Neural Network from Scratch

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Reference: Online blogs, towardsdatascience.com

Consider Y, X1, X2, X2

A1 i.e. value to input at the first neuron on in the network is

f(X1,X2,X3)=W11*X1* + *W12*X2 + W13*X3 + b11

This is further activated by

A1 = g(f(X1,X2,X3))=g(W11*X1* + *W12*X2 + W13*X3 + b11)

Similarly

A2 = g(f(X1,X2,X3))=g(W11*X1* + *W12*X2 + W13*X3 + b12)

A3 = g(f(X1,X2,X3))=g(W11*X1* + *W12*X2 + W13*X3 + b13)

g is an activation function. Here are a few examples of activation functions

If there are two hidden layers, activated valued on the hidden layers are given by A and B.

A = W1 X + b1

B = W2 A + b2

Y_pred = W3 B + b3

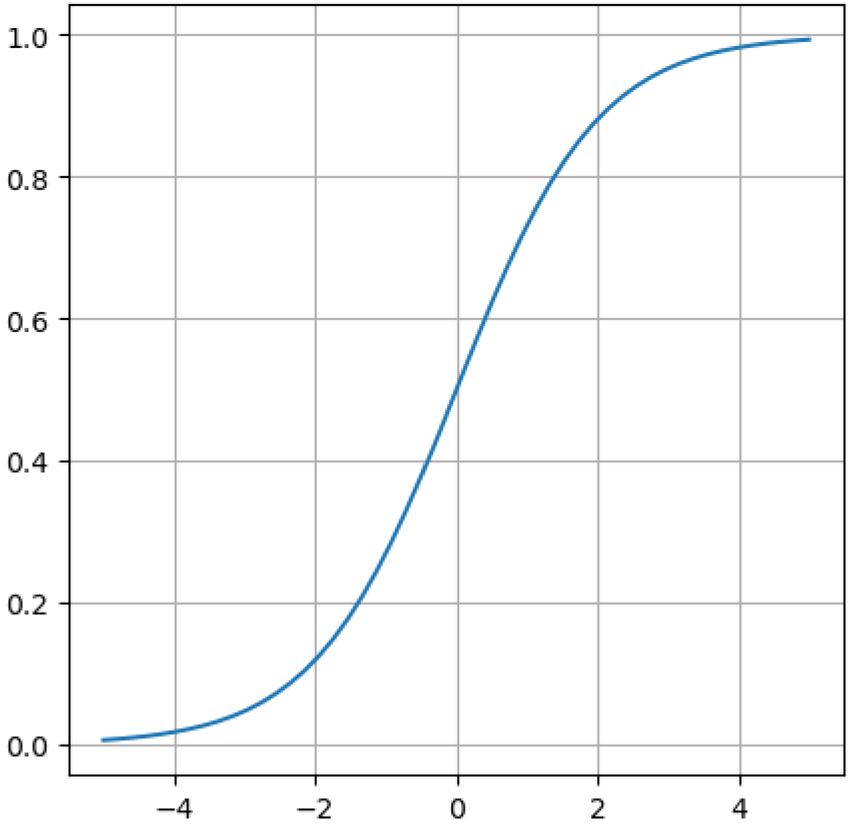
W1, W2, W3 will be matrices of appropriate sizes.

Sigmoid function in Python

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
```

In [2]:
```
x = np.linspace(-5, 5, 50)
z = 1/(1 + np.exp(-x))

plt.subplots(figsize=(5, 5))
plt.plot(x, z)
plt.grid()
plt.show()
```

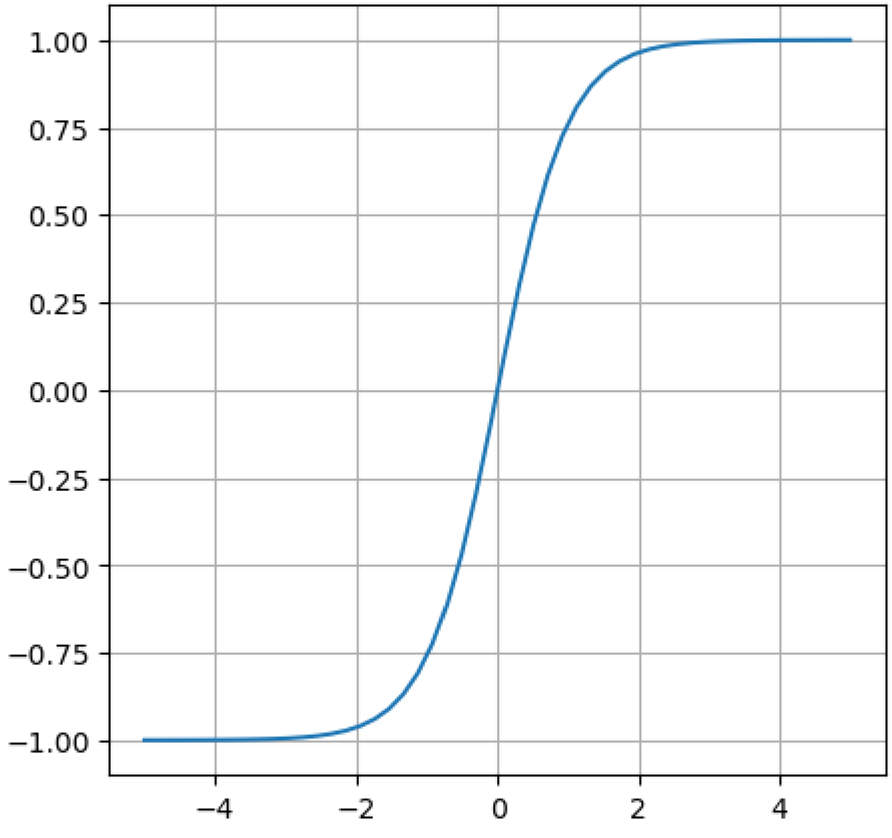


tanh function in Python

In [3]:
```
# tanh function in Python
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-5, 5, 50)
z = np.tanh(x)

