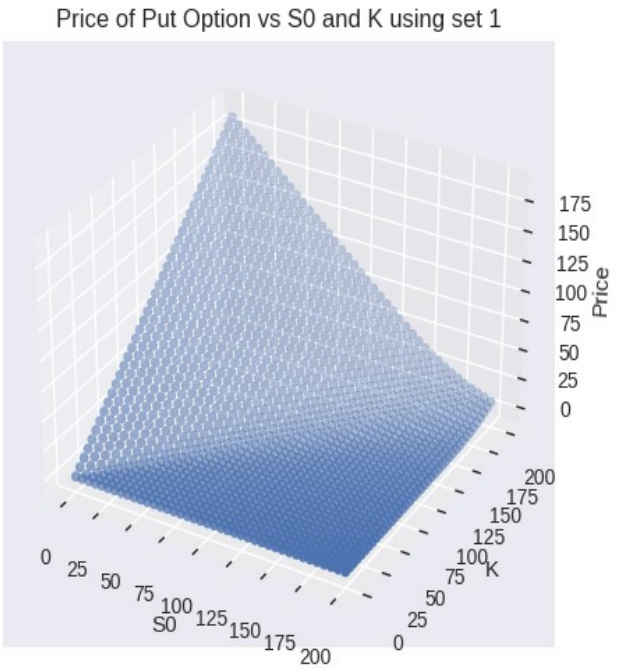

Price of Call Option vs M using set 2 and K = 105
Price
M
11.5
11.0
10.5
10.0
9.5
9.0
0
25
50
75
100
125
150
175
200

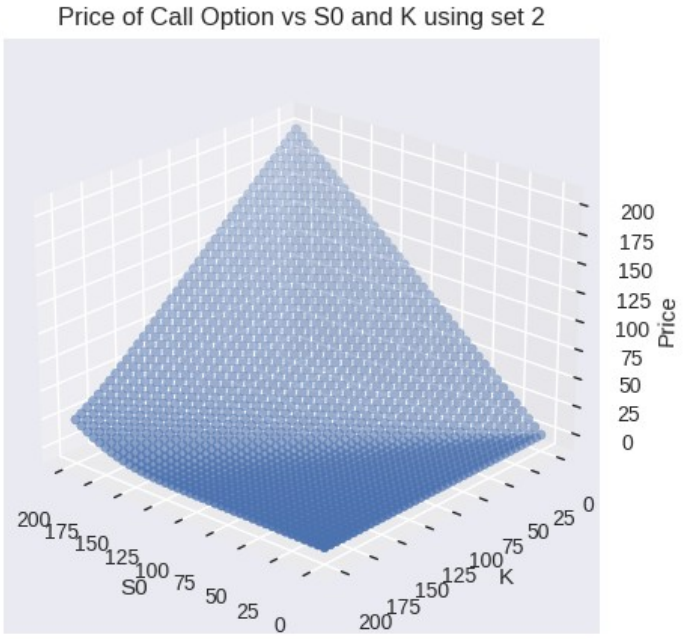

Price of Put Option vs M using set 2 and k = 105
Price
M
8.0
7.5
7.0
6.5
6.0
0
25
50
75
100
125
150
175
200

