Problem Set 2

Write a program that solves the neoclassical growth model with technology shocks through value function iteration.

In the repository you will find some basic code that solves the deterministic model in which a representative household solves

$$\max_{\{(C_t,K_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the resource constraint $K_{t+1} + C_t = K_t^\alpha + (1-\delta) K_t$ and a given K_0.

Modify this program to account for random technology shocks. The production function is now

$$Y_t = A_t K_t^\alpha$$

where A_t is random and not known when K_t is chosen. Let's keep things simple by assuming that $\{A_t\}$ follows a Markov process with only two states, A^h and A^l.

We can calibrate the model to the US economy by picking $\delta = 0.025$ (targeting an annual depreciation rate of around 10%), and a capital share of income of $.35 = \alpha$. Now we need to pick the elements of the transition matrix for A_t denoted $\Pi = \begin{bmatrix} \pi^{hh} & 1-\pi^{hh} \\ 1-\pi^{ll} & \pi^{ll} \end{bmatrix}$ and the values for A^h and A^l. For the transition probabilities we can look at the average frequency and duration of recessions in US data as determined by the NBER. With that information we get $\pi^{hh} = 0.977$ and $\pi^{ll} = 0.926$. We would like to pick the last two parameters to satisfy two conditions: first, we want to normalize the long-run mean of A_t to be 1. Second, we would like the size of the fluctuations in output to roughly match the size of the fluctuations in US GDP, that is we would like the standard deviation of Y_t in the model to be similar to the one in the data. But it is hard to know what that means for the values of A without solving the model first! Therefore, we start with a guess of $A^h = 1.1$ (note that this implies a value of $A^l = 678$ from the invariant distribution of Π).[1] Once we have solved the model and find that the output fluctuations are too large we can still adjust those values – in other words, we calibrate A^h "inside the model".

1. State the dynamic programming problem by writing down the functional equation. Clearly state which ones are the state variable(s) and control variable(s) of the problem.

2. Calibrate the values ofr A^h and A^l such that the

3. Plot the value function over K for each state of A. Is it increasing and concave?

[1] If $\bar{\pi}_h$ and $\bar{\pi}_l = 1-\bar{\pi}_h$ are the long-run probabilities of the invariant distribution associated with Π then the requirement that the long-run mean of $A_t = 1$ implies $\pi_h A^h + \pi_l A^l = 1$ from which we can back out the value for A^l.

4. Plot the policy function (ie capital K' next period) over K for each state of A. Is it increasing in K and A? Plot savings over K for each A. Is it increasing in K and A?

5. Extra credit: Solve the problem under uncertainty using EITHER a different programming language OR by using for loops instead of vectorization withing Matlab. Compare the speed differences.