plt.subplots(figsize=(5, 5))
plt.plot(x, z)
plt.grid()
plt.show()
```



Softmax function in Python

Softmax is a generalization sigmoid i.e. Softmax is used in multiple dimensions. And is generally used as an activation function in the output layer.

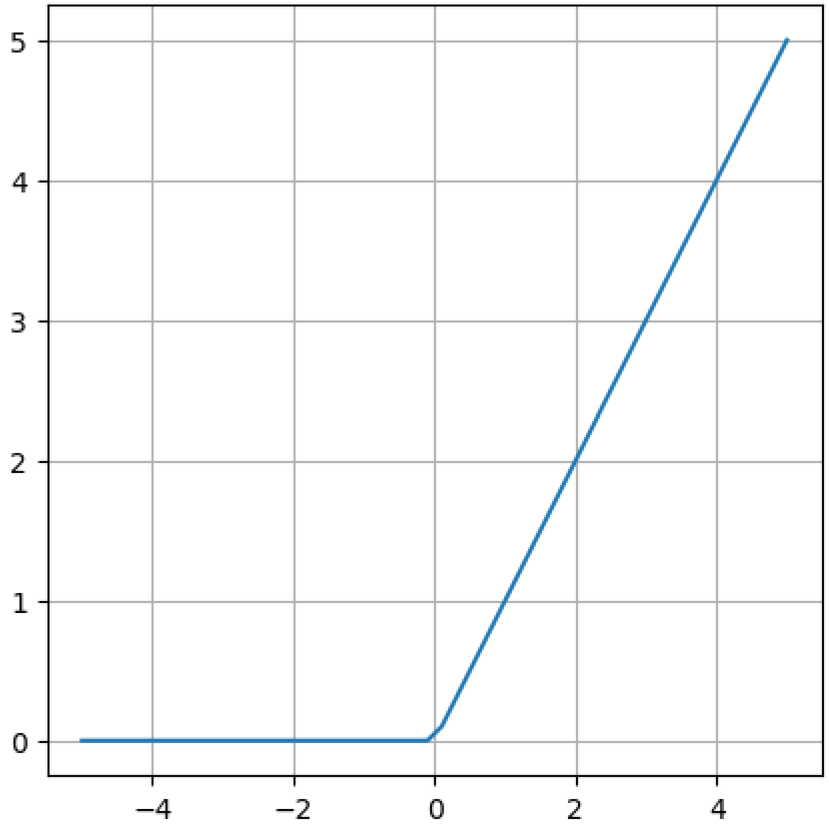
Rectified Linear Unit (ReLU)

In [4]:

```
# ReLU in Python
import matplotlib.pyplot as plt
import numpy as np


x = np.linspace(-5, 5, 50)
z = [max(0, i) for i in x]

plt.subplots(figsize=(5, 5))
plt.plot(x, z)
plt.grid()
plt.show()
```



In [5]:
```
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
```

Define ReLU function

In [6]:
```
def relu(z): # takes a numpy array as input and returns activated array
    a = np.maximum(0,z)
    return a
```

Initialize parameters

In [7]:
```
def initialize_params(layer_sizes): #takes a list of the layer sizes as input and returns initialized parameters
    params = {}
```

```
    for i in range(1, len(layer_sizes)):
        params['W' + str(i)] = np.random.randn(layer_sizes[i], layer_sizes[i-1])*0.01
        # rand(d0,d1) will return an array of size d0 x d1.
        params['B' + str(i)] = np.random.randn(layer_sizes[i],1)*0.01
    return params
```

Loss or cost is defined as $J(W,B) = \frac{1}{2m} * \Sigma(Y_{pred} - Y_{true})^2$

W and B represent weight and bias matrices. m is the number of observastion i.e. data points.

In [8]:

```
def compute_cost(values, Y_train): #takes true values and dictionary having activations of
# all layers as input and returns cost
    layers = len(values)//2
    Y_pred = values['A' + str(layers)]
    cost = 1/(2*len(Y_train)) * np.sum(np.square(Y_pred - Y_train))
    return cost
```

Define forward propagation

In [9]:

```
def forward_propagation(X_train, params):
    #takes input training features and parameters as input and returns a dictionary
# containining the numpy arrays of activations of all layers
    layers = len(params)//2
    values = {}
    for i in range(1, layers+1):
        if i==1: # transformation from input layer
            values['Z' + str(i)] = np.dot(params['W' + str(i)], X_train) + params['B' + str(i)]
            values['A' + str(i)] = relu(values['Z' + str(i)])
        else: # transformation from non-input layer
            values['Z' + str(i)] = np.dot(params['W' + str(i)], values['A' + str(i-1)]) + params['B' + str(i)]
            if i==layers: # Do not use activation function in the output layer
                values['A' + str(i)] = values['Z' + str(i)]
            else: # use activation function for non-output layers
                values['A' + str(i)] = relu(values['Z' + str(i)])
    return values
```

Define function for backward propagation

In [10]:
```
def backward_propagation(params, values, X_train, Y_train):
    #takes parameters, activations, training set as input and returns gradients wrt parameters
    layers = len(params)//2
    m = len(Y_train)
    grads = {}
    for i in range(layers,0,-1):
        if i==layers:
            dA = 1/m * (values['A' + str(i)] - Y_train)
            dZ = dA
        else:
            dA = np.dot(params['W' + str(i+1)].T, dZ)
            dZ = np.multiply(dA, np.where(values['A' + str(i)]>=0, 1, 0))
        if i==1:
            grads['W' + str(i)] = 1/m * np.dot(dZ, X_train.T)
            grads['B' + str(i)] = 1/m * np.sum(dZ, axis=1, keepdims=True)
        else:
            grads['W' + str(i)] = 1/m * np.dot(dZ,values['A' + str(i-1)].T)
            grads['B' + str(i)] = 1/m * np.sum(dZ, axis=1, keepdims=True)
    return grads
```

Function for updating paramters (this is done after backpropagation)

In [11]:
```
def update_params(params, grads, learning_rate):
    #takes parameters, gradients and learning rate as input and returns updated parameters
    layers = len(params)//2
    params_updated = {}
    for i in range(1,layers+1):
        params_updated['W' + str(i)] = params['W' + str(i)] - learning_rate * grads['W' + str(i)]
        params_updated['B' + str(i)] = params['B' + str(i)] - learning_rate * grads['B' + str(i)]
    return params_updated
```

Define function for accuracy

In [25]:
```
def compute_accuracy(X_train, X_test, Y_train, Y_test, params, layer_sizes): #compute accuracy on test and training data given le
    values_train = forward_propagation(X_train.T, params)
    values_test = forward_propagation(X_test.T, params)
    train_acc = np.sqrt(mean_squared_error(Y_train, values_train['A' + str(len(layer_sizes)-1)].T))
    test_acc = np.sqrt(mean_squared_error(Y_test, values_test['A' + str(len(layer_sizes)-1)].T))
    return train_acc, test_acc
```

Define function for predicting

In [13]:
```
def predict(X, params):  #predict on new array X given learnt parameters
    values = forward_propagation(X.T, params)
    predictions = values['A' + str(len(values)//2)].T
    return predictions
```

Assemble the model i.e. create the model i.e. setup the model

In [14]:
```
#MSE_Train
def model(X_train, Y_train, layer_sizes, num_iters, learning_rate): #trains the model
    params = initialize_params(layer_sizes)
    for i in range(num_iters):
        values = forward_propagation(X_train.T, params)
        cost = compute_cost(values, Y_train.T)
        print(f'cost of {i}th iteration: {cost}')
        grads = backward_propagation(params, values,X_train.T, Y_train.T)
        params = update_params(params, grads, learning_rate)
        #print('Cost at iteration ' + str(i+1) + ' = ' + str(cost) + '\n')
    return params
```

Run the model

In [33]:
```
# import pandas for importing csv files
import pandas as pd
from sklearn.preprocessing import normalize
df = pd.read_csv('dummy_data.csv', header=None)
df.head()
```

Out[33]:

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

In [34]:

```
num_cols = df.shape[1]
df.columns = ['Column' + str(i) for i in range(num_cols)]
```

In [35]:

```
df
```

Out[35]:

	Column0	Column1	Column2	Column3	Column4	Column5	Column6	Column7	Column8	Column9	Column10	Column11	Column12	Column1
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	.
501	0.06263	0.0	11.93	0.0	0.573	6.593	69.1	2.4786	1.0	273.0	21.0	391.99	9.67	22.
502	0.04527	0.0	11.93	0.0	0.573	6.120	76.7	2.2875	1.0	273.0	21.0	396.90	9.08	20.
503	0.06076	0.0	11.93	0.0	0.573	6.976	91.0	2.1675	1.0	273.0	21.0	396.90	5.64	23.
504	0.10959	0.0	11.93	0.0	0.573	6.794	89.3	2.3889	1.0	273.0	21.0	393.45	6.48	22.
505	0.04741	0.0	11.93	0.0	0.573	6.030	80.8	2.5050	1.0	273.0	21.0	396.90	7.88	11.

506 rows × 14 columns

In [37]:
```
# Separate features (X) and target variable (y)
X, Y = df.iloc[:, :-1], df.iloc[:, -1]