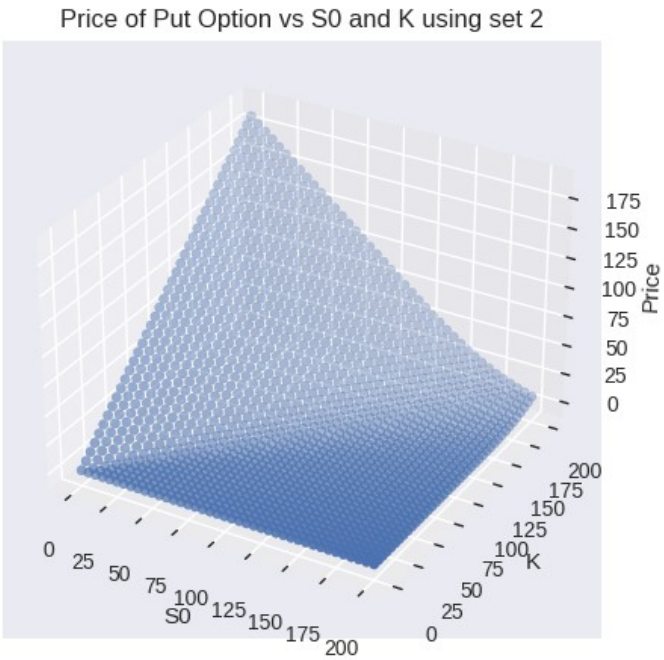
3D-Plots

Variation with S_0 and K

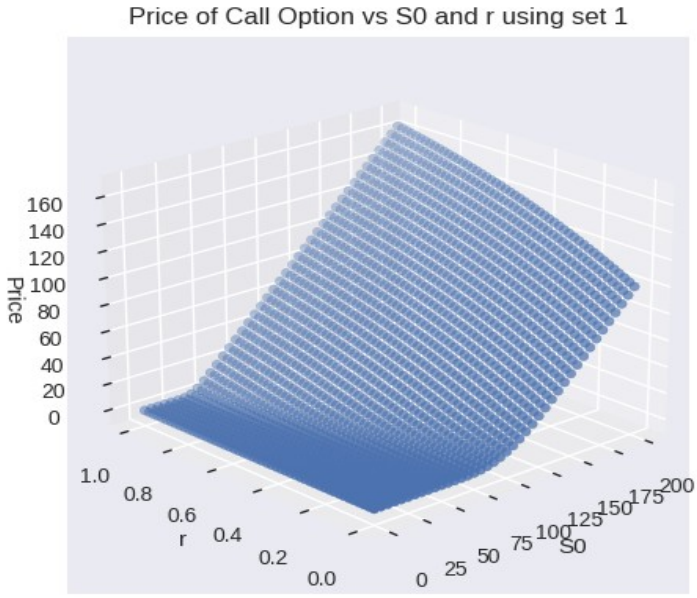


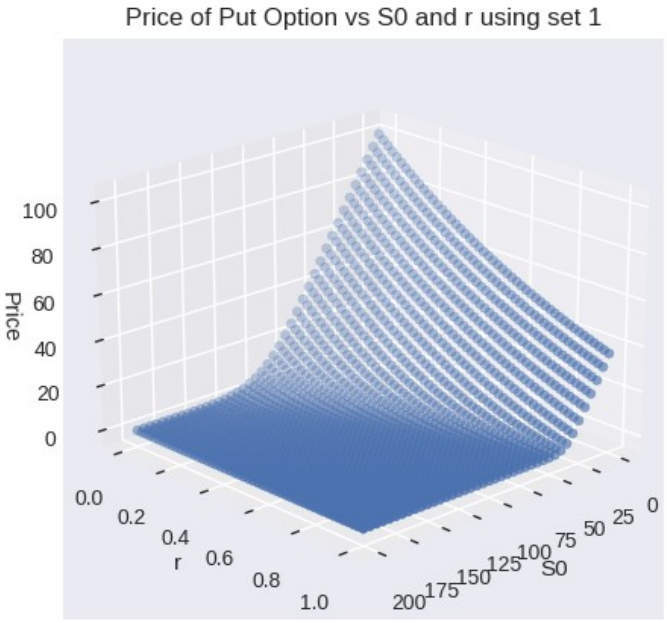


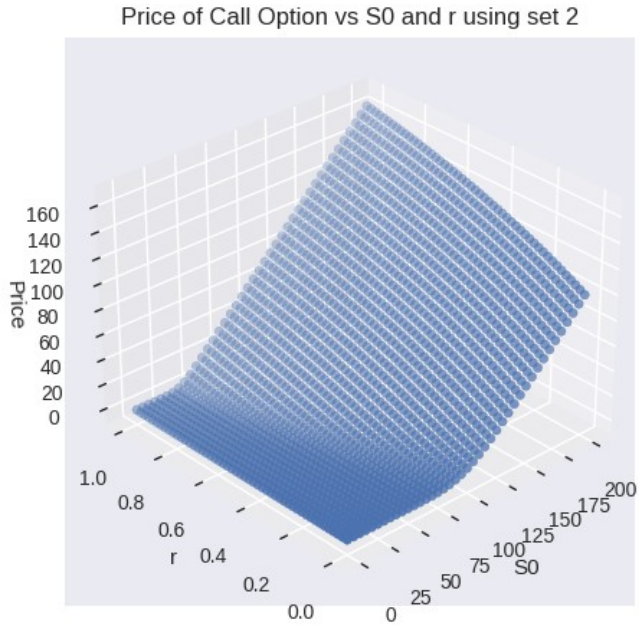


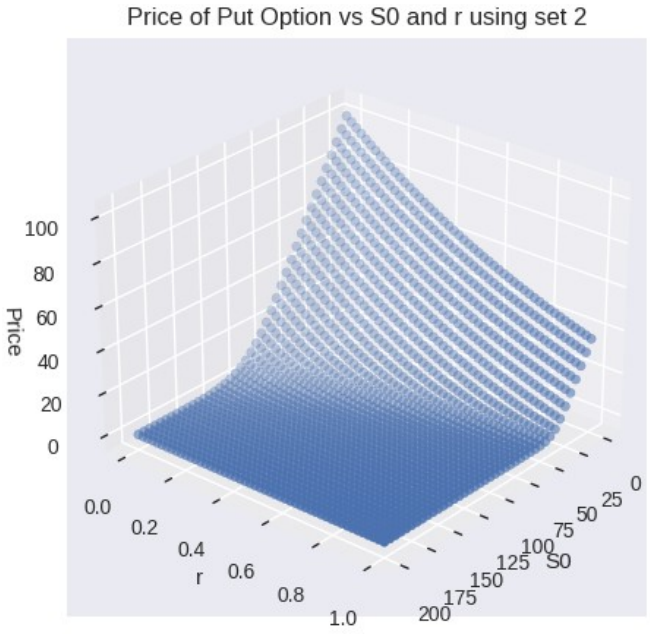


Variation with S_0 and r

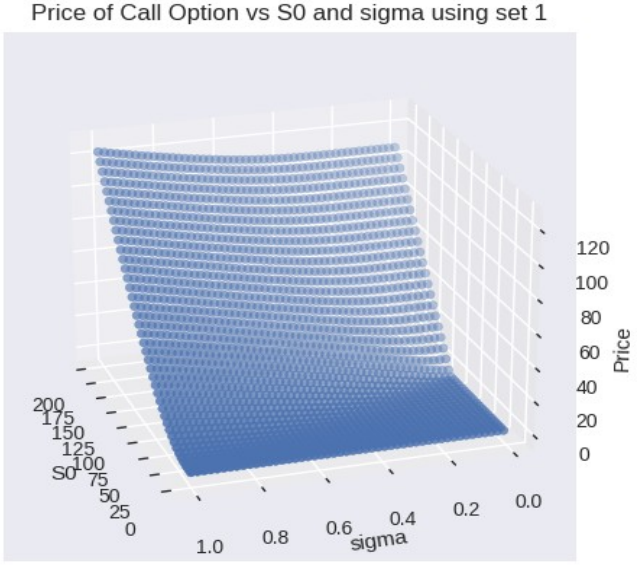


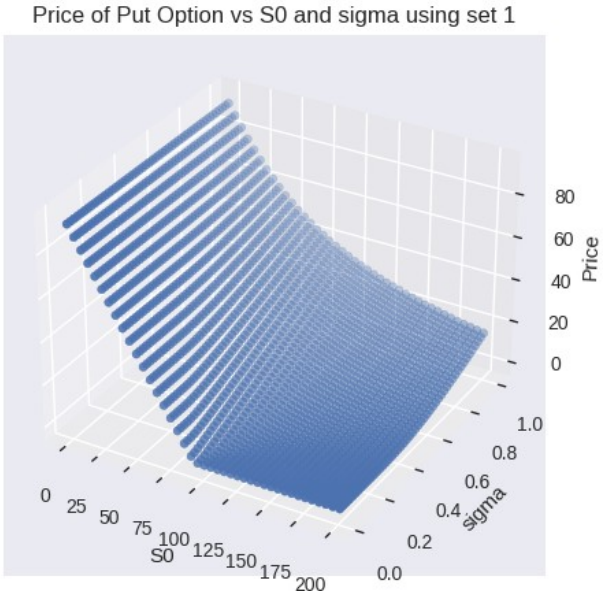


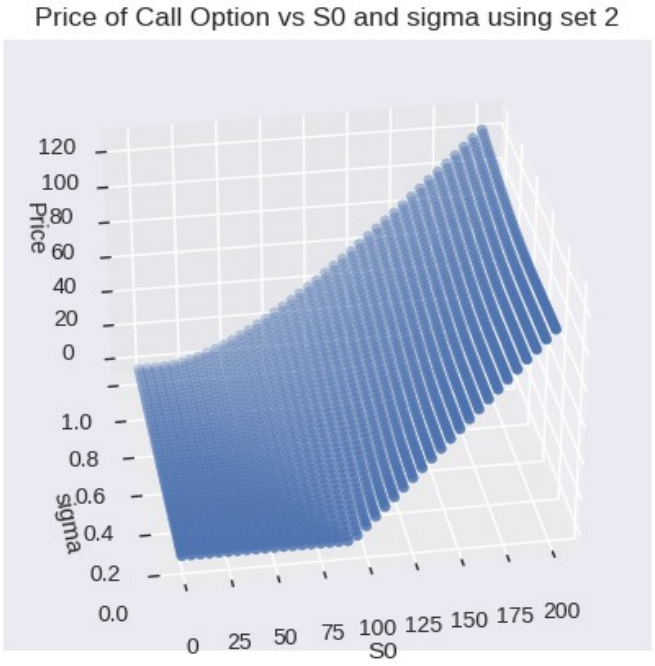


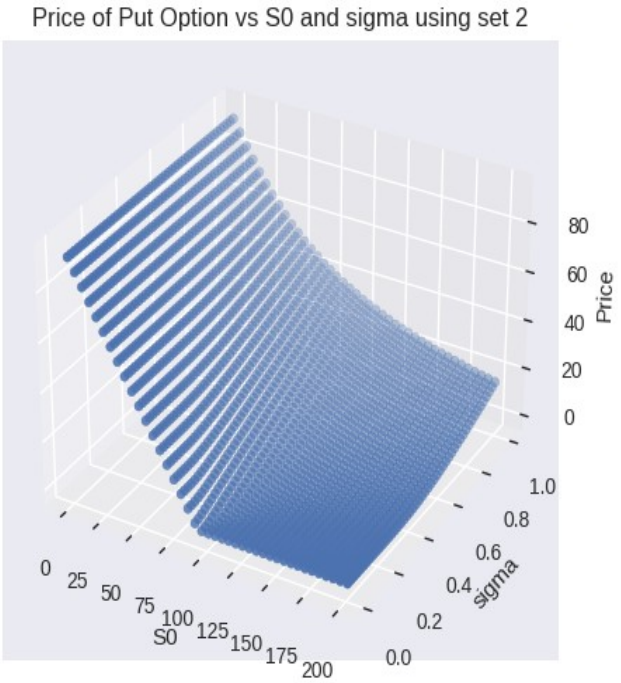


Variation with S_0 and σ

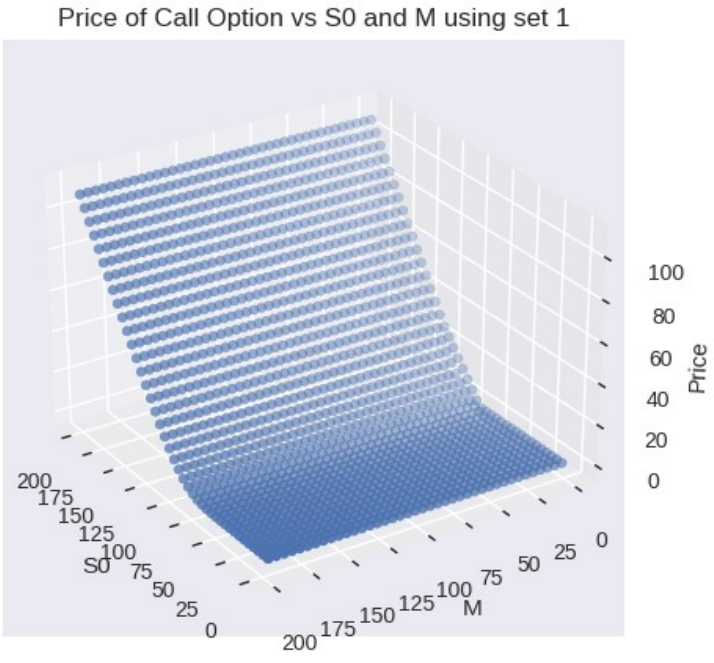


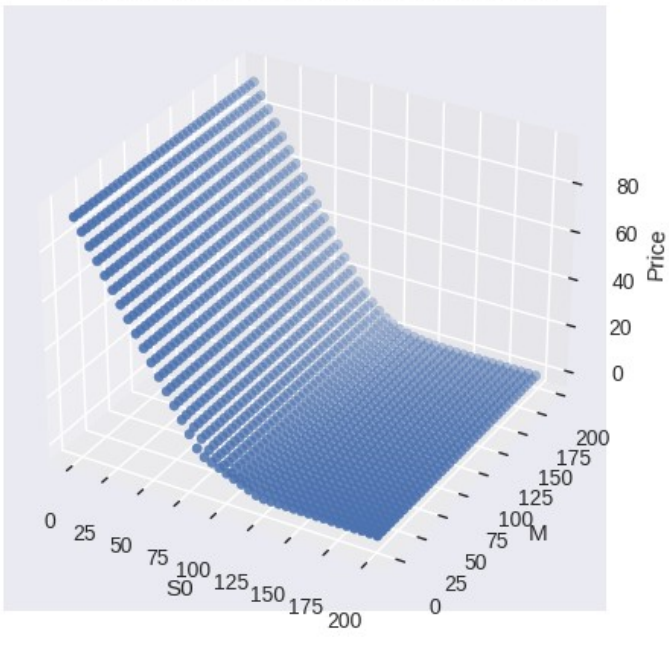


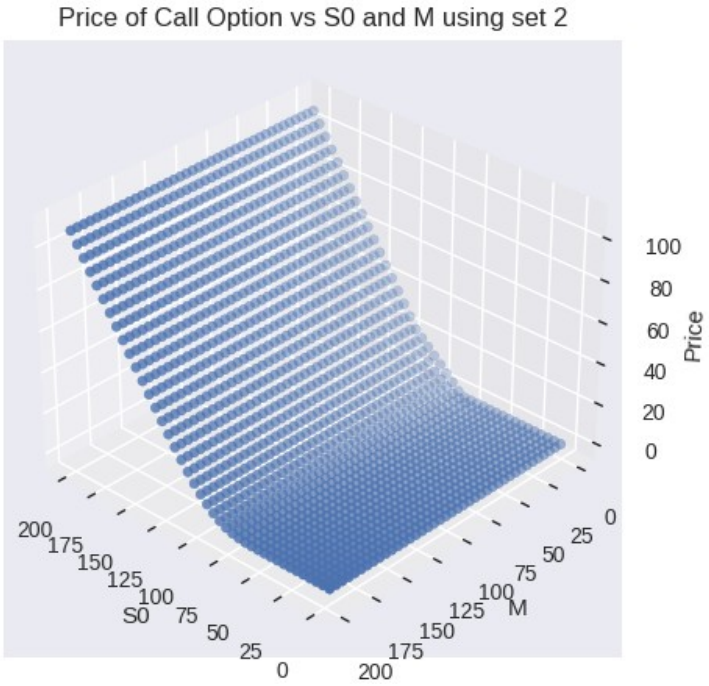


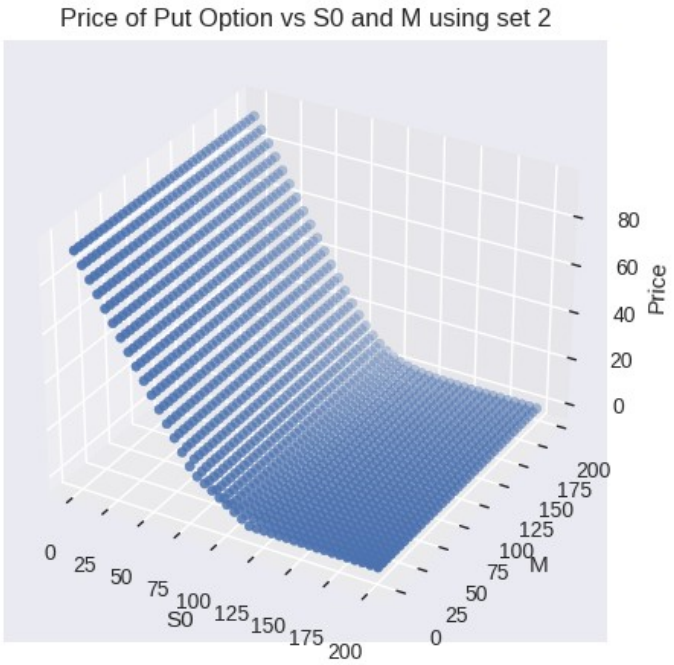


Variation with S_0 and M

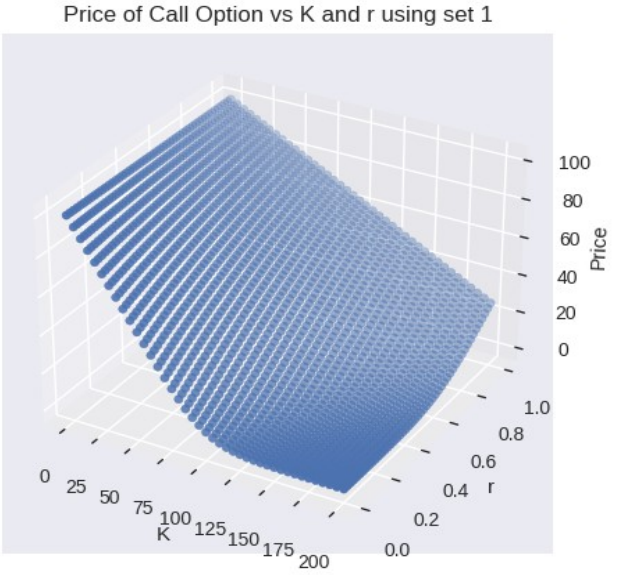


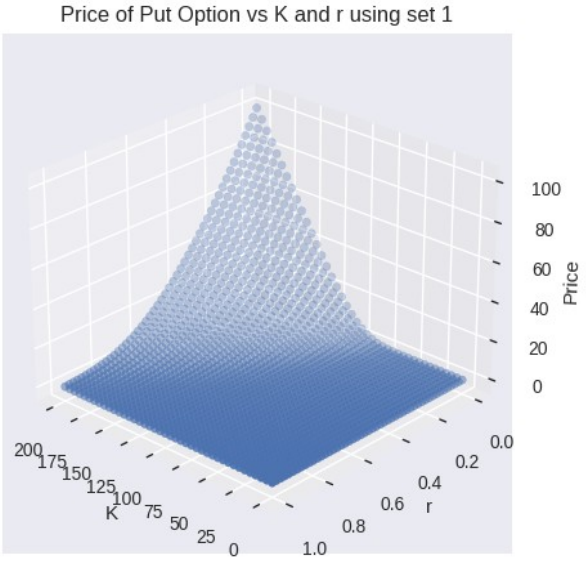


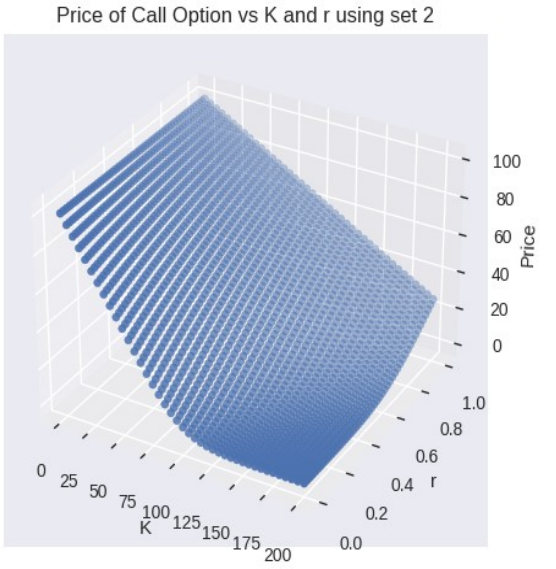


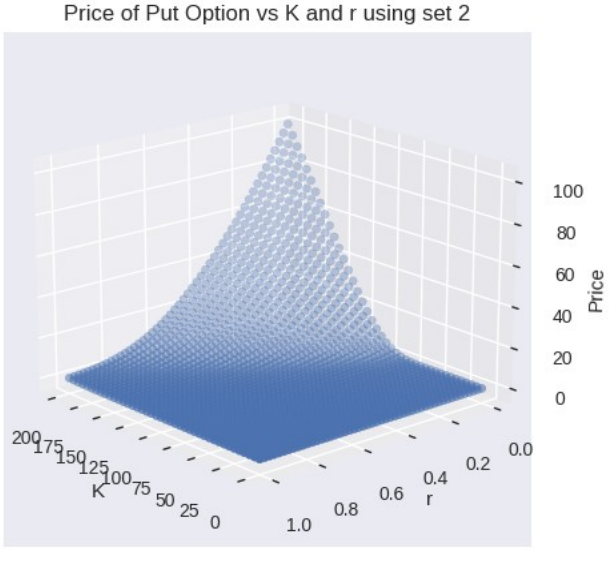


Variation with K and r

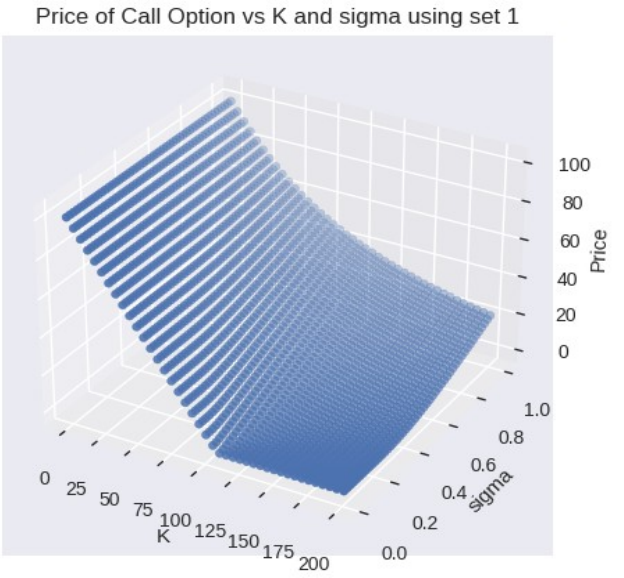


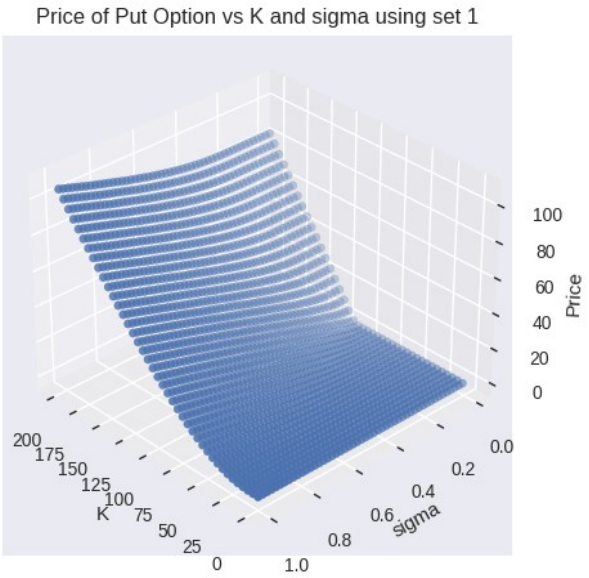


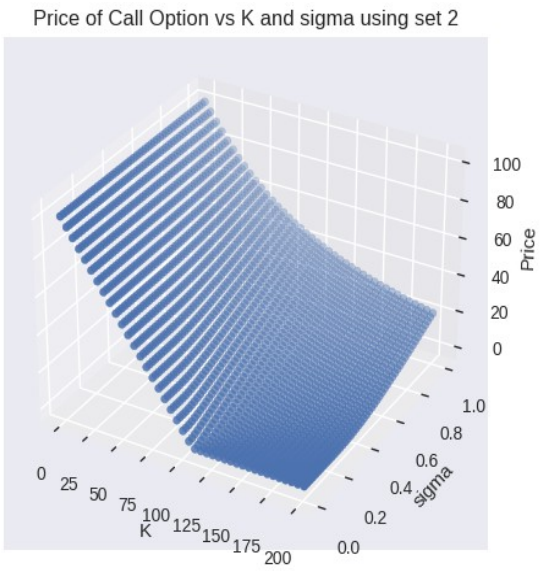


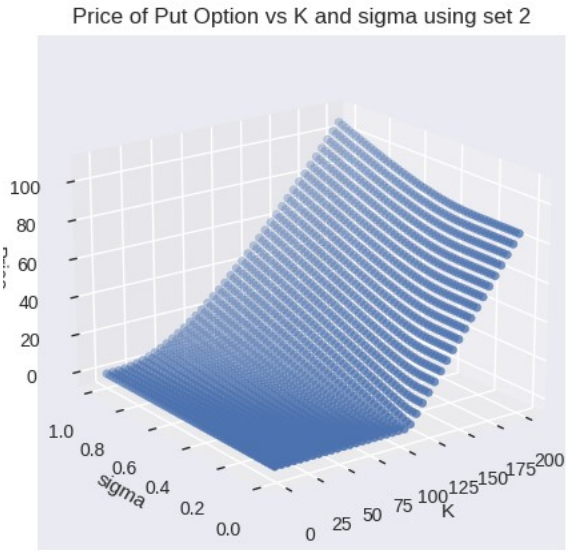


Variation with K and σ

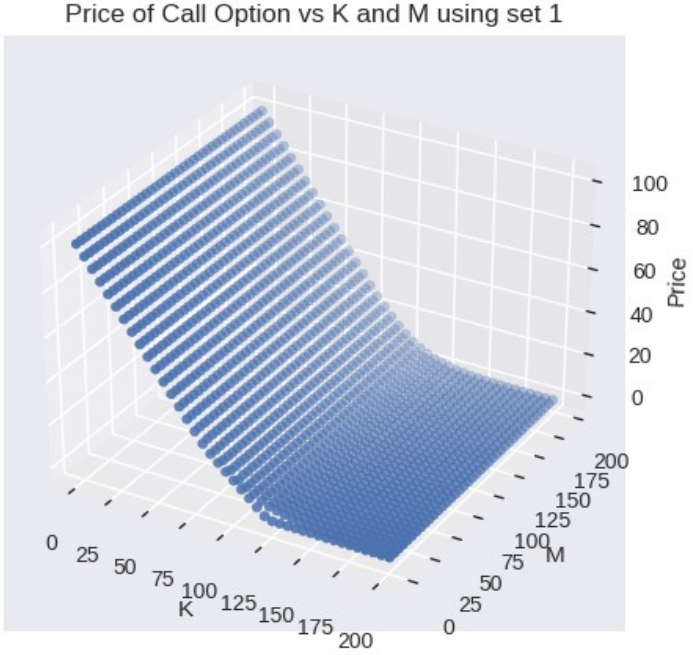


