TTK4190 Guidance and Control of Vehicles

Assignment 1

Satellite Attitude Control

Fall 2015

Objective

With Assignment 1 you will learn how to implement and simulate the kinematics and kinetics of a satellite and design an attitude controller using Euler angles and unit quaternions.
The code you will produce is the most important result and has the greatest weight on the grade. Whether your code works fine or not, please include inline comments that can help us understand your intentions and figure out possible mistakes.

Deadline and Delivery Details

The answer must be handed in **by 19:00 on Thursday 10 September 2015**. This assignment accounts for 10 % of the overall course grade. Teamwork is forbidden and you must deliver individual reports and Matlab files. Simulink *cannot* be used. The written report should not be more than 6 pages. Matlab m-files should not be included in the report. The code you will have to hand in must work as is, so I should just run it once in order to obtain *all* the required plots. Make sure all plots clearly show the required data (label and tag the axes, set the grid on...). Further information regarding the required Matlab code is given in the corresponding tasks. The report together with all the required code has to be handed in via **Itslearning**. You may write the report on your PC using your favorite editor (LaTeX, Word, Pages...) or hand in a scanned version, but in the end it has to be a **PDF** document. Paper versions will not be accepted.

1 6-DOF Kinematics and Kinetics (10%)

Kinematics describes the geometrical aspects of motion. For rigid bodies, the kinematics depends on how you choose to describe rotations. The most commonly used methods are:

1. Orientation given as a vector of Euler angles
2. Orientation given as a unit quaternion
3. Orientation given as a rotation matrix

In 6 DOF, define $\mathbf{p} \in \mathbb{R}^3$ as the position and $\mathbf{Q} \in \mathbb{R}^{m \times n}$ as the orientation (attitude) of a satellite. For a rigid body with linear velocity $\mathbf{v} \in \mathbb{R}^3$ and angular velocity $\boldsymbol{\omega} \in \mathbb{R}^3$ (both represented in a body-fixed reference frame), the kinematics is

$$\dot{\mathbf{p}} = \mathbf{U}(\mathbf{Q})\mathbf{v} \tag{1}$$

$$\dot{\mathbf{Q}} = \mathbf{T}(\mathbf{Q}, \boldsymbol{\omega}) \tag{2}$$

where $\mathbf{U} : \mathbb{R}^{m \times n} \mapsto \mathbb{R}^{3 \times 3}$ is a function of $\mathbf{Q}$, and $\mathbf{T} : \mathbb{R}^{m \times n} \times \mathbb{R}^3 \mapsto \mathbb{R}^{m \times n}$ is a function of $\mathbf{Q}$ and $\boldsymbol{\omega}$. $\mathbf{U}$ and $\mathbf{T}$ also depend on which reference frames you use.

Task 1.1 *Write down the expressions for Q, $\mathbf{U}(\mathbf{Q})$ and $\mathbf{T}(\mathbf{Q}, \boldsymbol{\omega})$ in (1) and (2) for each of the three above-mentioned orientation representations. The inertial reference frame is the north-east-down (NED) reference frame (considered inertial), and $\mathbf{v}$ and $\boldsymbol{\omega}$ are given in the body-fixed forward-right-down reference frame. For the Euler angles, use the standard roll-pitch-yaw parametrization.*

Task 1.2 *Show that the kinetic equations of a satellite in space can be written as:*

$$\boldsymbol{I}_{CG}\dot{\boldsymbol{\omega}} - (\boldsymbol{I}_{CG}\boldsymbol{\omega}) \times \boldsymbol{\omega} = \boldsymbol{\tau} \tag{3}$$

where $\boldsymbol{I}_{CG}$ is the inertia matrix in CG and $\boldsymbol{\tau}$ is the control inputs. It is assumed that the satellite has uniform mass distribution. Under what assumptions is the satellite model (3) *valid?*

Task 1.3 *What are the main advantages and disadvantages of describing rotations using Euler angles, unit quaternions and rotation matrices?*

2 Attitude Control using Euler Angles (45%)

Task 2.1 *Assume that the inertia matrix is $\boldsymbol{I}_{CG} = mr^2\boldsymbol{I}_3$, where $m = 80$ kg, $r = 1.2$ m (radius of gyration), and $\boldsymbol{I}_3$ is the identity matrix of dimension 3. Design a <u>nonlinear</u> PD controller for set-point regulation using the Euler angle representation. Explain how you choose the controller gains, preferably using a design method. Simulate your controller in Matlab by choosing your own initial conditions and desired values and plot the results. Make sure you specify what your initial and desired values are. Does the controller work for any possible desired values? If not, show simulation results that demonstrate this limitation.*

Code hand-in: *Hand in an m-file named* `run_task_2_1.m` *that will initialize the simulation, run the simulation, and generate the necessary plots. Include any other files necessary to execute* `run_task_2_1.m`. *Place all Matlab files in a folder entitled "task_2.1".*

Task 2.2 *What kind of disturbances will a satellite in space be exposed to? Include mathematical models of the disturbances in* (3) *and design a* <u>*nonlinear*</u> *PID controller for attitude stabilization. Explain how you choose the controller gains, preferably using a design method. If necessary, low-pass filter the measurements before you feed the signals to the attitude controller. Explain how you choose the filter time constants. Simulate your controller in Matlab by choosing your own initial conditions and desired values. Does the controller counteract the disturbances in a satisfactory way?*

Code hand-in: *Hand in an m-file named* `run_task_2_2.m` *that will initialize the simulation, run the simulation, and generate the necessary plots. Include any other files necessary to execute* `run_task_2_2.m`. *Place all Matlab files in a folder entitled "task_2.2".*

3 Attitude Control using Unit Quaternions (45%)

Task 3.1 *Consider the* <u>*nonlinear*</u> *PD controller for set-point regulation:*

$$\boldsymbol{\tau} = -\mathbf{K}_d\boldsymbol{\omega} - \mathbf{K}_p\mathbf{T}_q^\top(\mathbf{q})\tilde{\mathbf{q}} \tag{4}$$

where $\mathbf{K}_p$ *and* $\mathbf{K}_d$ *are the controller gains and* $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$ *is the quaternion tracking error. The matrix* $\mathbf{T}_q(\mathbf{q})$ *should be chosen such that (see Task 1.1):*

$$\mathbf{T}(\mathbf{Q}, \boldsymbol{\omega}) := \mathbf{T}_q(\mathbf{q})\boldsymbol{\omega} \tag{5}$$

Compute the desired unit quaternion $\mathbf{q}_d$ *corresponding to the Euler angles* $\boldsymbol{\Theta}_d = [10^o, 20^o, -10^o]$. *Simulate your controller in Matlab by using zero initial conditions. Simulate the Euler angle feedback controller of Task 2.1 using the same set-points and initial conditions and plot the Euler angle tracking errors of the two controllers in the same plot. Compare the results and comment them.*

Code hand-in: *Hand in an m-file named* `run_task_3_1.m` *that will initialize the simulation, run the simulation, and generate the necessary plots. Include any other files necessary to execute* `run_task_3_1.m`. *Place all Matlab files in a folder entitled "task_3.1".*