# Split the data into training and testing sets
X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size=0.2, random_state=42)
```

In [38]:
```
X_train
```

Out[38]:

	Column0	Column1	Column2	Column3	Column4	Column5	Column6	Column7	Column8	Column9	Column10	Column11	Column12
477	15.02340	0.0	18.10	0.0	0.6140	5.304	97.3	2.1007	24.0	666.0	20.2	349.48	24.91
15	0.62739	0.0	8.14	0.0	0.5380	5.834	56.5	4.4986	4.0	307.0	21.0	395.62	8.47
332	0.03466	35.0	6.06	0.0	0.4379	6.031	23.3	6.6407	1.0	304.0	16.9	362.25	7.83
423	7.05042	0.0	18.10	0.0	0.6140	6.103	85.1	2.0218	24.0	666.0	20.2	2.52	23.29
19	0.72580	0.0	8.14	0.0	0.5380	5.727	69.5	3.7965	4.0	307.0	21.0	390.95	11.28
...	...	...	...	...	...	...	...	...	...	...	...	...	...
106	0.17120	0.0	8.56	0.0	0.5200	5.836	91.9	2.2110	5.0	384.0	20.9	395.67	18.66
270	0.29916	20.0	6.96	0.0	0.4640	5.856	42.1	4.4290	3.0	223.0	18.6	388.65	13.00
348	0.01501	80.0	2.01	0.0	0.4350	6.635	29.7	8.3440	4.0	280.0	17.0	390.94	5.99
435	11.16040	0.0	18.10	0.0	0.7400	6.629	94.6	2.1247	24.0	666.0	20.2	109.85	23.27
102	0.22876	0.0	8.56	0.0	0.5200	6.405	85.4	2.7147	5.0	384.0	20.9	70.80	10.63

404 rows × 13 columns

In [39]:
```
# Convert to NumPy arrays
X_train = X_train.values
X_test = X_test.values
Y_train = Y_train.values
Y_test = Y_test.values
```

In [40]:
```
model1 = model(X_train, Y_train, [13, 5, 5, 1], 1000, 0.01)
```

```
cost of 0th iteration: 303.40762067089514
cost of 1th iteration: 303.3940351515662
cost of 2th iteration: 303.38047833158305
cost of 3th iteration: 303.3669474046098
cost of 4th iteration: 303.35344228011786
cost of 5th iteration: 303.33996128215
cost of 6th iteration: 303.3265190322308
cost of 7th iteration: 303.31311603674607
cost of 8th iteration: 303.2997401363282
cost of 9th iteration: 303.2863880819605
cost of 10th iteration: 303.273048839966
cost of 11th iteration: 303.2597186680538
cost of 12th iteration: 303.24639554587554
cost of 13th iteration: 303.23308082981856
cost of 14th iteration: 303.2197749515043
cost of 15th iteration: 303.20647995161943
cost of 16th iteration: 303.19319076250036
cost of 17th iteration: 303.17990295277485
cost of 18th iteration: 303.166615164679
cost of 19th iteration: 303.1533290789401
cost of 20th iteration: 303.1400466904957
cost of 21th iteration: 303.1267683816923
cost of 22th iteration: 303.11349588965936
cost of 23th iteration: 303.10022863352776
cost of 24th iteration: 303.0869661384078
cost of 25th iteration: 303.07370968250274
cost of 26th iteration: 303.0604598668872
cost of 27th iteration: 303.04721425999463
cost of 28th iteration: 303.03397368327046
cost of 29th iteration: 303.02074091366933
cost of 30th iteration: 303.0075174453762
cost of 31th iteration: 302.99430981464167
cost of 32th iteration: 302.9811250914443
cost of 33th iteration: 302.96796437778613
cost of 34th iteration: 302.9548179700092
cost of 35th iteration: 302.9416879978968
cost of 36th iteration: 302.9285802916436
cost of 37th iteration: 302.9154895838772
cost of 38th iteration: 302.9024210015948
cost of 39th iteration: 302.88937762212146
cost of 40th iteration: 302.8763550360319
cost of 41th iteration: 302.86334558115794
cost of 42th iteration: 302.8503468793319
cost of 43th iteration: 302.83736058255266
```

```
cost of 44th iteration: 302.82439393802804
cost of 45th iteration: 302.81143914875423
cost of 46th iteration: 302.7984982896917
cost of 47th iteration: 302.7855688057226
cost of 48th iteration: 302.77264820768545
cost of 49th iteration: 302.75974003574055
cost of 50th iteration: 302.746842948311
cost of 51th iteration: 302.73395723490245
cost of 52th iteration: 302.7210826873193
cost of 53th iteration: 302.70821680215244
cost of 54th iteration: 302.69535557052683
cost of 55th iteration: 302.68250037271486
cost of 56th iteration: 302.6696496048557
cost of 57th iteration: 302.6568009800967
cost of 58th iteration: 302.6439557195531
cost of 59th iteration: 302.63111371911094
cost of 60th iteration: 302.6182773518296
cost of 61th iteration: 302.6054449602023
cost of 62th iteration: 302.5926148667318
cost of 63th iteration: 302.57978578476127
cost of 64th iteration: 302.56695733962925
cost of 65th iteration: 302.5541295313093
cost of 66th iteration: 302.54130232641705
cost of 67th iteration: 302.52847573133187
cost of 68th iteration: 302.51564977258187
cost of 69th iteration: 302.5028244501386
cost of 70th iteration: 302.4899997565896
cost of 71th iteration: 302.4771756678831
cost of 72th iteration: 302.46435214816114
cost of 73th iteration: 302.45152924104207
cost of 74th iteration: 302.4387069264273
cost of 75th iteration: 302.4258852036778
cost of 76th iteration: 302.4130641156592
cost of 77th iteration: 302.4002436623401
cost of 78th iteration: 302.3874238436891
cost of 79th iteration: 302.3746046596746
cost of 80th iteration: 302.36178611026537
cost of 81th iteration: 302.34896819543
cost of 82th iteration: 302.33615091513695
cost of 83th iteration: 302.32333426935486
cost of 84th iteration: 302.3105182580524
cost of 85th iteration: 302.297702881198
cost of 86th iteration: 302.28488813876044
cost of 87th iteration: 302.2720740307082
```