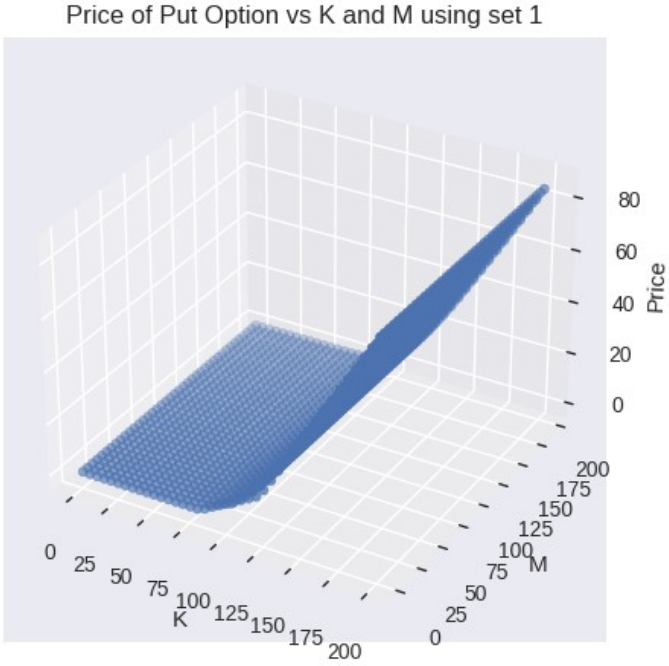


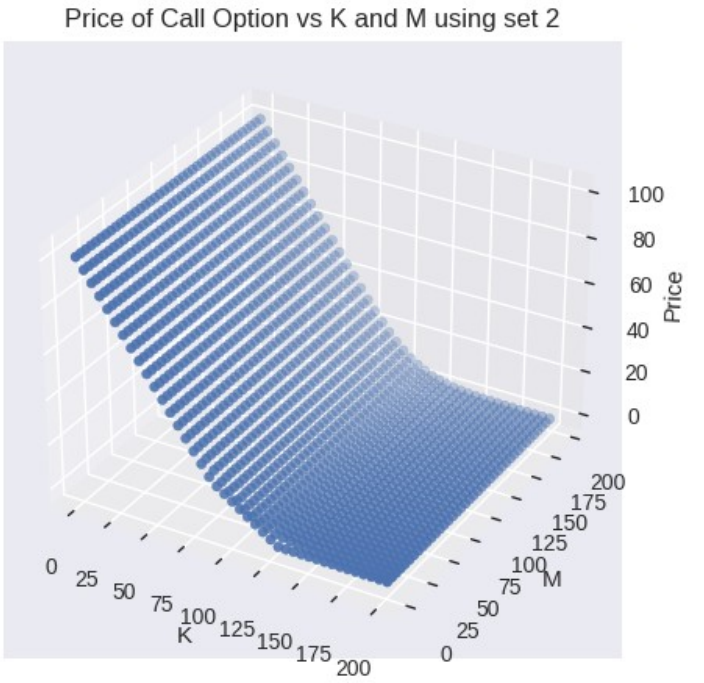


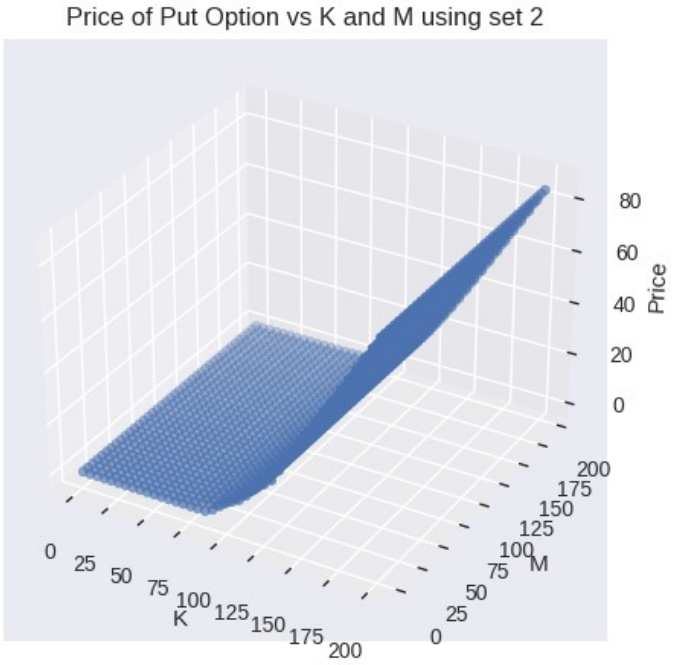


Variation with K and M

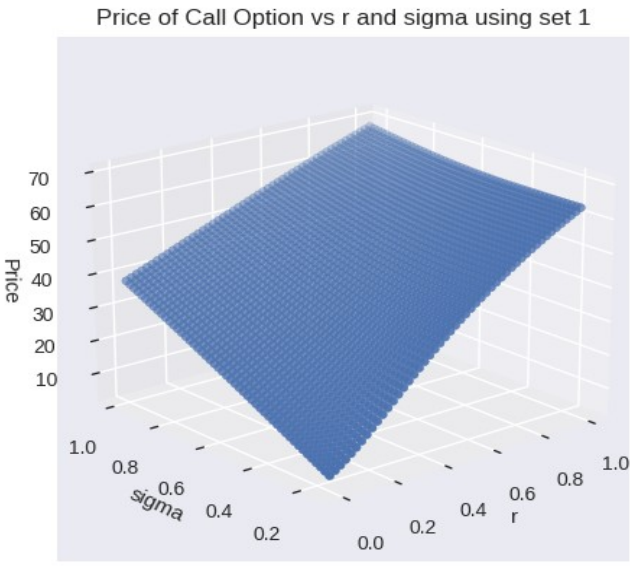


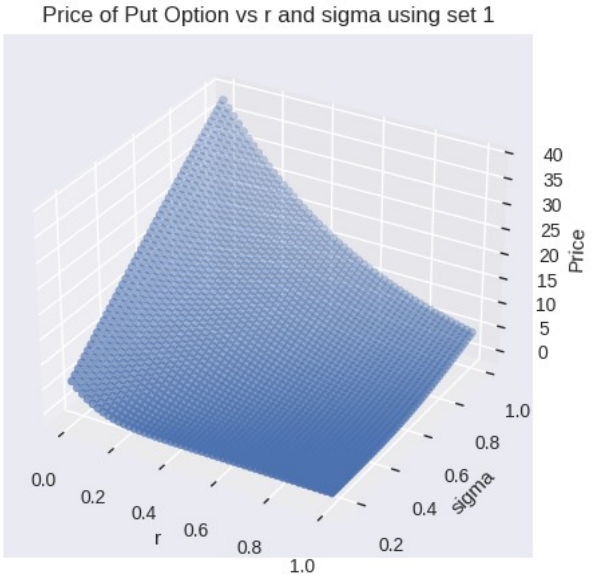


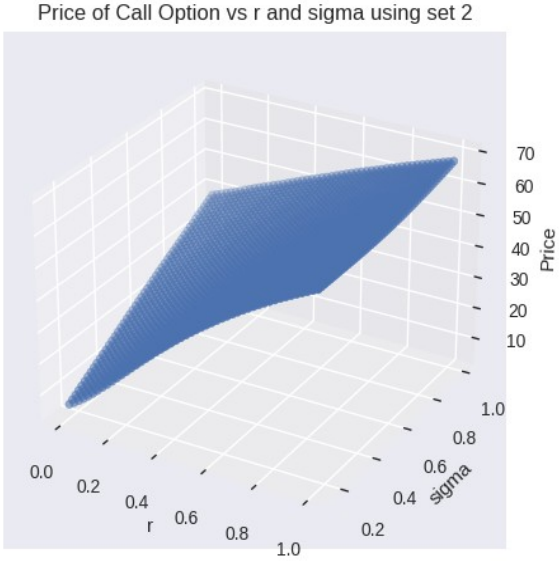


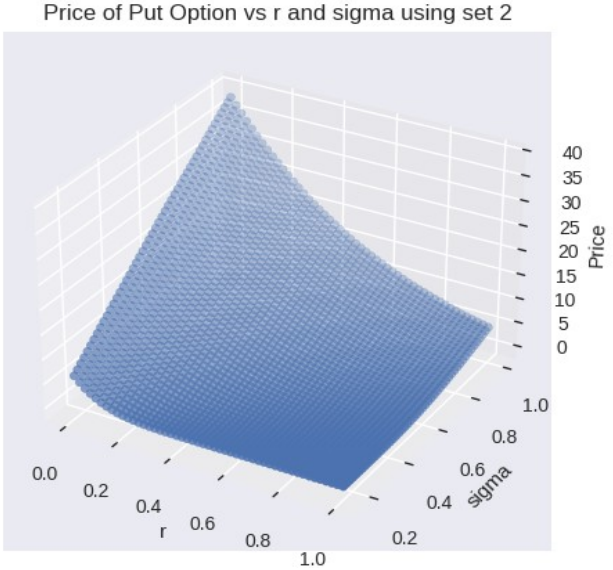


Variation with r and σ

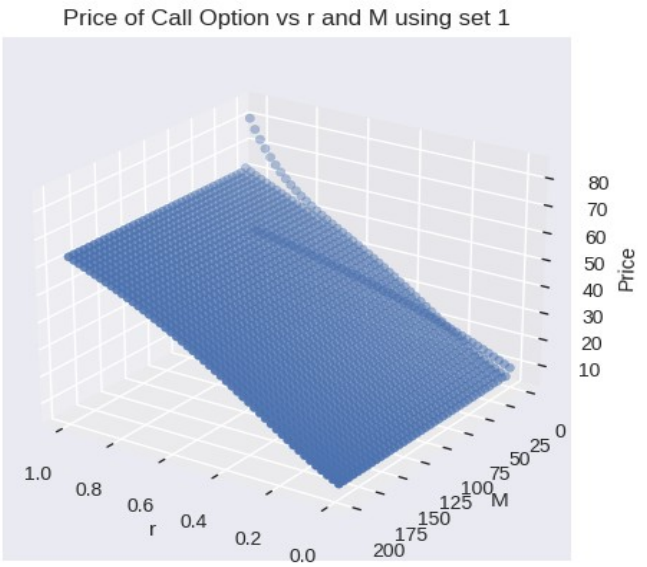


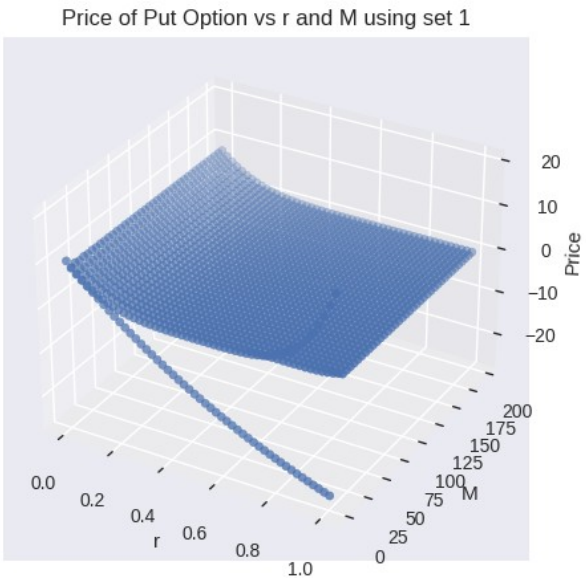


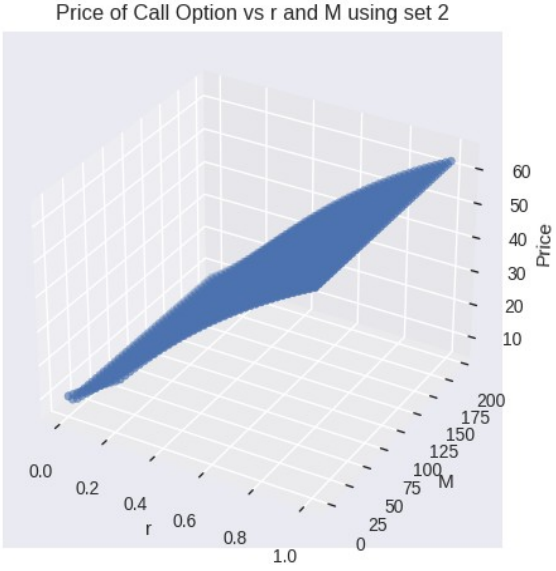


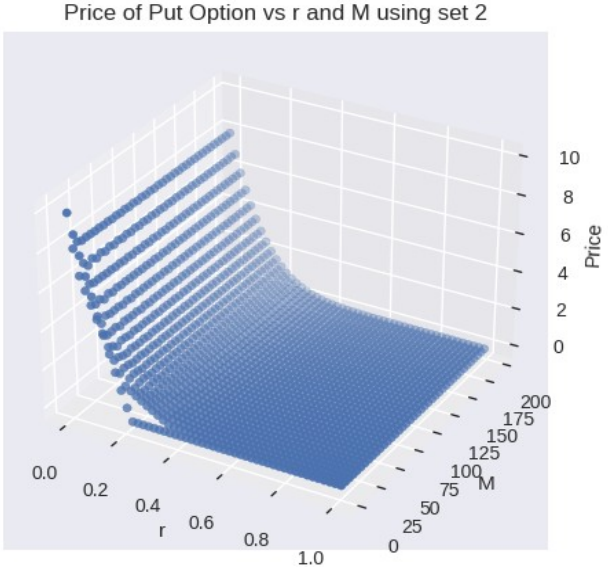


Variation with r and M

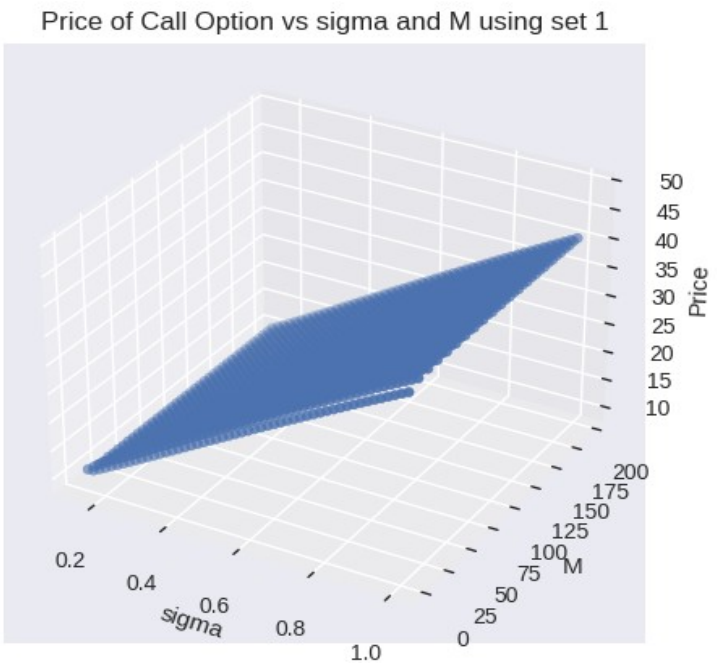


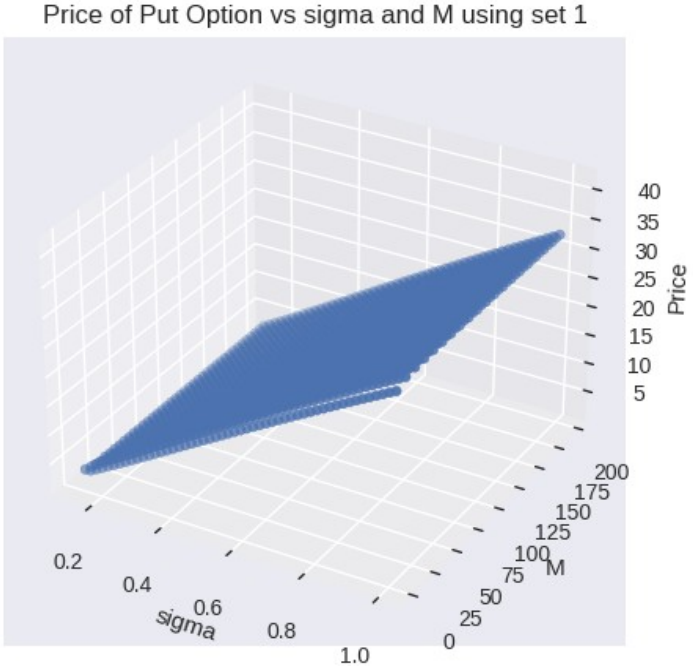


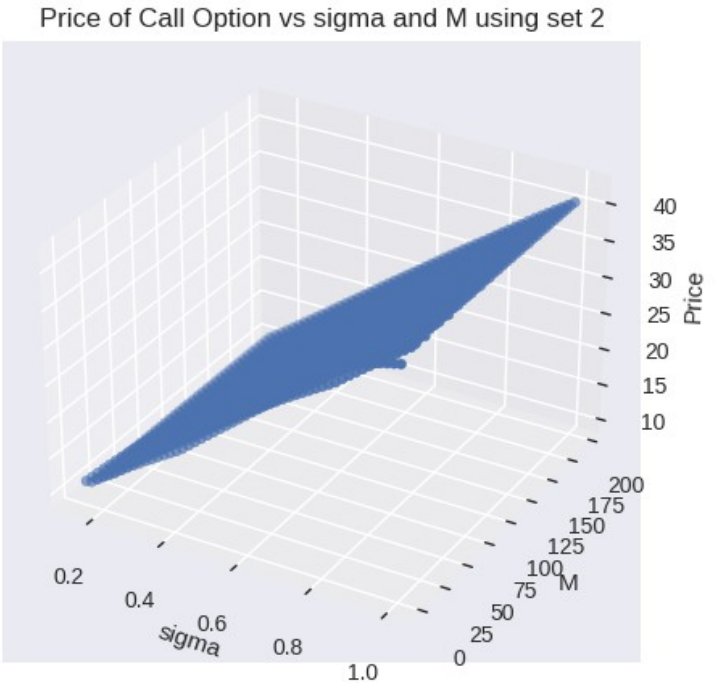


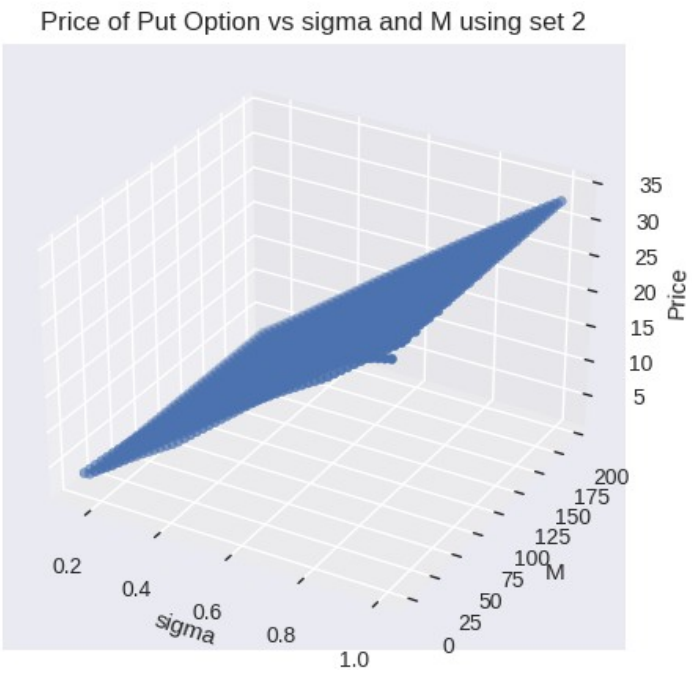


Variation with σ and M









Question-2

The path dependent derivative I chose for this problem is the Asian call and the Asian put option.It has the payoff as given below:-
For Asian Call Option = V(T) = max(S_{avg} -K,0)
For Asian Put Option = V(T) = max($K-S_{avg}$,0)

Here S_{avg} is the average of the stock price over its path. $S_{avg}=1/N\sum_{1}^{N}S_i$

