Ground Truth Dynamics

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1 Introduction

The ground truth model of a tennis ball trajectory given a set of initial conditions will be derived. This model will be used as a baseline to compare and assess the performance of the estimation algorithms.

2 Constants

$$
\begin{aligned}
d &= 0.067 \text{ m (tennis ball diameter)} \\
m &= 0.058 \text{ kg (tennis ball mass)} \\
C_D &= 0.53 \text{ (coefficient of drag)} \\
\rho &= 1.293\frac{kg}{m^3} \text{ (air density)} \\
e &= 0.73 \text{ coefficient of restitution} \\
g &= -9.81\frac{m}{s^2} \text{ (gravitational acceleration)}
\end{aligned}
$$

3 Notation

X = system state
$\vec{r} =< r_x, r_y, r_z > =$ position
$\vec{v} =< v_x, v_y, v_z > =$ velocity
Define z-axis pointing in the vertical, x-axis pointing along the length of the court, and y-axis pointing along the width of the court

4 Initial Conditions

Define δ as azimuth angle, θ as elevation angle. Then we have:
$\vec{r} =< r_{x,0}, r_{y,0}, r_{z,0} >$
$\vec{v} =< |\vec{v}|\cos\theta\cos\delta, |\vec{v}|\cos\theta\sin\delta, |\vec{v}|\sin\theta >$

5 Dynamics Equations

Here we will show the flight dynamics of the ball through the air considering gravitational forces and air resistance forces. We will ignore forces due to spin (Magnus effect).

$$\frac{dX}{dt} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ -\frac{1}{2m}\rho C_D A|\vec{v}|v_x \\ -\frac{1}{2m}\rho C_D A|\vec{v}|v_y \\ -g - \frac{1}{2m}\rho C_D A|\vec{v}|v_z \end{bmatrix}$$

Note at impact with the court, the z component of velocity changes in sign and magnitude. We treat this impact as an inelastic collision:

$$v_z = -ev_z$$