```
cost of 88th iteration: 302.25926055700984
cost of 89th iteration: 302.2464477176341
cost of 90th iteration: 302.2336355125494
cost of 91th iteration: 302.2208239417245
cost of 92th iteration: 302.20801300512784
cost of 93th iteration: 302.19520270272824
cost of 94th iteration: 302.18239303449405
cost of 95th iteration: 302.169584000394
cost of 96th iteration: 302.1567756003968
cost of 97th iteration: 302.1439678344709
cost of 98th iteration: 302.131160702585
cost of 99th iteration: 302.1183542047077
cost of 100th iteration: 302.10554834080756
cost of 101th iteration: 302.09274311085323
cost of 102th iteration: 302.07993851481336
cost of 103th iteration: 302.0671345526565
cost of 104th iteration: 302.0543312243513
cost of 105th iteration: 302.04152852986647
cost of 106th iteration: 302.0287264691705
cost of 107th iteration: 302.0159250422321
cost of 108th iteration: 302.00312424901983
cost of 109th iteration: 301.9903240895023
cost of 110th iteration: 301.9775245636483
cost of 111th iteration: 301.96472567142627
cost of 112th iteration: 301.95192741280493
cost of 113th iteration: 301.9391297877529
cost of 114th iteration: 301.92633279623885
cost of 115th iteration: 301.9135364382314
cost of 116th iteration: 301.9007407136992
cost of 117th iteration: 301.8879456226108
cost of 118th iteration: 301.875151164935
cost of 119th iteration: 301.86235734064024
cost of 120th iteration: 301.8495641496954
cost of 121th iteration: 301.836771592069
cost of 122th iteration: 301.82397966772965
cost of 123th iteration: 301.81118837664604
cost of 124th iteration: 301.79839771878676
cost of 125th iteration: 301.78560769412064
cost of 126th iteration: 301.7728183026162
cost of 127th iteration: 301.76002954424206
cost of 128th iteration: 301.74724141896695
cost of 129th iteration: 301.7344539267595
cost of 130th iteration: 301.72166706758844
cost of 131th iteration: 301.70888084142234
```

```
cost of 132th iteration: 301.69609524822994
cost of 133th iteration: 301.68331028797985
cost of 134th iteration: 301.6705259606407
cost of 135th iteration: 301.6577422661813
cost of 136th iteration: 301.64495920457017
cost of 137th iteration: 301.63217677577603
cost of 138th iteration: 301.61939497976755
cost of 139th iteration: 301.60661381651346
cost of 140th iteration: 301.5938332859824
cost of 141th iteration: 301.58105338814295
cost of 142th iteration: 301.568274122964
cost of 143th iteration: 301.55549549041405
cost of 144th iteration: 301.5427174904618
cost of 145th iteration: 301.5299401230761
cost of 146th iteration: 301.5171633882254
cost of 147th iteration: 301.5043872858785
cost of 148th iteration: 301.4916118160042
cost of 149th iteration: 301.47883697857094
cost of 150th iteration: 301.46606277354766
cost of 151th iteration: 301.4532892009029
cost of 152th iteration: 301.44051626060536
cost of 153th iteration: 301.42774395262376
cost of 154th iteration: 301.4149722769269
cost of 155th iteration: 301.40220123348325
cost of 156th iteration: 301.38943082226183
cost of 157th iteration: 301.376661043231
cost of 158th iteration: 301.3638918963597
cost of 159th iteration: 301.35112338161656
cost of 160th iteration: 301.33835549897026
cost of 161th iteration: 301.3255882483895
cost of 162th iteration: 301.31282162984314
cost of 163th iteration: 301.3000556432997
cost of 164th iteration: 301.287290288728
cost of 165th iteration: 301.2745255660967
cost of 166th iteration: 301.26176147537456
cost of 167th iteration: 301.24899801653027
cost of 168th iteration: 301.2362351895325
cost of 169th iteration: 301.2234729943501
cost of 170th iteration: 301.2107114309517
cost of 171th iteration: 301.19795049930605
cost of 172th iteration: 301.1851901993819
cost of 173th iteration: 301.17243053114794
cost of 174th iteration: 301.1596714945728
cost of 175th iteration: 301.1469130896255
```