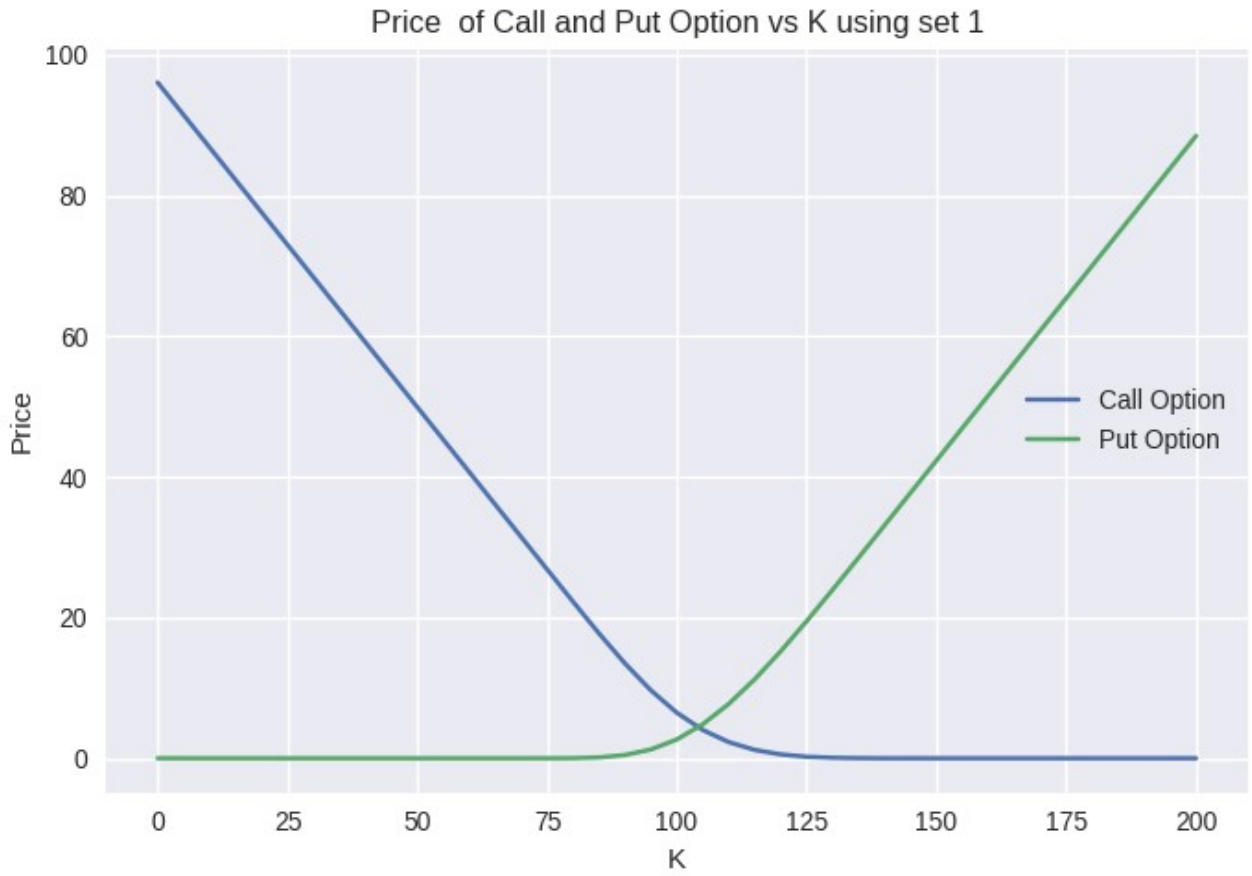
After using the above formulae, I used the algorithm for binomial model to calculate the option prices i.e. $V(T_i)=e^{-r\Delta}(\hat{p}*V(T_iH)+\hat{q}*V(T_iT))$. **Also note that since all paths are distinct and do not depend only on the number of heads and tails so the time complexity in this case is O($m2^m$) and hence the value of m I chose was equal to 10 whereas all the other parameters are same as given in the first problem**. Below are the answer to the problems in similar format as given in the first problem:-

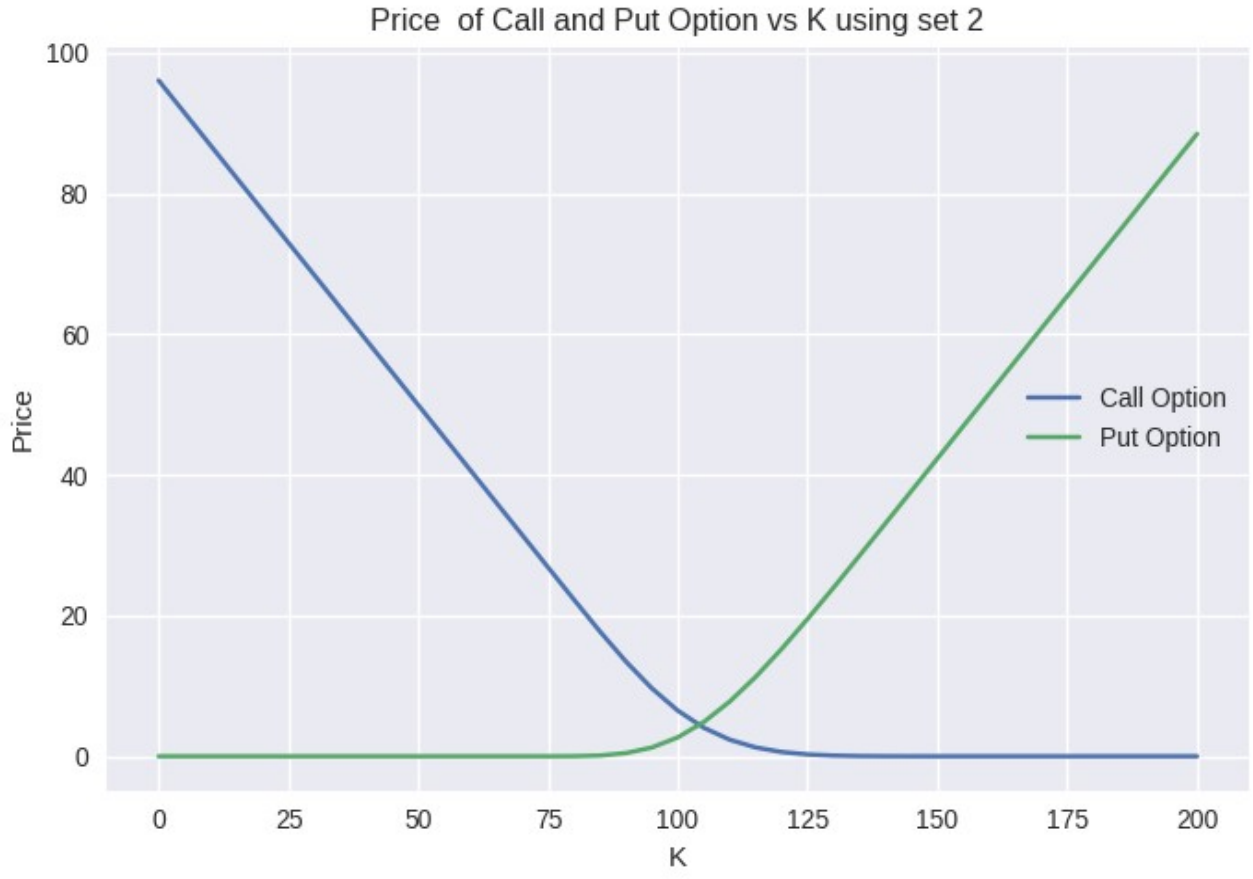
Set Number	Type of Option	Price
1	Asian Call	6.476003047446576
1	Asian Put	2.6779455899442346
2	Asian Call	6.49002937764339
2	Asian Put	2.6919719201410497

