



Department of Computer Science and Engineering (Data Science)

Subject: Artificial Intelligence (DJ19DSC502)

AY: 2023-24

Experiment 4

(Solution Space)

**Aim:** Find the solution of a SAT (Satisfiability) problem using Variable Neighborhood Descent.

**Theory:**

### The SAT problem

Given a Boolean formula made up of a set of propositional variables  $V = \{a, b, c, d, e, \dots\}$  each of which can be *true* or *false*, or 1 or 0, to find an assignment for the variables such that the given formula evaluates to *true* or 1.

For example,  $F = ((a \vee \sim e) \wedge (e \vee \sim c)) \supset (\sim c \vee \sim d)$  can be made *true* by the assignment  $\{a=true, c=true, d=false, e=false\}$  amongst others.

Very often SAT problems are studied in the *Conjunctive Normal Form (CNF)*. For example, the following formula has five variables (a,b,c,d,e) and six clauses.

$$(b \vee \sim c) \wedge (c \vee \sim d) \wedge (\sim b) \wedge (\sim a \vee \sim e) \wedge (e \vee \sim c) \wedge (\sim c \vee \sim d)$$

### Solution Space Search and Perturbative methods

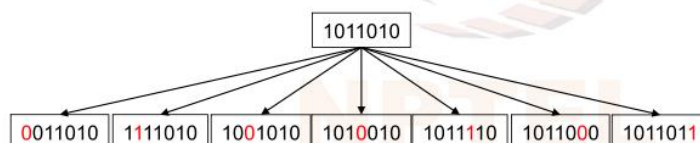
The Solution Space is the space of candidate solutions.

A local search method generates the neighbours of a candidate by applying some perturbation to the given candidate

MoveGen function = neighbourhood function

A SAT problem with N variables has  $2^N$  candidates  
 - where each candidate is a N bit string

When  $N = 7$ , a *neighbourhood function* may change **one** bit.





## Variable Neighbourhood Descent

```
VariableNeighbourhoodDescent()
1   node ← start
2   for i ← 1 to n
3       do moveGen ← MoveGen(i)
4       node ← HillClimbing(node, moveGen)
5   return node
```

The algorithm assumes that the function *moveGen* can be passed as a parameter. It assumes that there are *N* *moveGen* functions sorted according to the density of the neighbourhoods produced.

### Lab Assignment to do:

Solve the following SAT problems using VND

1.  $F = (A \vee \sim B) \wedge (B \vee \sim C) \wedge (\sim B) \wedge (\sim C \vee E) \wedge (A \vee C) \wedge (\sim C \vee \sim D)$
2.  $F = (A \vee B) \wedge (A \wedge \sim C) \wedge (B \wedge D) \wedge (A \vee \sim E)$

### CODE FOR QUESTION 1 :

```
import random

def sat_like_problem(A, B, C, D, E):
    clause1 = (A or (not B))
    clause2 = (B or (not C))
    clause3 = (not B)
    clause4 = ((not C) or E)
    clause5 = (A or C)
    clause6 = ((not C) or (not D))
    return clause1 and clause2 and clause3 and clause4 and clause5 and clause6

def hill_climbing_for_sat_like(initial_solution, max_iterations):
    current_solution = initial_solution
    for iteration in range(max_iterations):
        neighbors = movegen(current_solution)
        best_neighbor = max(neighbors, key=lambda x: sat_like_problem(*x))
        if sat_like_problem(*best_neighbor):
```



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```
        current_solution = best_neighbor
        return current_solution, iteration # Return the solution and
the iteration
    return current_solution, max_iterations # If no satisfying solution
is found

def movegen(solution):
    neighbors = []
    for i, var in enumerate(solution):
        flipped_solution = solution[:i] + (not var,) + solution[i+1:]
        neighbors.append(flipped_solution)
    return neighbors

if __name__ == "__main__":
    max_iterations = 1000
    while True:
        initial_solution = [random.choice([True, False]) for _ in
range(5)]
        best_solution, iteration =
hill_climbing_for_sat_like(tuple(initial_solution), max_iterations)
        if sat_like_problem(*best_solution):
            print("Best Solution:", best_solution)
            print("Satisfiability:", sat_like_problem(*best_solution))
            break
```

**SOLUTION**

```
Best Solution: (True, False, False, False, True)
Satisfiability: True
```

**CODE FOR QUESTION 2 :**

```
import random

def sat_like_problem(A, B, C, D, E):
    clause1 = (A or (not B))
    clause2 = (B or (not C))
    clause3 = (not B)
    clause4 = ((not C) or E)
    clause5 = (A or C)
    clause6 = ((not C) or (not D))
```



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```
    return clause1 and clause2 and clause3 and clause4 and clause5 and
clause6

def hill_climbing_for_sat_like(initial_solution, max_iterations):
    current_solution = initial_solution
    for iteration in range(max_iterations):
        neighbors = movegen(current_solution)
        best_neighbor = max(neighbors, key=lambda x: sat_like_problem(*x))
        if sat_like_problem(*best_neighbor):
            current_solution = best_neighbor
            return current_solution, iteration # Return the solution and
the iteration
    return current_solution, max_iterations # If no satisfying solution
is found

def movegen(solution):
    neighbors = []
    for i, var in enumerate(solution):
        flipped_solution = solution[:i] + (not var,) + solution[i+1:]
        neighbors.append(flipped_solution)
    return neighbors

if __name__ == "__main__":
    max_iterations = 1000
    while True:
        initial_solution = [random.choice([True, False]) for _ in
range(5)]
        best_solution, iteration =
hill_climbing_for_sat_like(tuple(initial_solution), max_iterations)
        if sat_like_problem(*best_solution):
            print("Best Solution:", best_solution)
            print("Satisfiability:", sat_like_problem(*best_solution))
            break
```

**SOLUTION**

Best Solution: (False, True, False, True, False)

Satisfiability: True

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