```
cost of 176th iteration: 301.1341553162745
cost of 177th iteration: 301.12139817448855
cost of 178th iteration: 301.1086416642366
cost of 179th iteration: 301.0958857854871
cost of 180th iteration: 301.083130538209
cost of 181th iteration: 301.07037592237094
cost of 182th iteration: 301.05762193794175
cost of 183th iteration: 301.0448685848901
cost of 184th iteration: 301.03211586318474
cost of 185th iteration: 301.0193637727944
cost of 186th iteration: 301.0066123136878
cost of 187th iteration: 300.99386148583386
cost of 188th iteration: 300.9811112892011
cost of 189th iteration: 300.96836172375845
cost of 190th iteration: 300.95561278947457
cost of 191th iteration: 300.9428644863183
cost of 192th iteration: 300.93011681425827
cost of 193th iteration: 300.9173697732633
cost of 194th iteration: 300.9046233633021
cost of 195th iteration: 300.8918775843436
cost of 196th iteration: 300.8791324363564
cost of 197th iteration: 300.8663879193093
cost of 198th iteration: 300.85364403317107
cost of 199th iteration: 300.8409007779106
cost of 200th iteration: 300.82815815349636
cost of 201th iteration: 300.81541615989744
cost of 202th iteration: 300.80267479708243
cost of 203th iteration: 300.78993406502013
cost of 204th iteration: 300.77719396367934
cost of 205th iteration: 300.76445449302884
cost of 206th iteration: 300.7517156530374
cost of 207th iteration: 300.7389774436738
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cost of 710th iteration: 40.96957991893585
cost of 711th iteration: 40.96683254213835
cost of 712th iteration: 40.96411038471535
cost of 713th iteration: 40.96141286009717
cost of 714th iteration: 40.95873939519862
cost of 715th iteration: 40.95608943057598
cost of 716th iteration: 40.9534624204047
cost of 717th iteration: 40.95085783234107
cost of 718th iteration: 40.948275147310085
cost of 719th iteration: 40.945713859247164
cost of 720th iteration: 40.943173474812575
cost of 721th iteration: 40.94065351309076
cost of 722th iteration: 40.93815350528271
cost of 723th iteration: 40.935672994396725
cost of 724th iteration: 40.93321153494085
cost of 725th iteration: 40.930768692619665
cost of 726th iteration: 40.92834404403629
cost of 727th iteration: 40.925937176400964
cost of 728th iteration: 40.92354768724638
cost of 729th iteration: 40.92117518740887
cost of 730th iteration: 40.91881929296617
cost of 731th iteration: 40.916479628721454
cost of 732th iteration: 40.91415583079324
cost of 733th iteration: 40.91184754430761
cost of 734th iteration: 40.909554423155996
cost of 735th iteration: 40.90727612975998
cost of 736th iteration: 40.905012334842624
cost of 737th iteration: 40.90276271720624
cost of 738th iteration: 40.90052696351657
cost of 739th iteration: 40.89830476809298
cost of 740th iteration: 40.896095832704574
cost of 741th iteration: 40.89389986637215
cost of 742th iteration: 40.891716585175686
cost of 743th iteration: 40.8895457120673
cost of 744th iteration: 40.887386976689534
cost of 745th iteration: 40.885240115198634
cost of 746th iteration: 40.88310487009304
cost of 747th iteration: 40.88098099004649
```

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cost of 748th iteration: 40.878868229746
cost of 749th iteration: 40.876766349734275
cost of 750th iteration: 40.87467511625669
cost of 751th iteration: 40.87259430111246
cost of 752th iteration: 40.870523681510036
cost of 753th iteration: 40.8684630399266
cost of 754th iteration: 40.86641216397146
cost of 755th iteration: 40.86437084625323
cost of 756th iteration: 40.86233888425084
cost of 757th iteration: 40.86031608018803
cost of 758th iteration: 40.858302240911414
cost of 759th iteration: 40.85629717777199
cost of 760th iteration: 40.85430070650983
cost of 761th iteration: 40.85231264714201
cost of 762th iteration: 40.85033282385374
cost of 763th iteration: 40.848361064892465
cost of 764th iteration: 40.84639720246478
cost of 765th iteration: 40.84444107263637
cost of 766th iteration: 40.84249251523463
cost of 767th iteration: 40.840551373753954
cost of 768th iteration: 40.838617495263634
cost of 769th iteration: 40.836690730318296
cost of 770th iteration: 40.834770932870825
cost of 771th iteration: 40.832857960187575
cost of 772th iteration: 40.830951672765984
cost of 773th iteration: 40.82905193425443
cost of 774th iteration: 40.82715861137415
cost of 775th iteration: 40.82527157384346
cost of 776th iteration: 40.82339069430384
cost of 777th iteration: 40.82151584824816
cost of 778th iteration: 40.819646913950784
cost of 779th iteration: 40.817779090159235
cost of 780th iteration: 40.81589416549767
cost of 781th iteration: 40.814015219603924
cost of 782th iteration: 40.812142144517104
cost of 783th iteration: 40.81027483192029
cost of 784th iteration: 40.80841317402946
cost of 785th iteration: 40.80655706416754
cost of 786th iteration: 40.80470639712927
cost of 787th iteration: 40.802861069407236
cost of 788th iteration: 40.801020979326026
cost of 789th iteration: 40.799186027115496
cost of 790th iteration: 40.797356114944094
cost of 791th iteration: 40.79553114692576
```