Plots

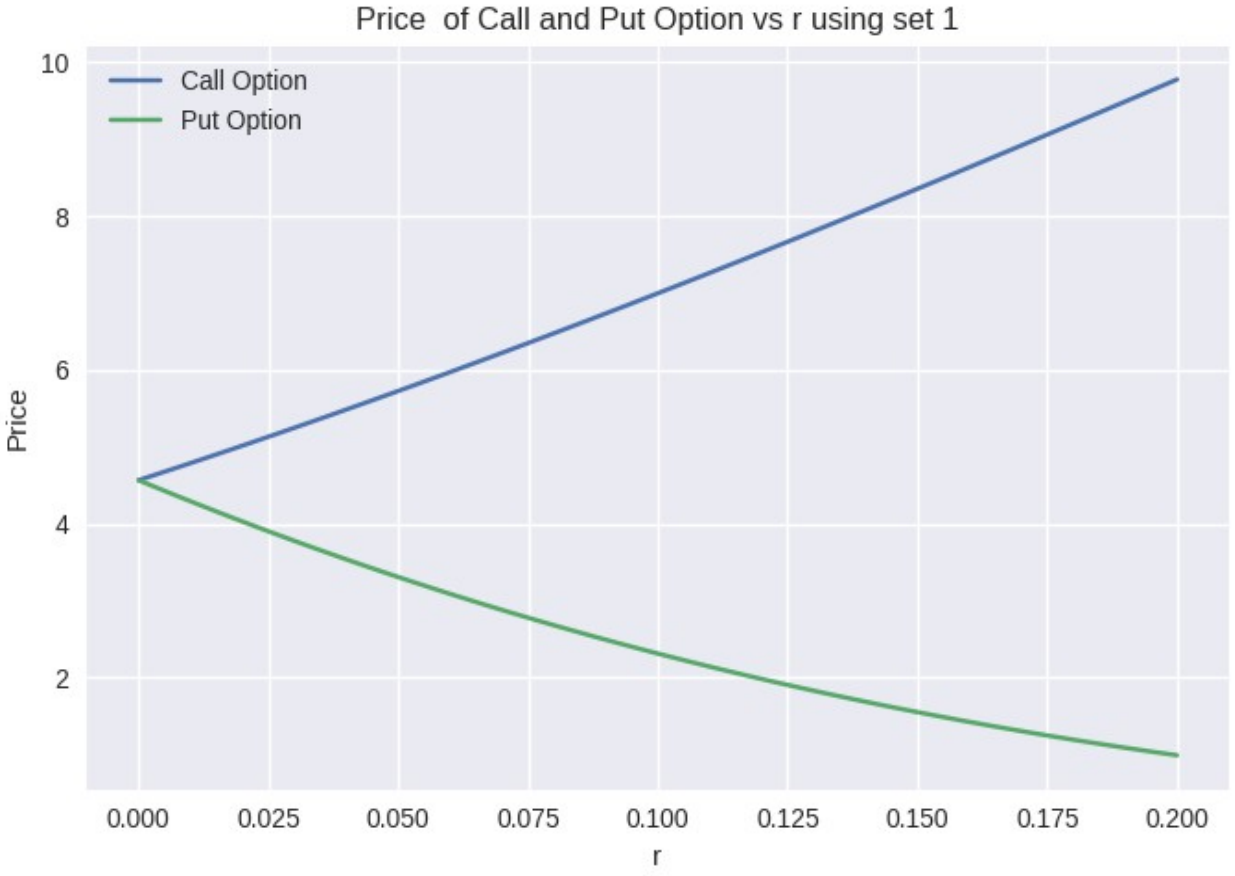
Variation with S_0

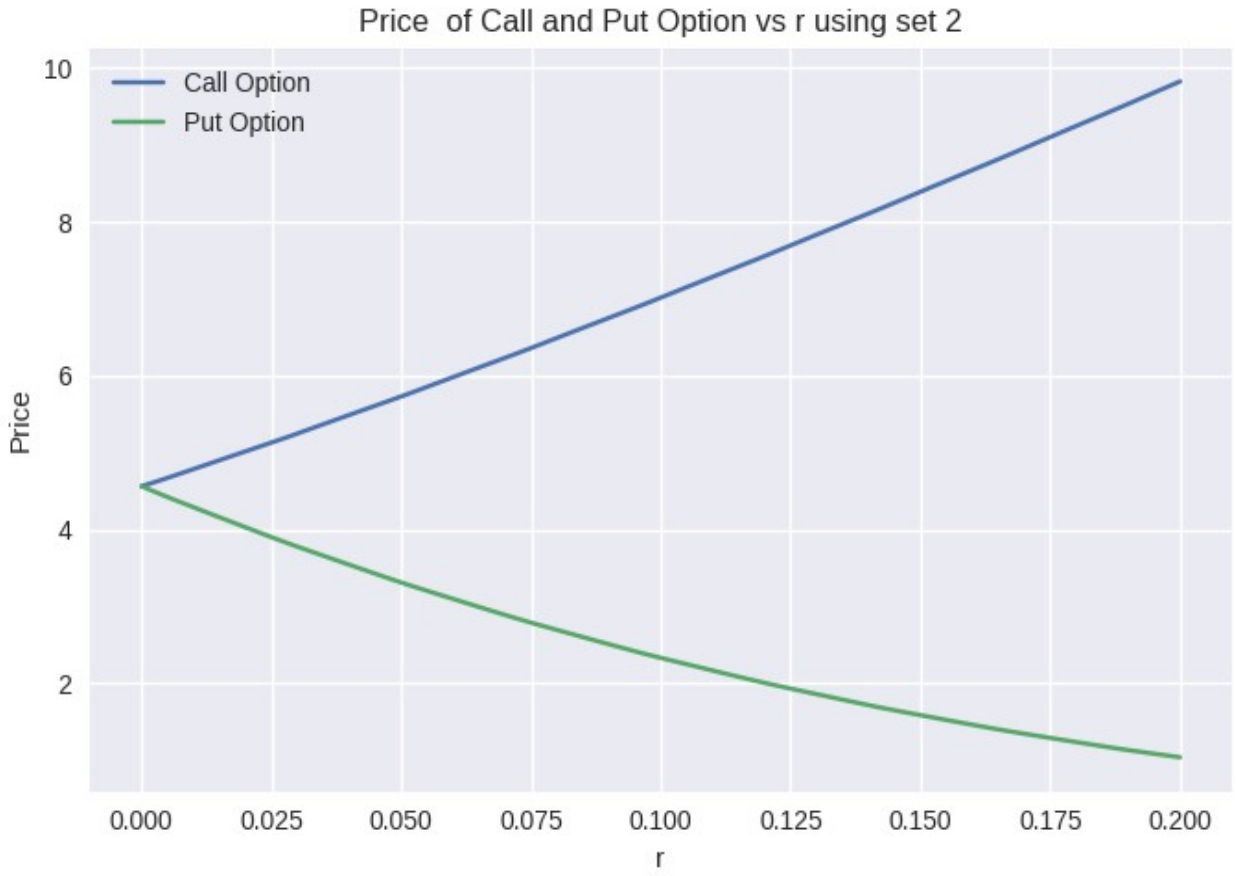
Variation with K



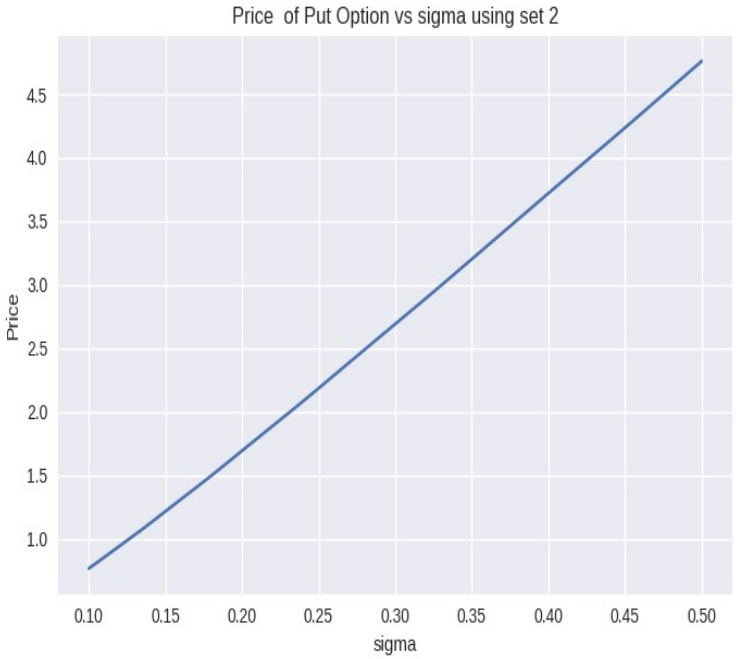


Variation with r



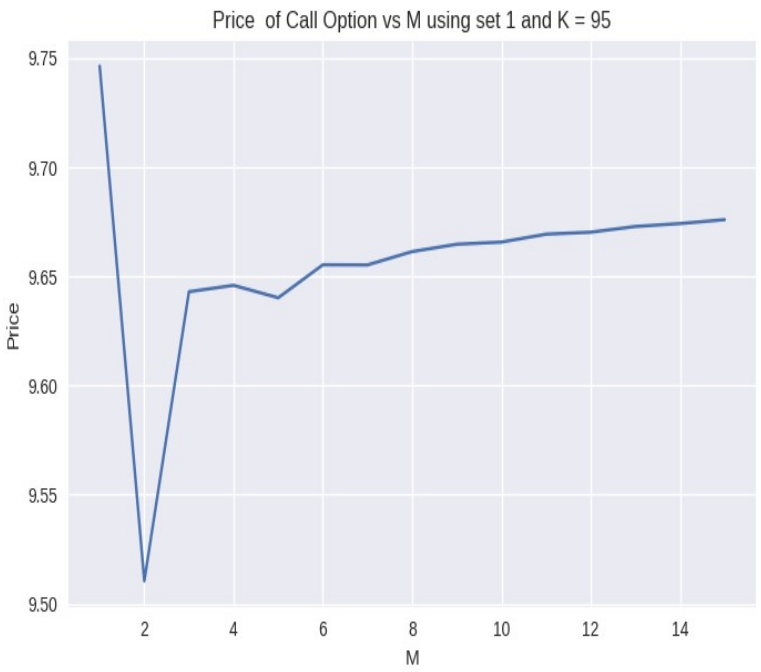


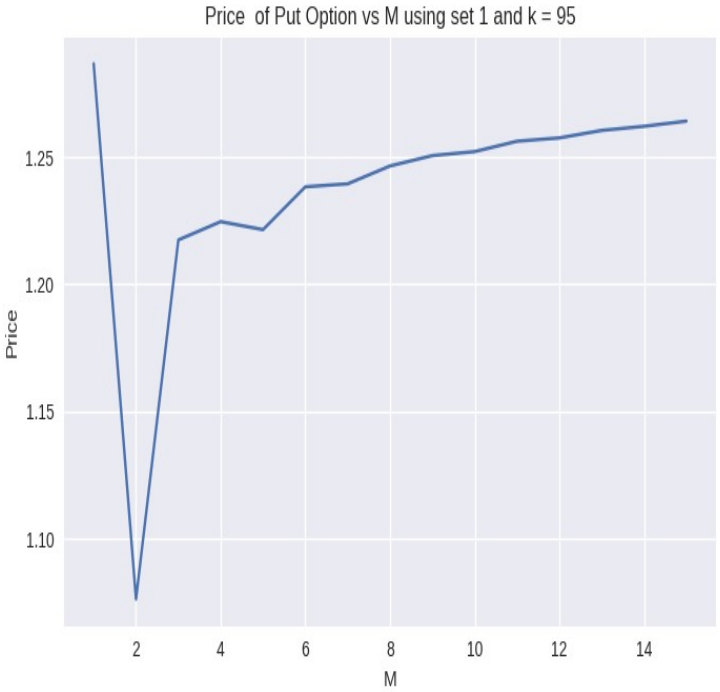
Variation with σ

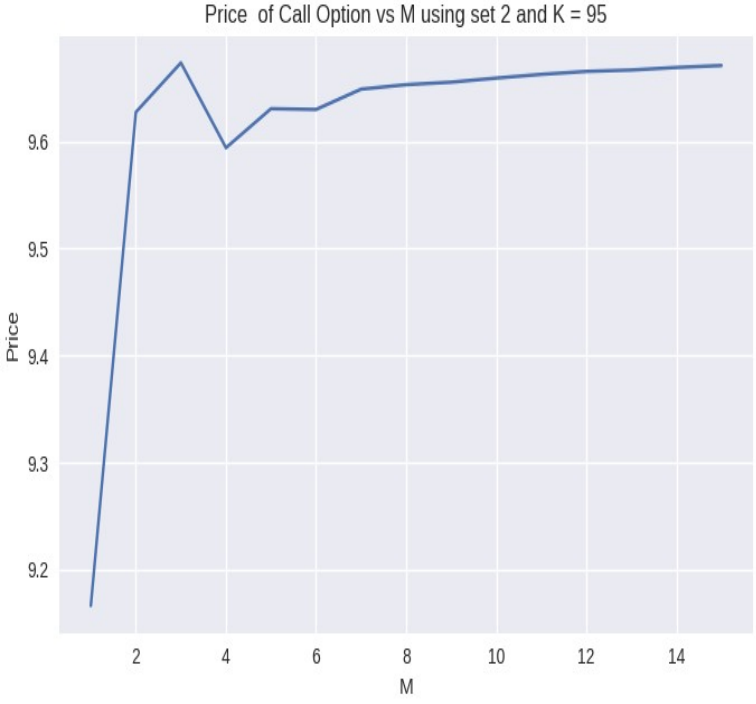


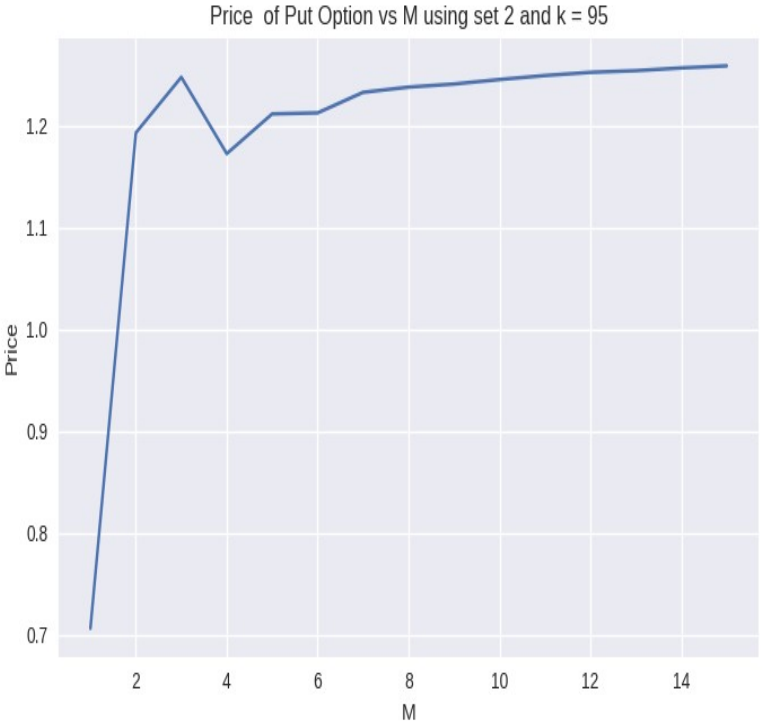
Variation with M

K=95

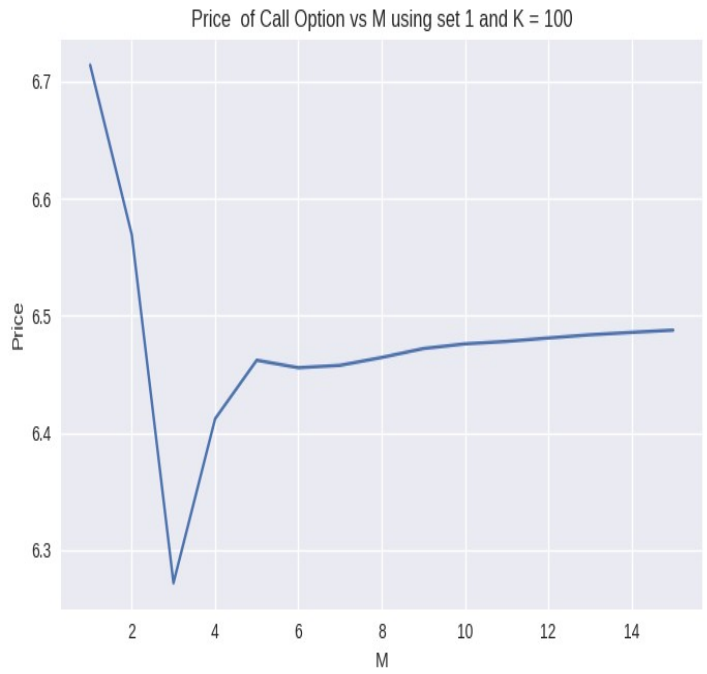


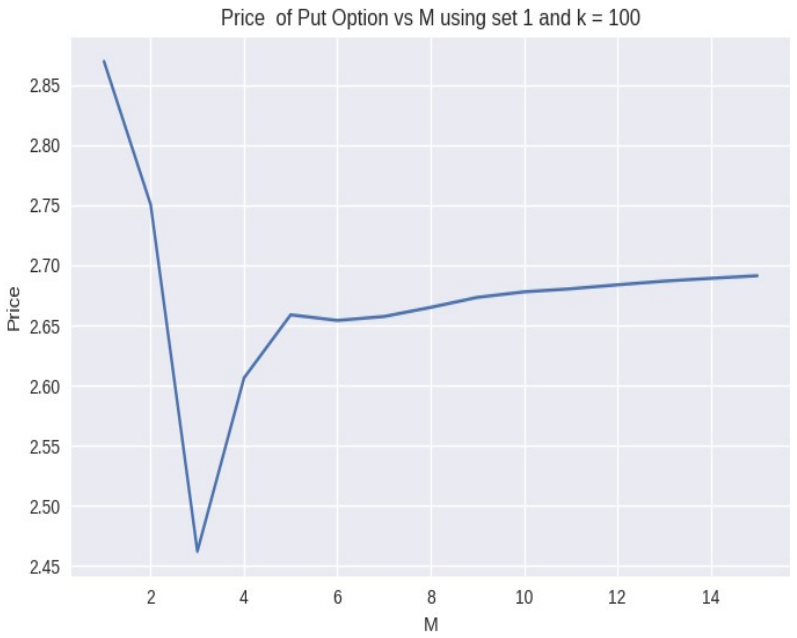


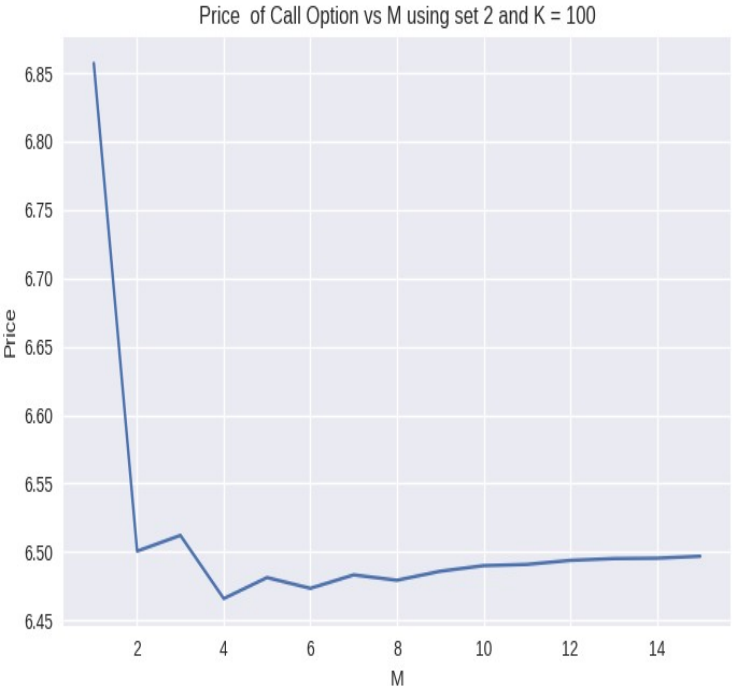


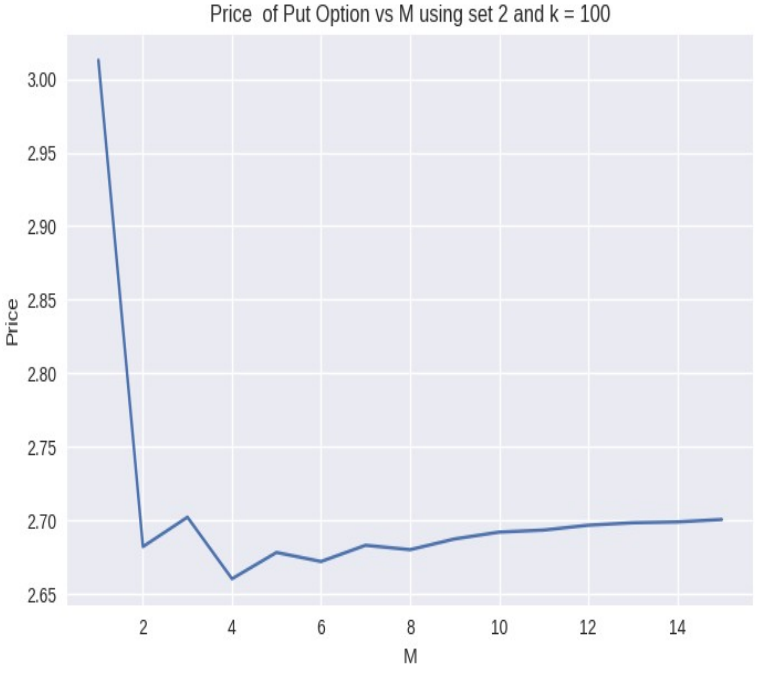


K=100

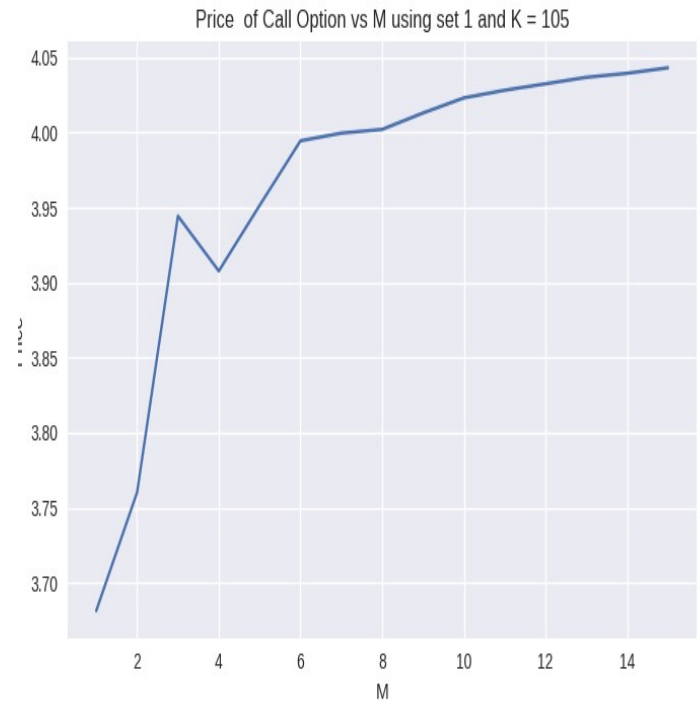

Price of Call Option vs M using set 1 and K = 100
Price
M
6.7
6.6
6.5
6.4
6.3
2
4
6
8
10
12
14


Price of Put Option vs M using set 1 and k = 100
Price
M
2.85
2.80
2.75
2.70
2.65
2.60
2.55
2.50
2.45
2
4
6
8
10
12
14

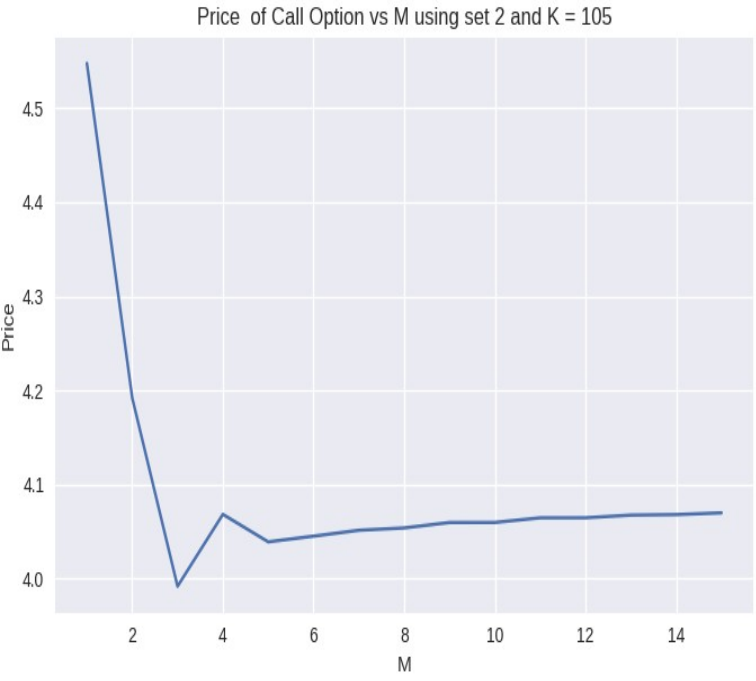

Price of Call Option vs M using set 2 and K = 100
Price
M
6.85
6.80
6.75
6.70
6.65
6.60
6.55
6.50
6.45
2
4
6
8
10
12
14

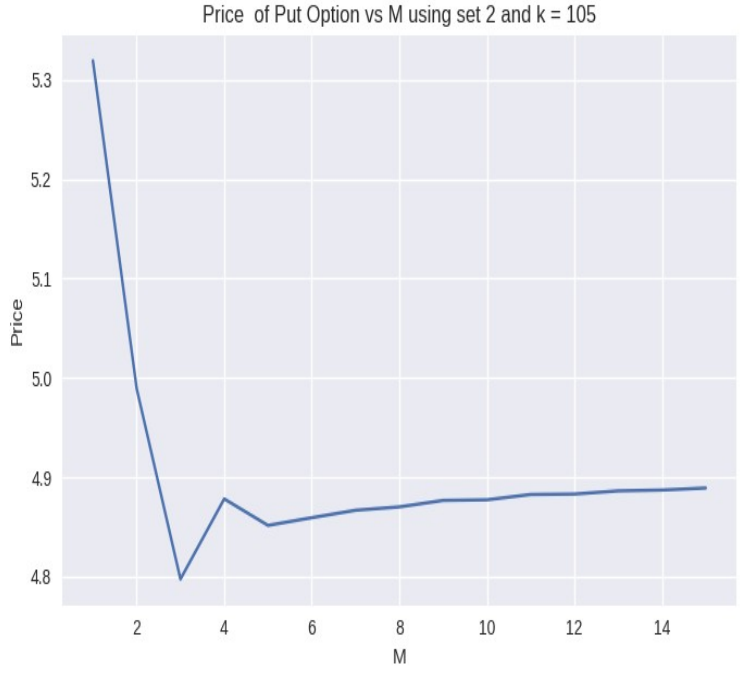

Price of Put Option vs M using set 2 and k = 100
Price
M
3.00
2.95
2.90
2.85
2.80
2.75
2.70
2.65
2
4
6
8
10
12
14

K=105


Price of Call Option vs M using set 1 and K = 105
4.05
4.00
3.95
3.90
3.85
3.80
3.75
3.70
2
4
6
8
10
12
14
M

