

Problem 1

color C can be expressed in the form of its primaries P_1, P_2, P_3 as follows.

$$C(X, Y, Z) = \alpha_1 P_1(x_1, y_1, z_1) + \alpha_2 P_2(x_2, y_2, z_2) + \alpha_3 P_3(x_3, y_3, z_3)$$

i) Normalized chromaticity co-ordinates of primaries P_1, P_2 & P_3

$$P_1 : \quad x_1 = \frac{x_1}{x_1 + y_1 + z_1} \quad y_1 = \frac{y_1}{x_1 + y_1 + z_1} \quad z_1 = \frac{z_1}{x_1 + y_1 + z_1}$$

$$P_2 : \quad x_2 = \frac{x_2}{x_2 + y_2 + z_2} \quad y_2 = \frac{y_2}{x_2 + y_2 + z_2} \quad z_2 = \frac{z_2}{x_2 + y_2 + z_2}$$

$$P_3 : \quad x_3 = \frac{x_3}{x_3 + y_3 + z_3} \quad y_3 = \frac{y_3}{x_3 + y_3 + z_3} \quad z_3 = \frac{z_3}{x_3 + y_3 + z_3}$$

Normalized chromaticity coordinates of color C is:

$$x = \frac{x}{x + y + z} \quad y = \frac{y}{x + y + z} \quad z = \frac{z}{x + y + z} \quad \text{--- (1)}$$

ii) Now based on the above the normalized chromaticity coordinates of Color C in terms of P_1, P_2, P_3 can be expressed as.

$$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \quad Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$$

$$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3 \quad - \quad (2)$$

substitute (2) in (1):

$$x = \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$y = \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$z = \frac{\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$(3) - \alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)$$

iii) Representing chromaticity coordinates of color C as linear combinations of respective primaries.

$$x = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

$$y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$$

$$z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3$$

$$X = \frac{x}{x+y+z} \quad \text{from (1)}$$

$$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \quad \text{from (2)}$$

$$X_1 = x_1 (x_1 + y_1 + z_1)$$

$$\alpha_1 X_1 = \alpha_1 x_1 (x_1 + y_1 + z_1) \quad -$$

$$\alpha_2 X_2 = \alpha_2 x_2 (x_2 + y_2 + z_2)$$

$$\alpha_3 X_3 = \alpha_3 x_3 (x_3 + y_3 + z_3).$$

From (3) we get

$$x = \frac{\alpha_1 (x_1 + y_1 + z_1) x_1 + \alpha_2 (x_2 + y_2 + z_2) x_2 + \alpha_3 (x_3 + y_3 + z_3) x_3}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

from above we get:

$$B_1 = \frac{\alpha_1 (x_1 + y_1 + z_1)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

$$B_2 = \frac{\alpha_2 (x_2 + y_2 + z_2)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

$$B_3 = \frac{\alpha_3 (x_3 + y_3 + z_3)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

similarly we can get for y, z .

Hence,

$$x = B_1 x_1 + B_2 x_2 + B_3 x_3.$$

$$y = B_1 y_1 + B_2 y_2 + B_3 y_3.$$

$$z = B_1 z_1 + B_2 z_2 + B_3 z_3.$$

Problem 2.

1) Given level 0 - 0.25

Values:

5.8, 6.2, 6.2, 7.2, 7.3, 7.3, 6.5, 6.8, 6.8, 6.8, 5.5, 5,
5.2, 5.2, 5.8, 6.2, 6.2, 6.2, 5.9, 6.3, 5.2, 4.2,
2.8, 2.8, 2.3, 2.9, 1.8, 2.5, 2.5, 3.3, 4.1, 4.9.

2) Levels After Quantization:

22, 24, 24, 28, 28, 28, 25, 26, 26, 26, 21,
19, 20, 20, 22, 24, 24, 24, 23, 24, 20,
16, 10, 10, 8, 11, 6, 9, 9, 12, 15, 19.

2) There are 32 levels hence we need 5 bits to represent all levels.

$$\begin{aligned}\text{Hence no. of transmitted bits} &= 5 \times 32 \\ &= \underline{\underline{160}}.\end{aligned}$$

3) DPCM:

2, 0, 4, 0, 0, -3, 1, 0, 0, -5, -2, 1, 0, 2
2, 0, 0, -1, 1, -4, -4, -6, 0, -2, 3, -5, 3, 0
3, 0, 3, 3, 4.