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cost of 792th iteration: 40.7937110291098
cost of 793th iteration: 40.79189566945971
cost of 794th iteration: 40.79008497782508
cost of 795th iteration: 40.78827886590906
cost of 796th iteration: 40.78647724723342
cost of 797th iteration: 40.78468003710225
cost of 798th iteration: 40.78288715256498
cost of 799th iteration: 40.781098512379494
cost of 800th iteration: 40.77931403697527
cost of 801th iteration: 40.777533648417204
cost of 802th iteration: 40.77575727036986
cost of 803th iteration: 40.77398482806244
cost of 804th iteration: 40.772216248254615
cost of 805th iteration: 40.770451459202775
cost of 806th iteration: 40.76869039062738
cost of 807th iteration: 40.766932973680824
cost of 808th iteration: 40.76517914091616
cost of 809th iteration: 40.76342882625647
cost of 810th iteration: 40.76168196496504
cost of 811th iteration: 40.759938493616175
cost of 812th iteration: 40.7581983500667
cost of 813th iteration: 40.75646147342818
cost of 814th iteration: 40.75472780403975
cost of 815th iteration: 40.7529972834416
cost of 816th iteration: 40.751269854349054
cost of 817th iteration: 40.74954546062731
cost of 818th iteration: 40.747824047266676
cost of 819th iteration: 40.74610556035854
cost of 820th iteration: 40.74438994707166
cost of 821th iteration: 40.742677155629316
cost of 822th iteration: 40.740967135286645
cost of 823th iteration: 40.739259836308776
cost of 824th iteration: 40.737555209949335
cost of 825th iteration: 40.73585320842945
cost of 826th iteration: 40.734153784917325
cost of 827th iteration: 40.73245689350809
cost of 828th iteration: 40.73076248920436
cost of 829th iteration: 40.729070527897015
cost of 830th iteration: 40.7273809663466
cost of 831th iteration: 40.72569376216493
cost of 832th iteration: 40.724008873797374
cost of 833th iteration: 40.72232626050531
cost of 834th iteration: 40.720645882349096
cost of 835th iteration: 40.71896770017135
```

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cost of 836th iteration: 40.717291675580725
cost of 837th iteration: 40.71561777093591
cost of 838th iteration: 40.713945949330025
cost of 839th iteration: 40.71227617457546
cost of 840th iteration: 40.7106084111889
cost of 841th iteration: 40.70894262437674
cost of 842th iteration: 40.70727878002094
cost of 843th iteration: 40.70561684466497
cost of 844th iteration: 40.70395678550027
cost of 845th iteration: 40.70229857035281
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cost of 848th iteration: 40.697334676521436
cost of 849th iteration: 40.69568352794481
cost of 850th iteration: 40.694034071588
cost of 851th iteration: 40.69238627882054
cost of 852th iteration: 40.69074012156107
cost of 853th iteration: 40.68909557226609
cost of 854th iteration: 40.687452603919084
cost of 855th iteration: 40.68581119001979
cost of 856th iteration: 40.68417130457372
cost of 857th iteration: 40.68253292208191
cost of 858th iteration: 40.680896017530905
cost of 859th iteration: 40.679260566382936
cost of 860th iteration: 40.677626544566316
cost of 861th iteration: 40.67599392846599
cost of 862th iteration: 40.67436269491443
cost of 863th iteration: 40.6727328211825
cost of 864th iteration: 40.67110428497078
cost of 865th iteration: 40.66947706440082
cost of 866th iteration: 40.667851138006746
cost of 867th iteration: 40.66622648472694
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cost of 870th iteration: 40.66135995885282
cost of 871th iteration: 40.65974019522017
cost of 872th iteration: 40.65812160518067
cost of 873th iteration: 40.656504169934216
cost of 874th iteration: 40.65488787103178
cost of 875th iteration: 40.65327269036839
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cost of 878th iteration: 40.648433681780936
cost of 879th iteration: 40.6468227996687
```

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cost of 880th iteration: 40.64521295019698
cost of 881th iteration: 40.643604117186406
cost of 882th iteration: 40.6419962847566
cost of 883th iteration: 40.64038943732033
cost of 884th iteration: 40.63878355957773
cost of 885th iteration: 40.63717863651058
cost of 886th iteration: 40.63557465337682
cost of 887th iteration: 40.63397159570503
cost of 888th iteration: 40.63236944928916
cost of 889th iteration: 40.63076820018327
cost of 890th iteration: 40.62916783469645
cost of 891th iteration: 40.62756833938779
cost of 892th iteration: 40.62596970106146
cost of 893th iteration: 40.6243719067619
cost of 894th iteration: 40.62277494376915
cost of 895th iteration: 40.62117879959418
cost of 896th iteration: 40.6195834619744
cost of 897th iteration: 40.617988918869194
cost of 898th iteration: 40.616395158455624
cost of 899th iteration: 40.61480216912414
cost of 900th iteration: 40.613209939474416
cost of 901th iteration: 40.611618458311284
cost of 902th iteration: 40.61002771464068
cost of 903th iteration: 40.608437697665806
cost of 904th iteration: 40.606848396783214
cost of 905th iteration: 40.60525980157904
cost of 906th iteration: 40.60367190182536
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cost of 909th iteration: 40.59891227570116
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cost of 911th iteration: 40.59574248940069
cost of 912th iteration: 40.594158557527564
cost of 913th iteration: 40.59257525441993
cost of 914th iteration: 40.590992571212794
cost of 915th iteration: 40.589410499197
cost of 916th iteration: 40.58782902981615
cost of 917th iteration: 40.58624815466376
cost of 918th iteration: 40.58466786548025
cost of 919th iteration: 40.58308815415016
cost of 920th iteration: 40.58150901269936
cost of 921th iteration: 40.57993043329228
cost of 922th iteration: 40.57835240822931
cost of 923th iteration: 40.57677492994405
```

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cost of 924th iteration: 40.57519799100087
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cost of 927th iteration: 40.57047033777532
cost of 928th iteration: 40.5688954843709
cost of 929th iteration: 40.567321135004384
cost of 930th iteration: 40.56574728297311
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cost of 932th iteration: 40.56260104467376
cost of 933th iteration: 40.56102864556193
cost of 934th iteration: 40.55945671809357
cost of 935th iteration: 40.55788525611478
cost of 936th iteration: 40.55631425357519
cost of 937th iteration: 40.554743704525926
cost of 938th iteration: 40.5531736031177
cost of 939th iteration: 40.55160394359885
cost of 940th iteration: 40.55003472031353
cost of 941th iteration: 40.54846592769981
cost of 942th iteration: 40.54689756028791
cost of 943th iteration: 40.545329612698424
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cost of 947th iteration: 40.539061916043494
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cost of 954th iteration: 40.52810844440979
cost of 955th iteration: 40.526545145725066
cost of 956th iteration: 40.52498220441039
cost of 957th iteration: 40.52341961618418
cost of 958th iteration: 40.52185737683248
cost of 959th iteration: 40.5202954822077
cost of 960th iteration: 40.51873392822733
cost of 961th iteration: 40.517172710872714
cost of 962th iteration: 40.515611826187815
cost of 963th iteration: 40.514051270278046
cost of 964th iteration: 40.51249103930908
cost of 965th iteration: 40.51093112950569
cost of 966th iteration: 40.50937153715067
cost of 967th iteration: 40.50781225858366
```