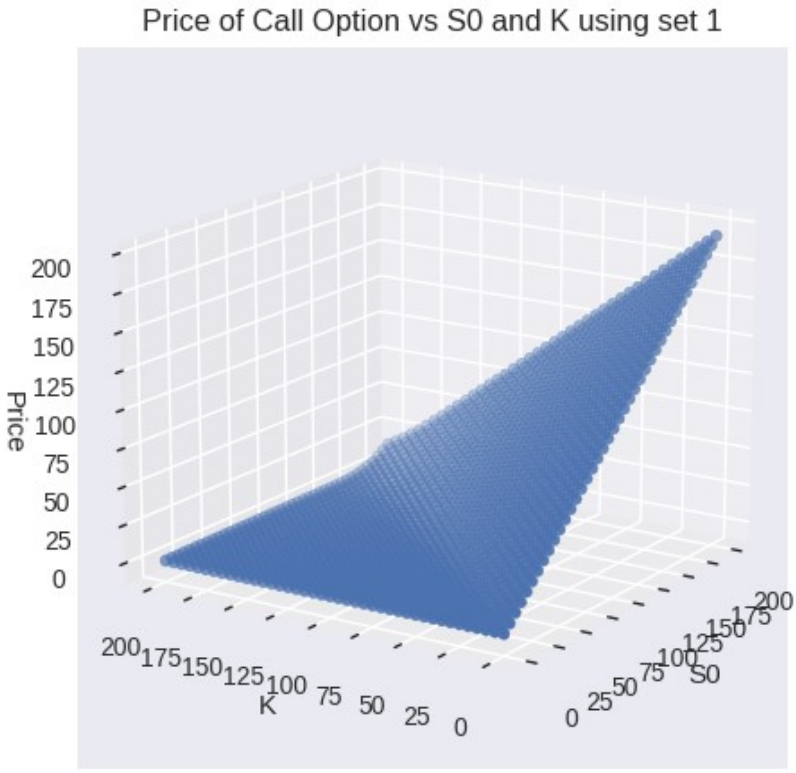

Price of Put Option vs M using set 1 and k = 105
4.8
4.7
4.6
4.5
Price
2
4
6
8
10
12
14
M

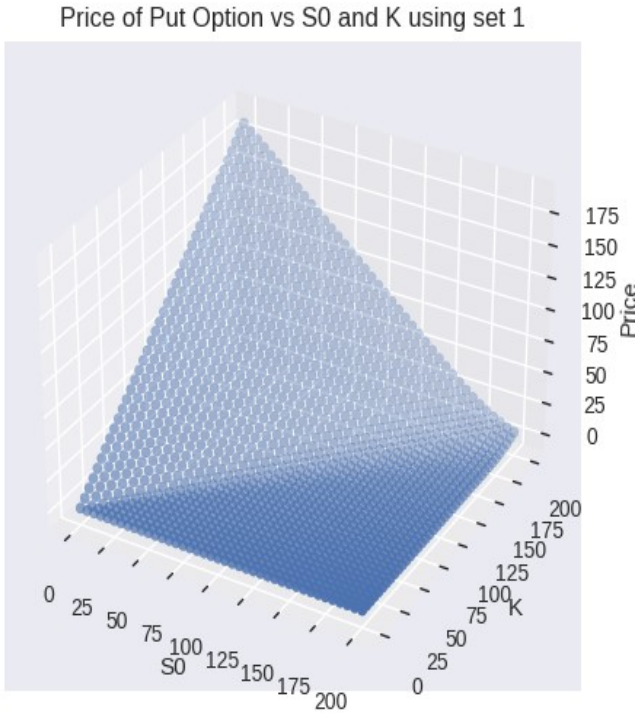

Price of Call Option vs M using set 2 and K = 105
4.5
4.4
4.3
4.2
4.1
4.0
Price
2
4
6
8
10
12
14
M

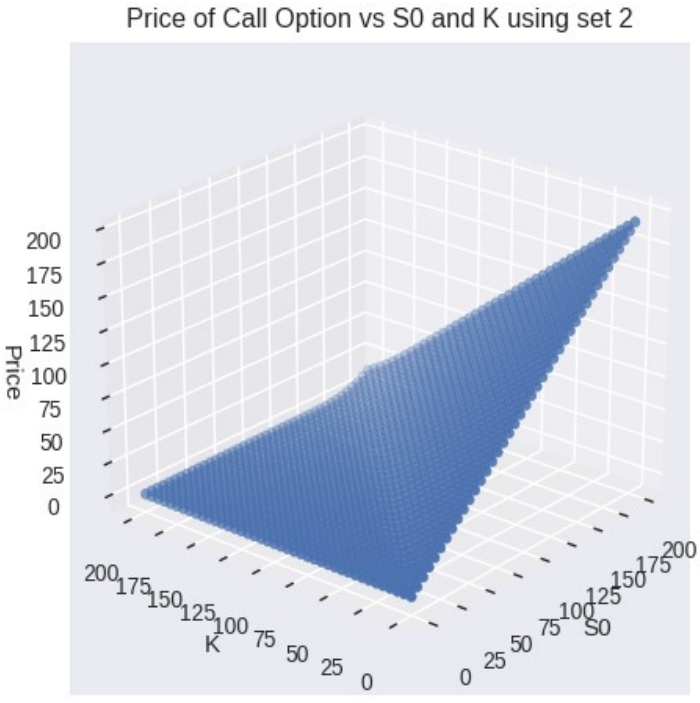

Price of Put Option vs M using set 2 and k = 105
5.3
5.2
5.1
5.0
4.9
4.8
Price
2
4
6
8
10
12
14
M