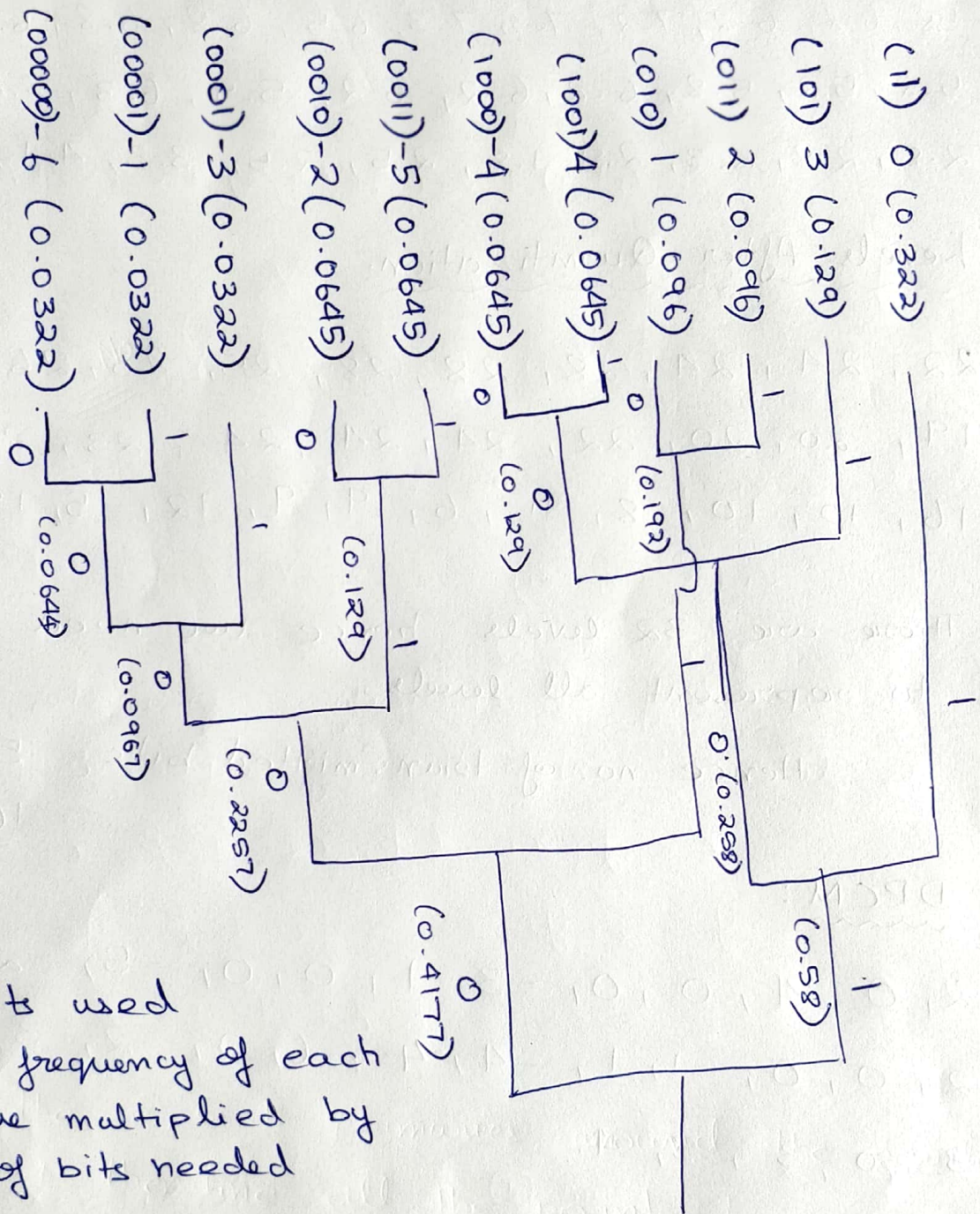
range is from 6 to -4
(max) (min).

there are 11 values. hence we need 4 bits.

$$\begin{aligned}\text{no. of transmitted bits} &= 4 \times 31 \\ &= 124\end{aligned}$$

4) compression ratio $\div \frac{4 \times 31}{5 \times 31} = \underline{\underline{0.8}} \Rightarrow \underline{\underline{1.25}}$.

5) Using Huffman encoding:



Total bits used
will be frequency of each
DPCM value multiplied by
the no. of bits needed

$$= (10 \times 2) + (3 \times 4) + (3 \times 3) + (3 \times 3) + 32 + 4 + 10$$

$$= \underline{\underline{96 \text{ bits}}}$$

we need 96 bits to encode the
sequence.