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cost of 968th iteration: 40.50625329020014
cost of 969th iteration: 40.50469462845032
cost of 970th iteration: 40.50313626983811
cost of 971th iteration: 40.50157821092008
cost of 972th iteration: 40.50002044830452
cost of 973th iteration: 40.498462978650366
cost of 974th iteration: 40.49690579866633
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cost of 979th iteration: 40.48912413196575
cost of 980th iteration: 40.48756862355311
cost of 981th iteration: 40.48601338308682
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cost of 984th iteration: 40.481349240314636
cost of 985th iteration: 40.47979504278849
cost of 986th iteration: 40.47824109898948
cost of 987th iteration: 40.476687406193115
cost of 988th iteration: 40.475133961712956
cost of 989th iteration: 40.47358076289988
cost of 990th iteration: 40.472027807141366
cost of 991th iteration: 40.470475091860834
cost of 992th iteration: 40.46892261451692
cost of 993th iteration: 40.467370372602865
cost of 994th iteration: 40.4658183636458
cost of 995th iteration: 40.464266585206175
cost of 996th iteration: 40.46271503487707
cost of 997th iteration: 40.4611637102836
cost of 998th iteration: 40.45961260908231
cost of 999th iteration: 40.45806172896061
```

In [41]:

```
model1
```

Out[41]:
```
{'W1': array([[-2.07398201e-03, -5.05655258e-03,  5.06171091e-03,
         -8.98977200e-03, -1.66050608e-04, -1.77405290e-02,
         -1.37106226e-02, -8.08802733e-03,  1.80351440e-03,
         -1.80793642e-02, -1.08912460e-02, -3.17880399e-02,
         -5.34414653e-03],
        [-7.93983038e-03,  3.05987098e-02, -6.81786315e-03,
         -6.27019452e-03, -1.45051022e-03, -1.33903560e-02,
          1.66428873e-02,  1.31131664e-02, -4.66012038e-03,
         -2.30267457e-05, -7.54146770e-03,  1.08699507e-01,
         -4.86204707e-03],
        [-2.25944524e-02,  1.16968148e-01, -8.32180593e-03,
         -1.53146999e-02, -1.81735141e-03,  3.14112780e-02,
          1.54879803e-02,  2.27490044e-02, -1.62198104e-02,
         -5.90608493e-03,  3.55055619e-02,  3.93140631e-01,
         -3.98615475e-02],
        [-1.78785674e-02,  2.33491783e-02, -6.62593405e-04,
         -1.38500902e-02,  2.32399511e-02,  6.85081038e-03,
         -7.36413395e-03,  7.03313542e-03,  1.31872932e-02,
         -1.33536730e-02, -1.88655812e-03,  1.25113474e-03,
          1.46430332e-02],
        [-5.66111334e-03, -4.63550096e-03, -4.59170939e-03,
         -5.26465621e-03, -7.64573006e-03,  1.06788897e-02,
          1.14602068e-02, -1.86806067e-02, -9.54313186e-03,
         -3.36280357e-02,  8.92692062e-04, -2.66183759e-02,
          1.55526380e-02]]),
 'B1': array([[-0.00704713],
        [ 0.00833008],
        [ 0.02012855],
        [ 0.00958591],
        [-0.00529336]]),
 'W2': array([[-0.00141452, -0.00774803, -0.06986662, -0.00300974, -0.00170811],
        [-0.00841959,  0.01205008, -0.02597685,  0.02308697, -0.00413424],
        [ 0.00869936, -0.02887072, -0.1369364 ,  0.00788175,  0.01042847],
        [-0.00382953, -0.0262278 , -0.10715744,  0.00516658, -0.00200058],
        [-0.00774879,  0.10652002,  0.37275573,  0.00690328, -0.01050026]]),
 'B2': array([[-0.00316497],
        [ 0.01303736],
        [-0.00482612],
        [-0.01496338],
        [ 0.02036788]]),
 'W3': array([[-8.96981687e-03, -3.84145258e-04, -1.98802881e-02,
         -1.51738384e-02,  3.87756548e-01]]),
 'B3': array([[0.24390751]])}
```

In [42]:
```
compute_accuracy(X_train, X_test, Y_train, Y_test, model1, [13,5,5,1])
```

Out[42]:
```
(8.995166598527916, 8.442594389694793)
```

In [ ]:
```

```