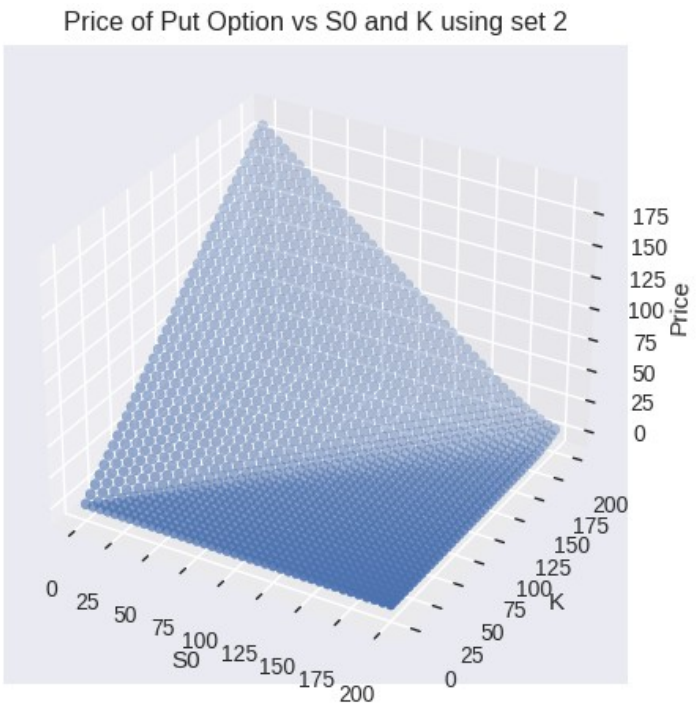
3D-Plots

Variation with S_0 and K

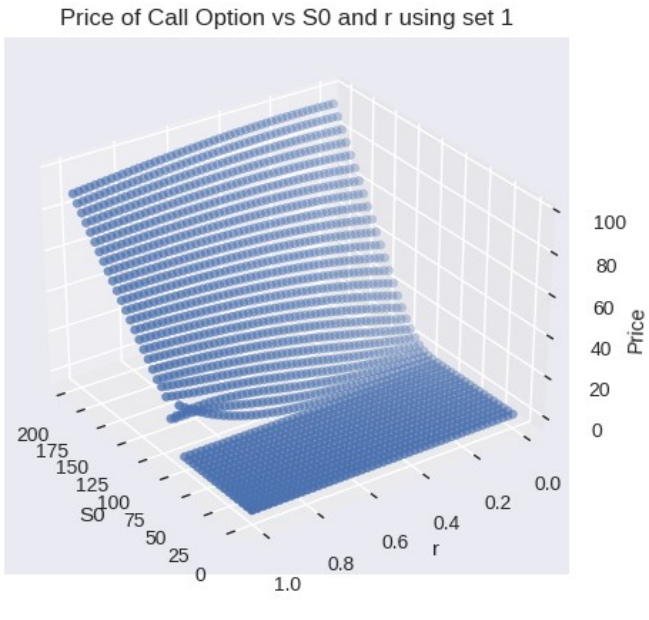


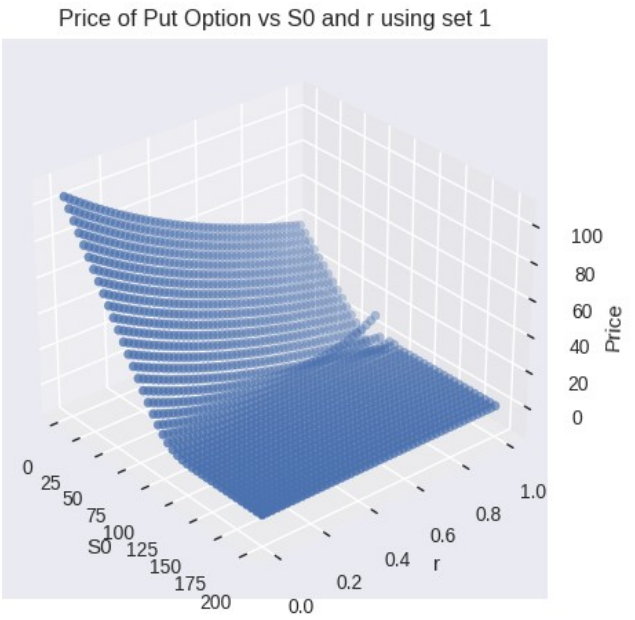


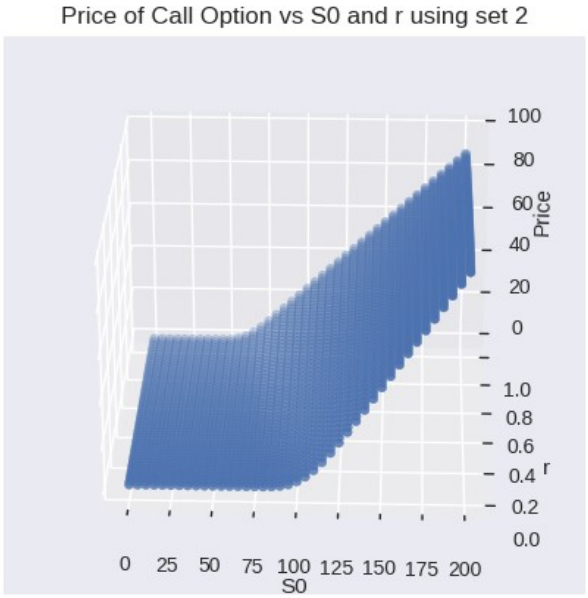


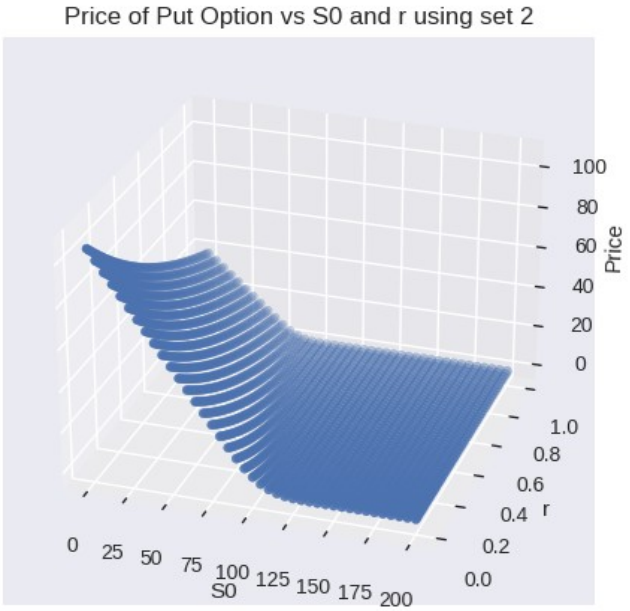


Variation with S_0 and r

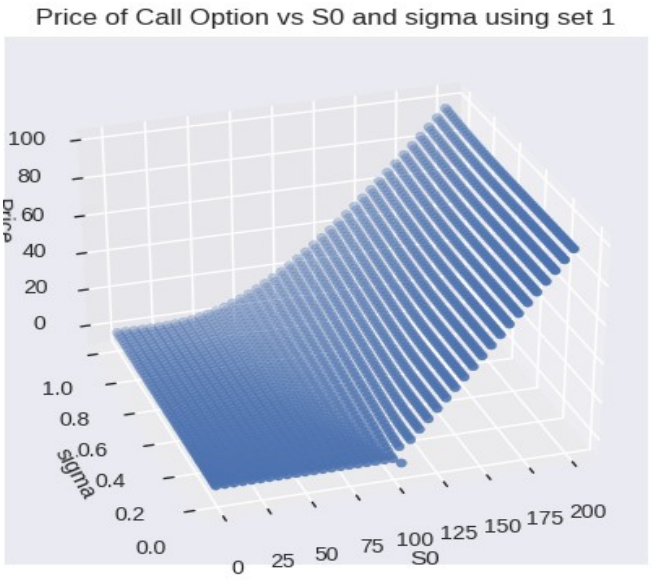


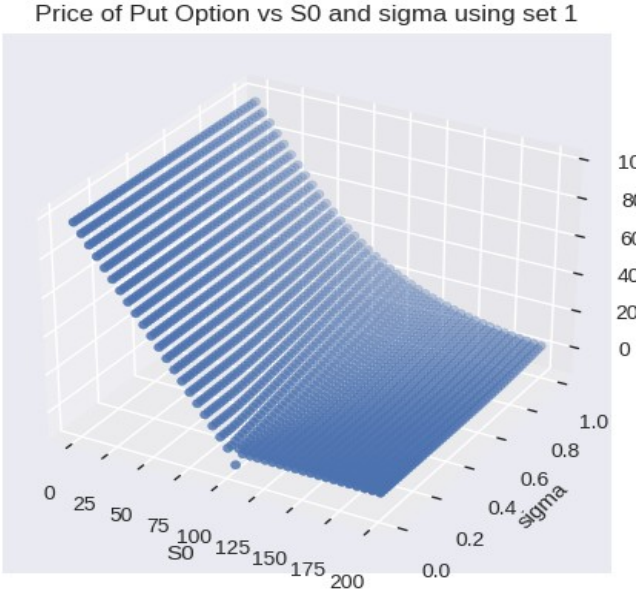


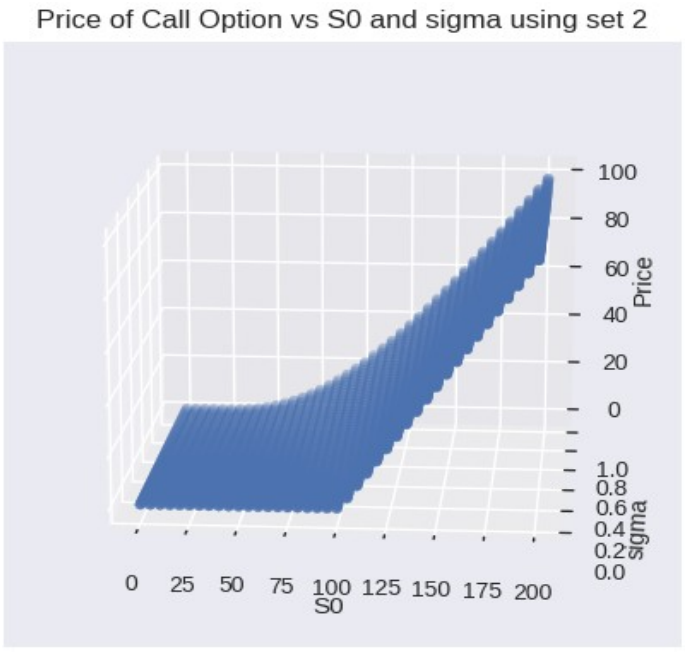


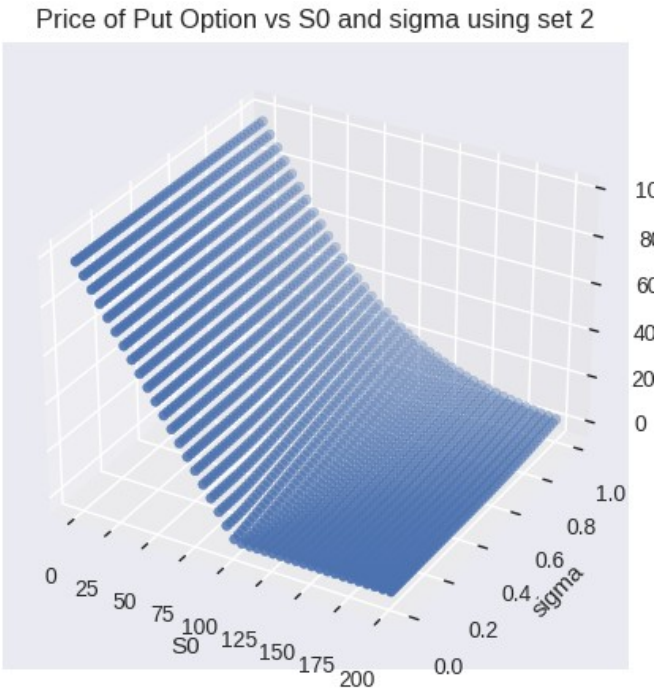


Variation with S_0 and σ

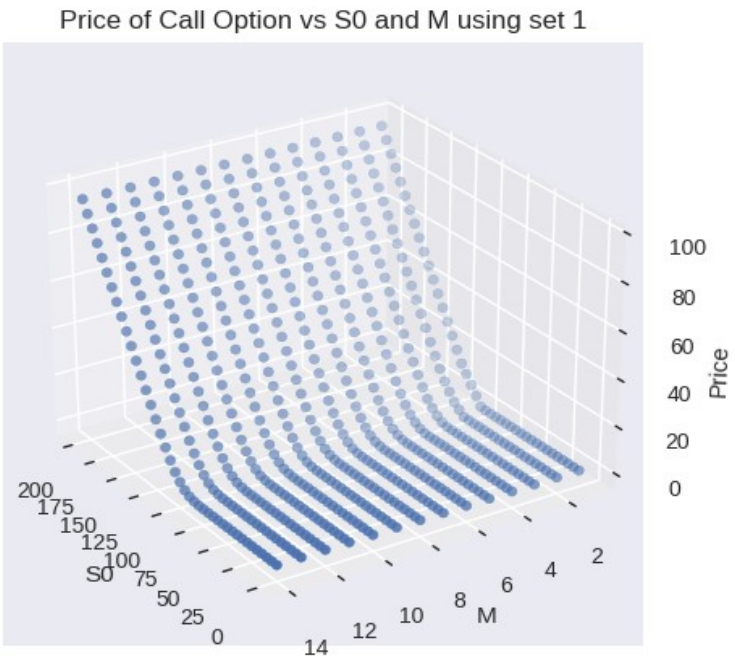


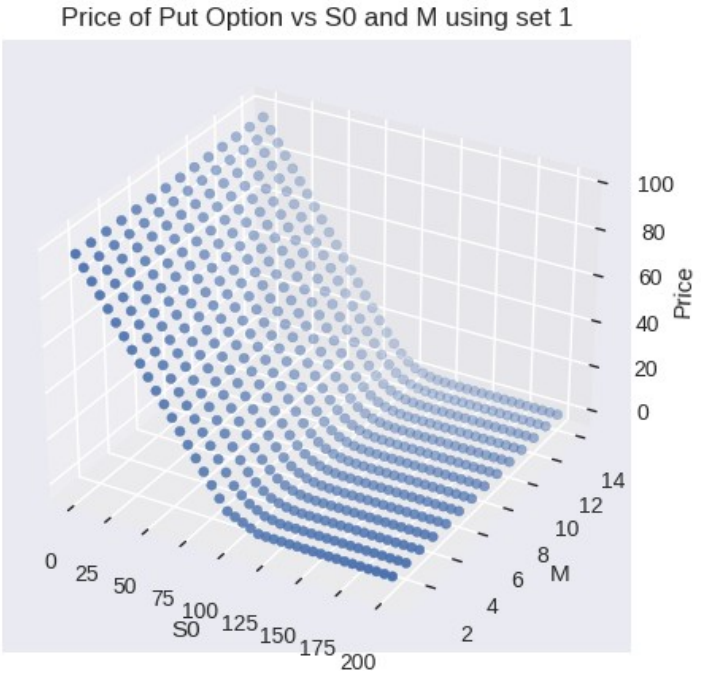




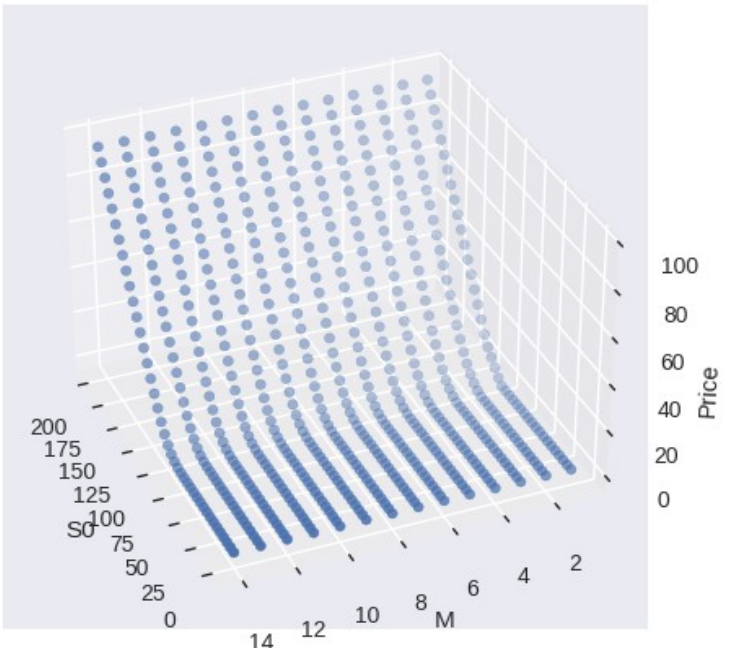


Variation with S_0 and M

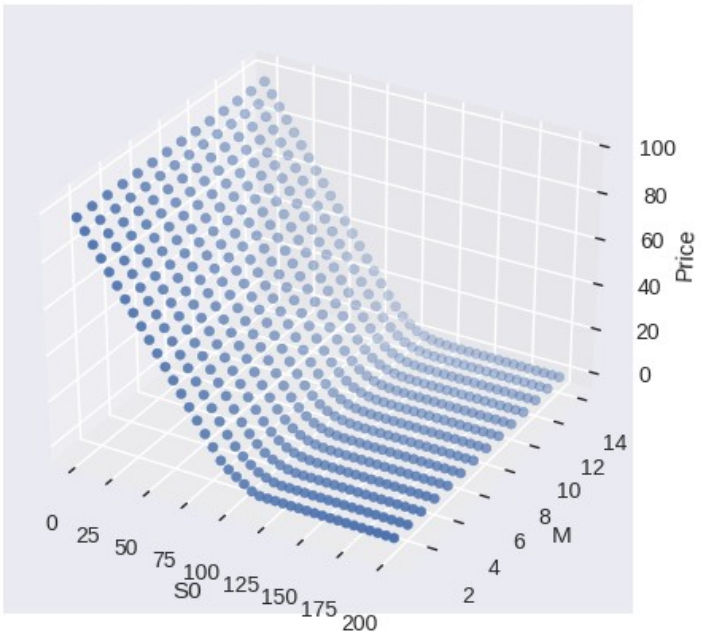




Price of Call Option vs S0 and M using set 2

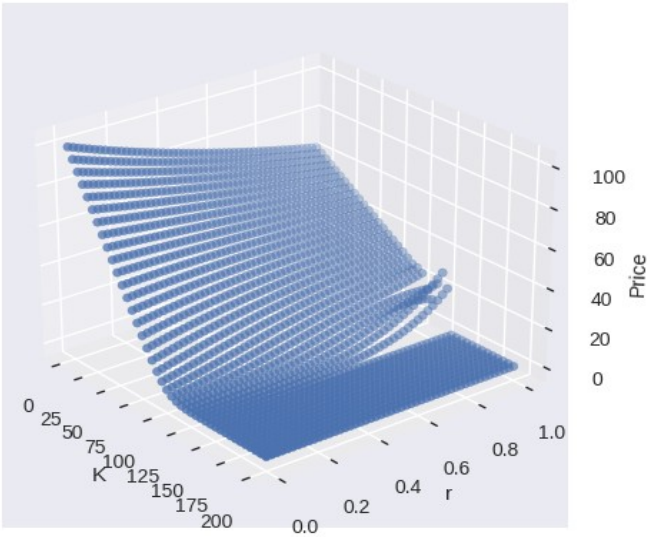


Price of Put Option vs S0 and M using set 2

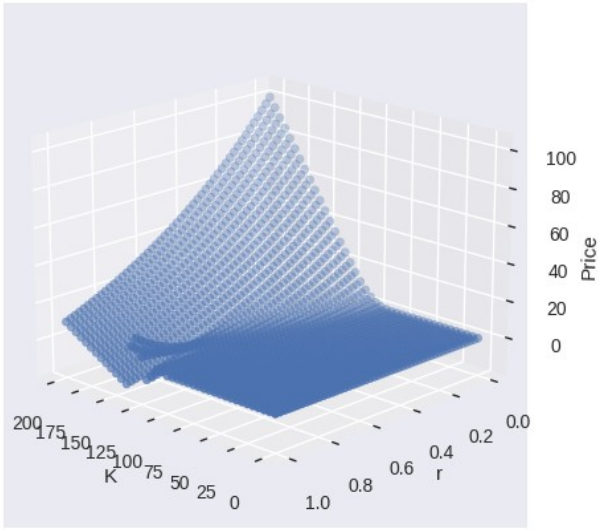


Variation with K and r

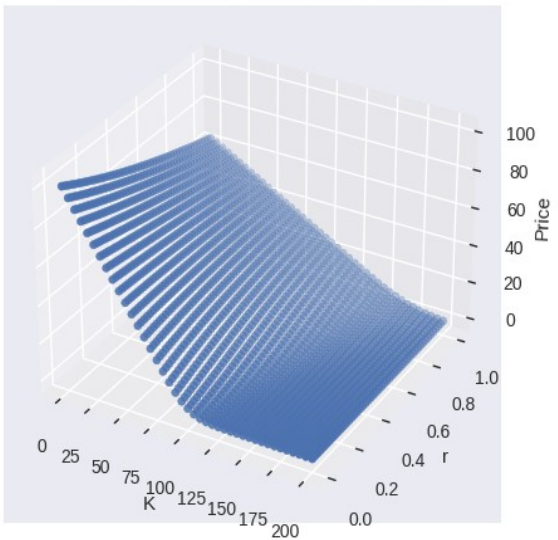
Price of Call Option vs K and r using set 1



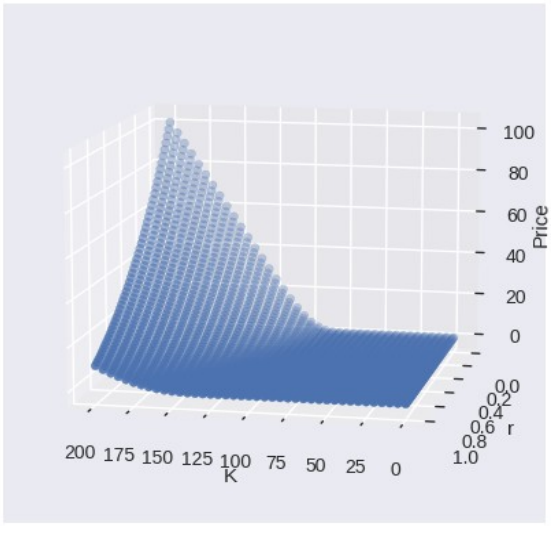
Price of Put Option vs K and r using set 1



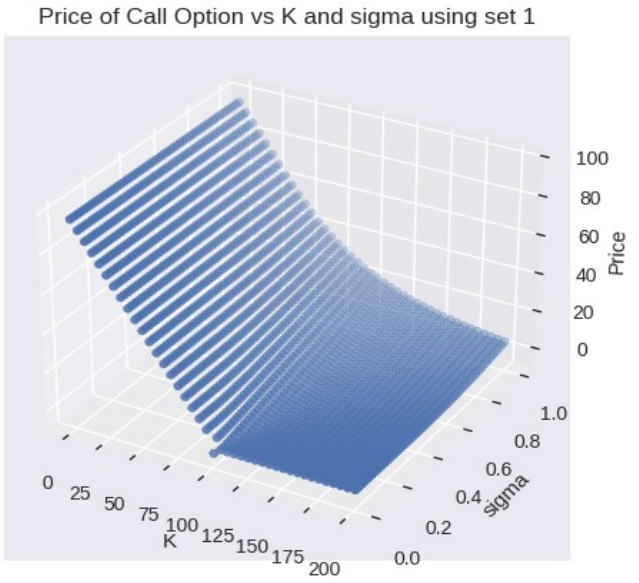
Price of Call Option vs K and r using set 2

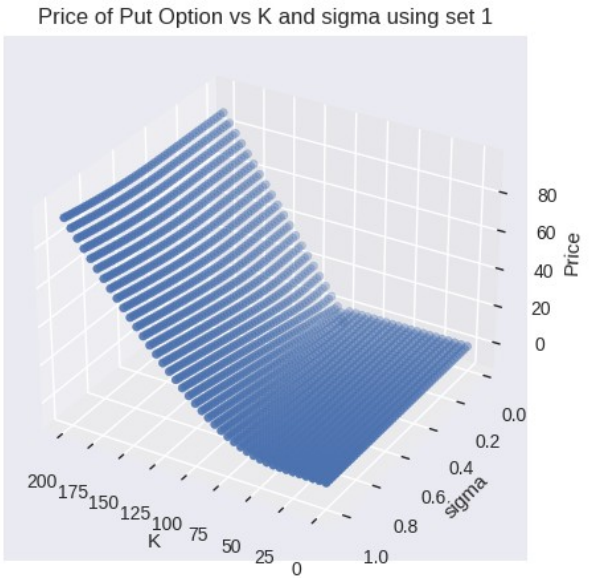


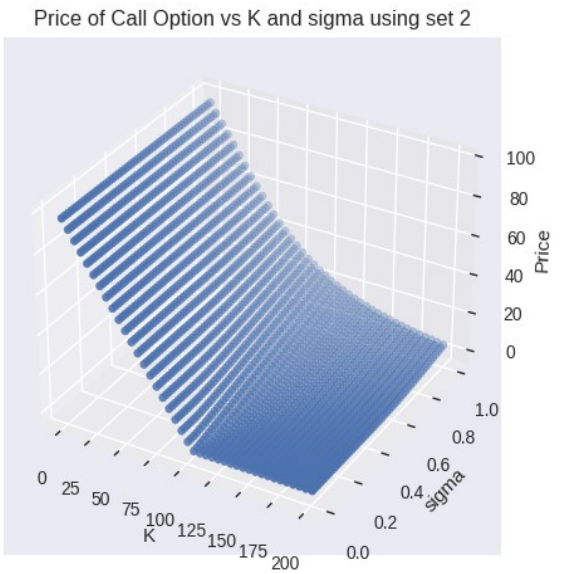
Price of Put Option vs K and r using set 2