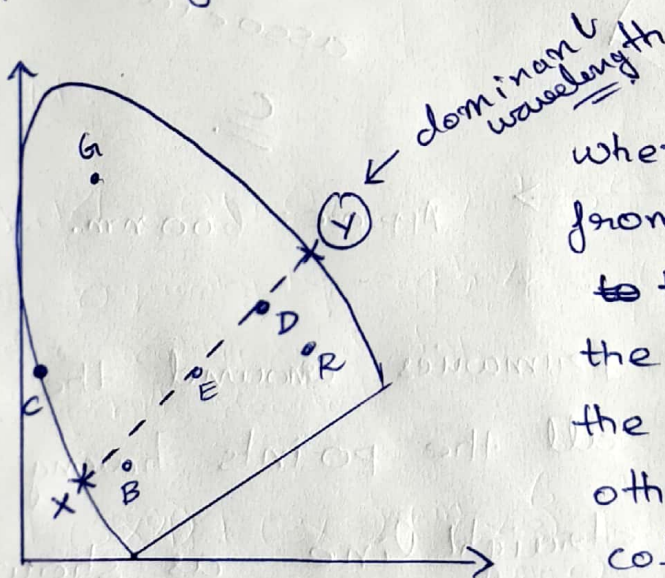
6) Compression ratio: $\frac{96}{5 \times 31} = 0.61 \Rightarrow \underline{\underline{1.63}}$

we notice, we get better compression using Huffman encoding over DPCM rather than just DPCM.

Problem-3

- 1) Dominant color as described in the question is the spectral color which can be mixed with white light to reproduce desired color C.

→ A straight line drawn between the point of given color when extrapolated to intersect the gamut perimeter through the equiluminous point gives the dominant color.



when line is ~~inter~~ extrapolated from D through E we get two colors X & Y, the one closest to D is the dominant color & other is complimentary color.

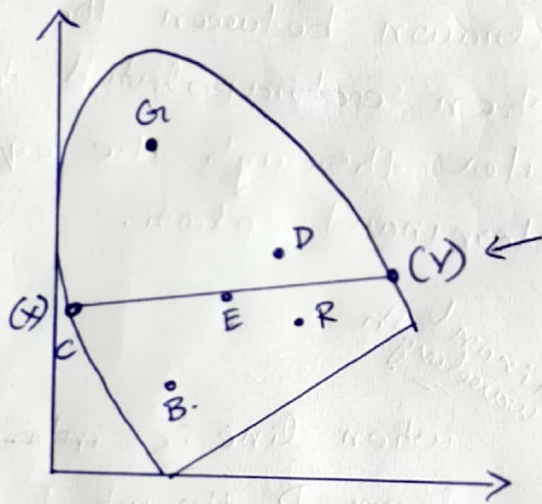
approx: 580 nm

2) ~~No~~ ^{not} all colors in the gamut will have a dominant wavelength associated with them.

In case of when the extrapolated line does not intersect the horse-shoe part of the gamut but rather intersects the straight line then the complementary wavelength is used to describe the color more accurately and it's called "complimentary dominant wavelength".

This happens more for colors lying in the Red space of gamut.

3)

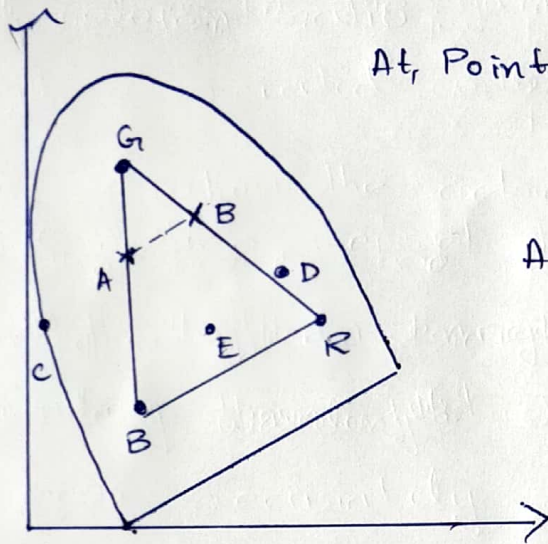


complimentary wavelength associated with

Approx = 600 nm.

4)

Given the R, G, B primaries around the equiluminance point E then ~~line~~ all the points having $G = 0.5$ will lie on a straight line as shown in the below figure.



At, Point A : $G = 0.5$
 $B = 0.5$
 $R = 0$

At, Point B : $G = 0.5$
 $R = 0.5$
 $B = 0$

line AB contains the
 set of points where
 $G = \underline{0.5}$

5). The locus $G = 0.5$ will be projected into
 RGB space as a plane with $G = 0.5$ &
 R & B values varying from 0 to 1.

The plane when projected onto the chromaticity
 RGB space will give the above line

A-B