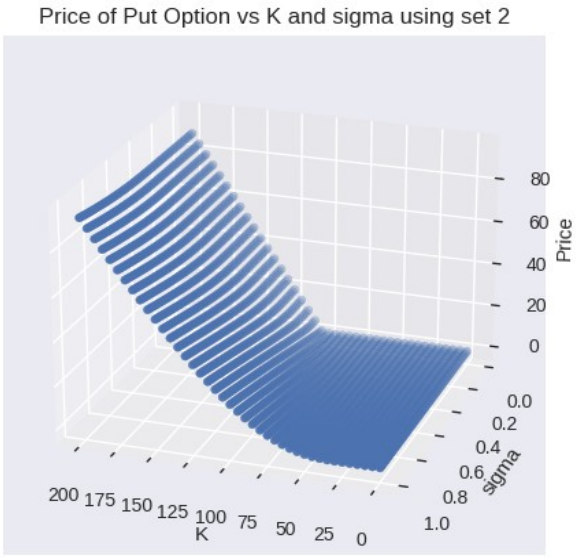


Variation with K and σ

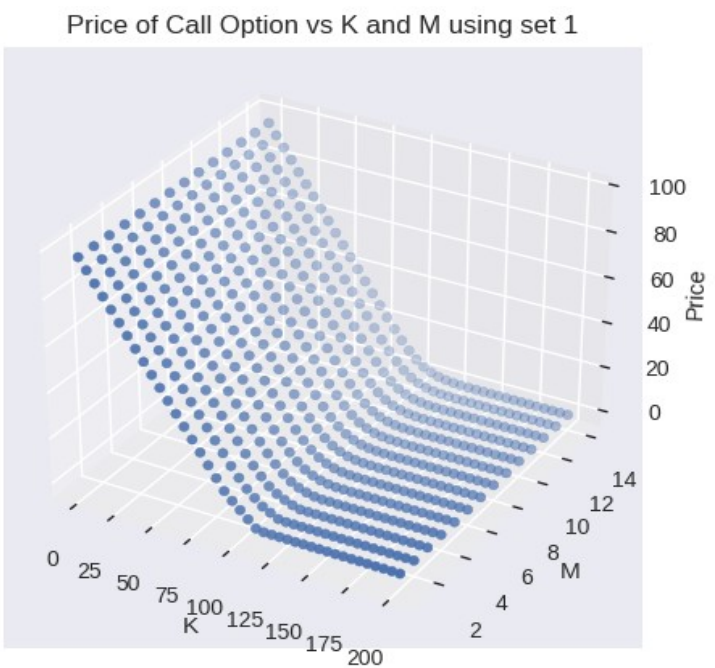


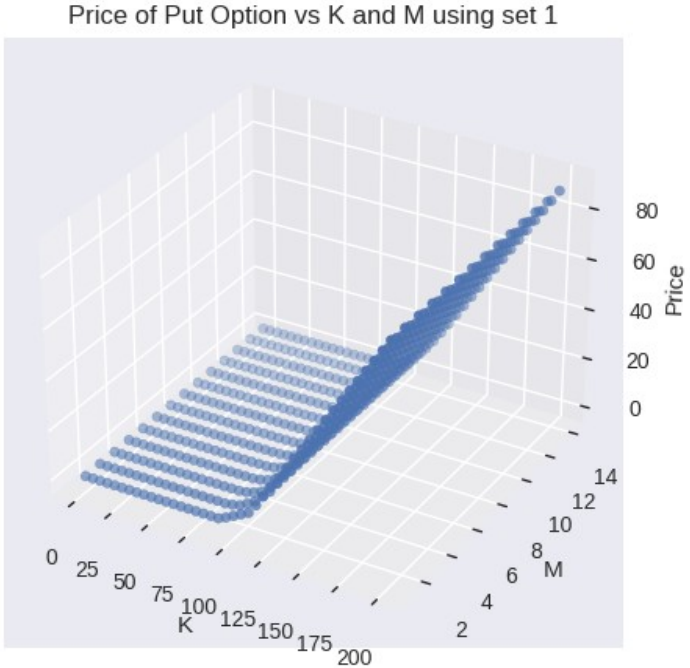




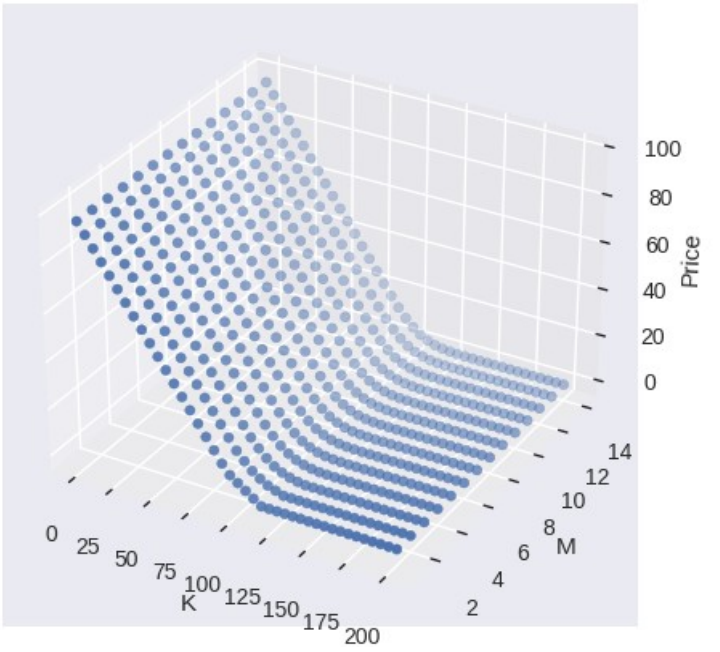


Variation with K and M

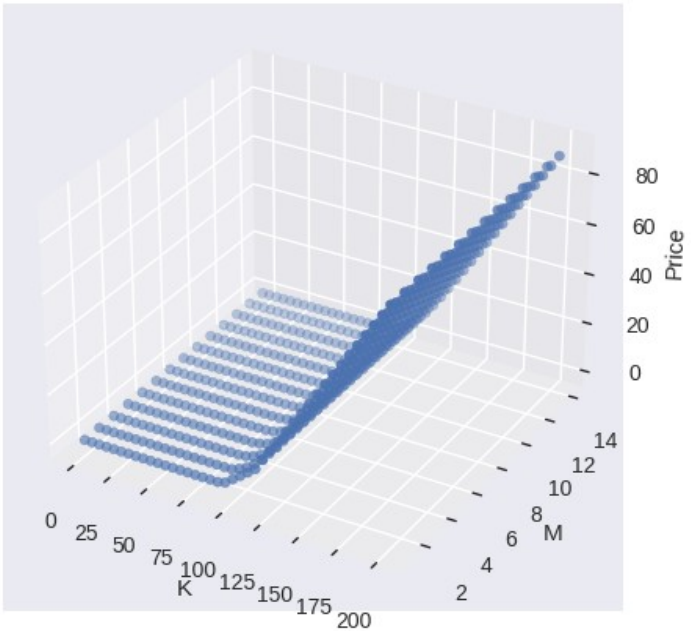




Price of Call Option vs K and M using set 2

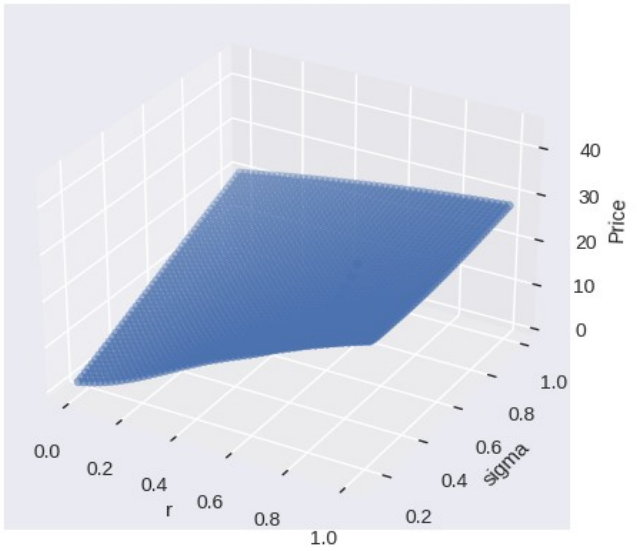


Price of Put Option vs K and M using set 2

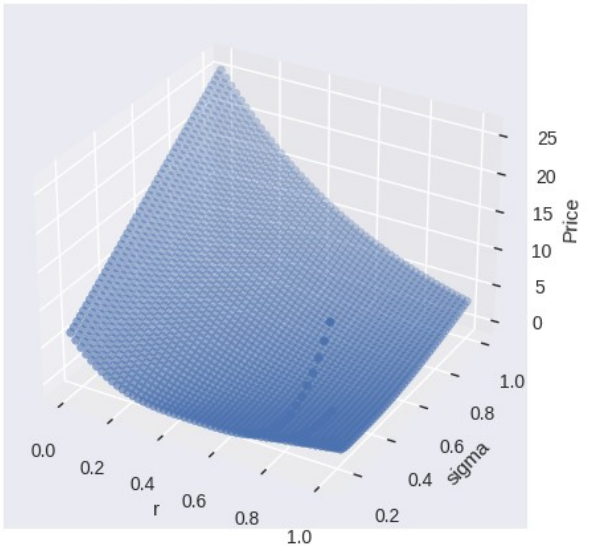


Variation with r and σ

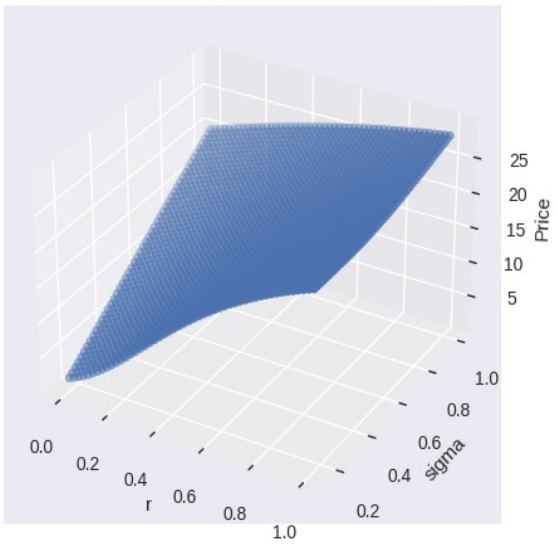
Price of Call Option vs r and sigma using set 1



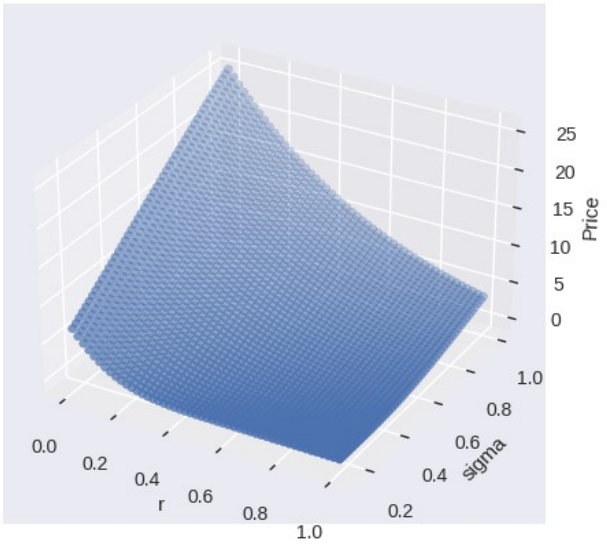
Price of Put Option vs r and sigma using set 1



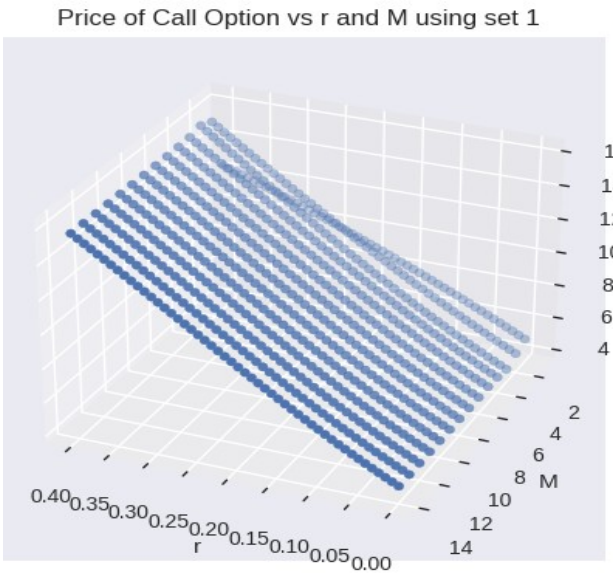
Price of Call Option vs r and sigma using set 2

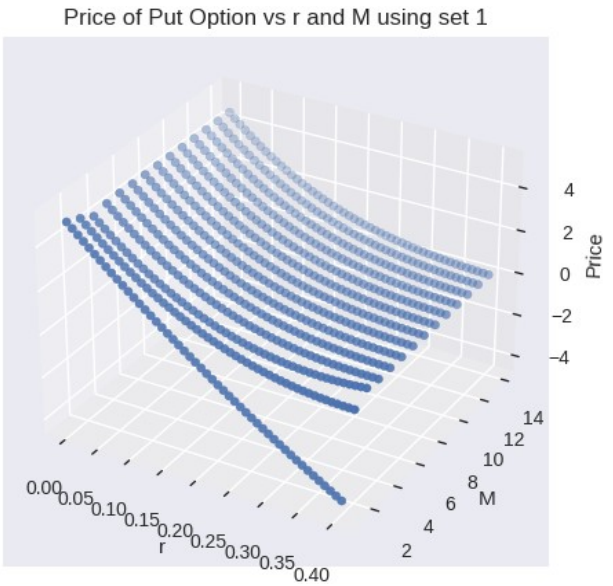


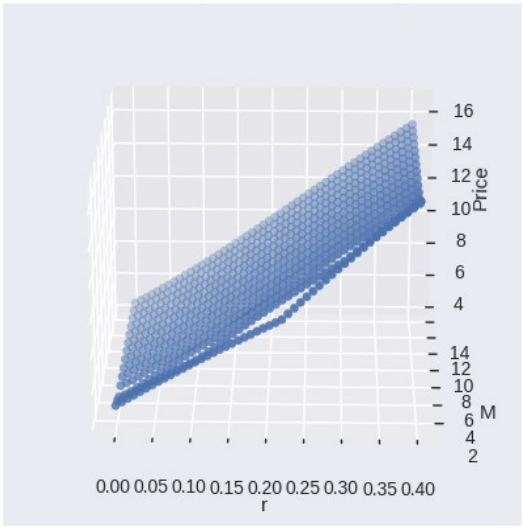
Price of Put Option vs r and sigma using set 2

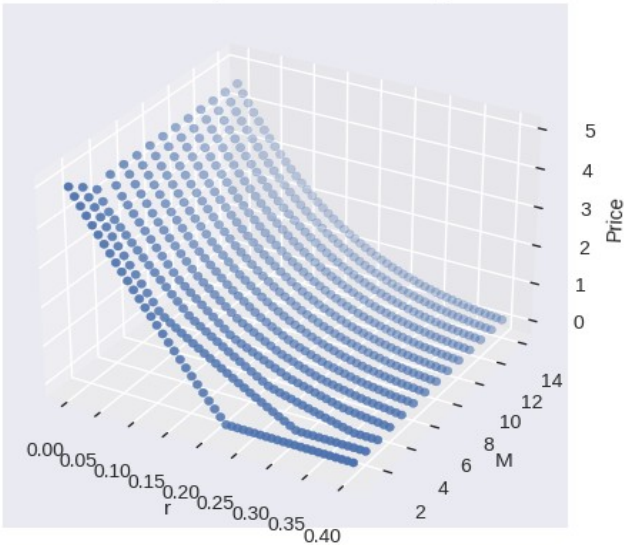


Variation with r and M

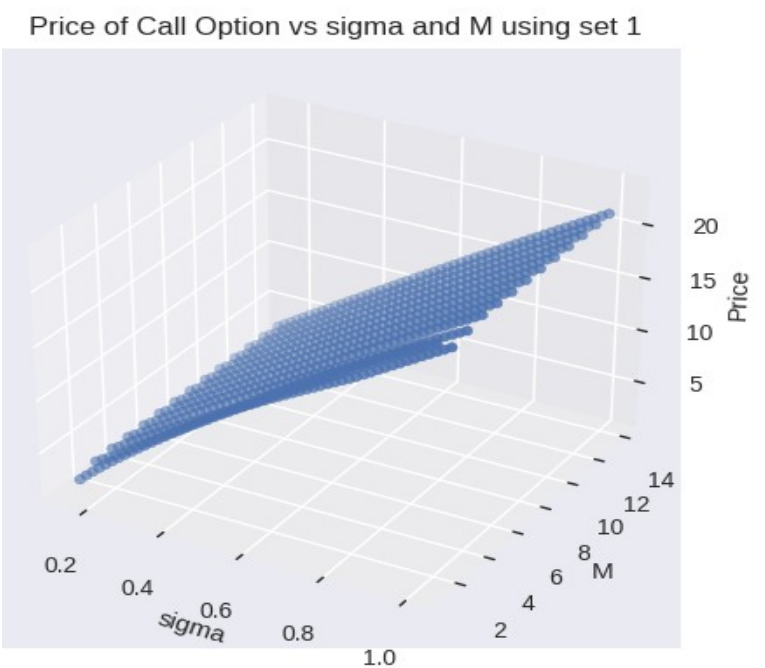


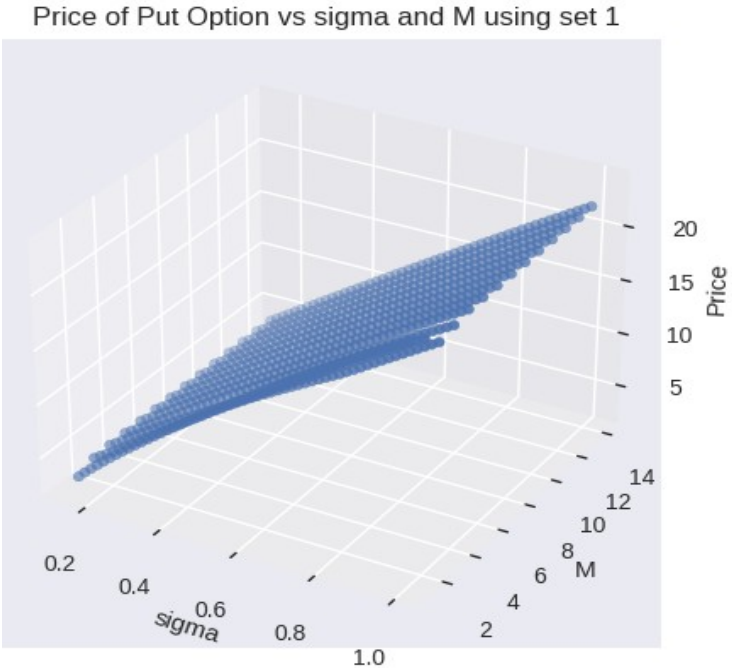




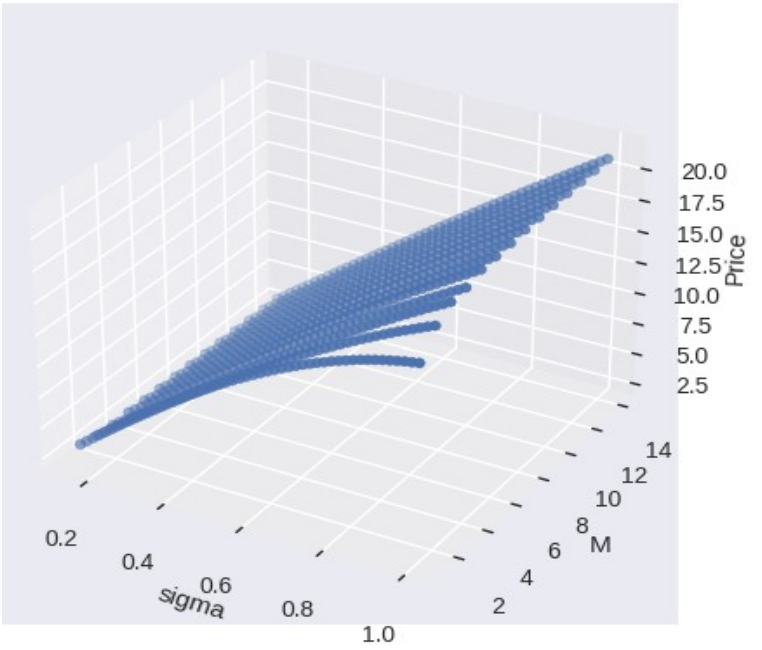


Variation with σ and M





Price of Call Option vs sigma and M using set 2



Price of Put Option vs sigma and M using set 2