Descriptive statistics

① measures of central tendancy

↳ mean, mode, median

② measures of dispersion / variation

↳ range, quartile, IQR, variation, std deviation

③ measures of asymmetry

↳ skewness, kurtosis

④ measures of relationship

↳ covariance & correlation, Regression

Measures Of Central Tendency

central value of series

Measures of Central Tendency

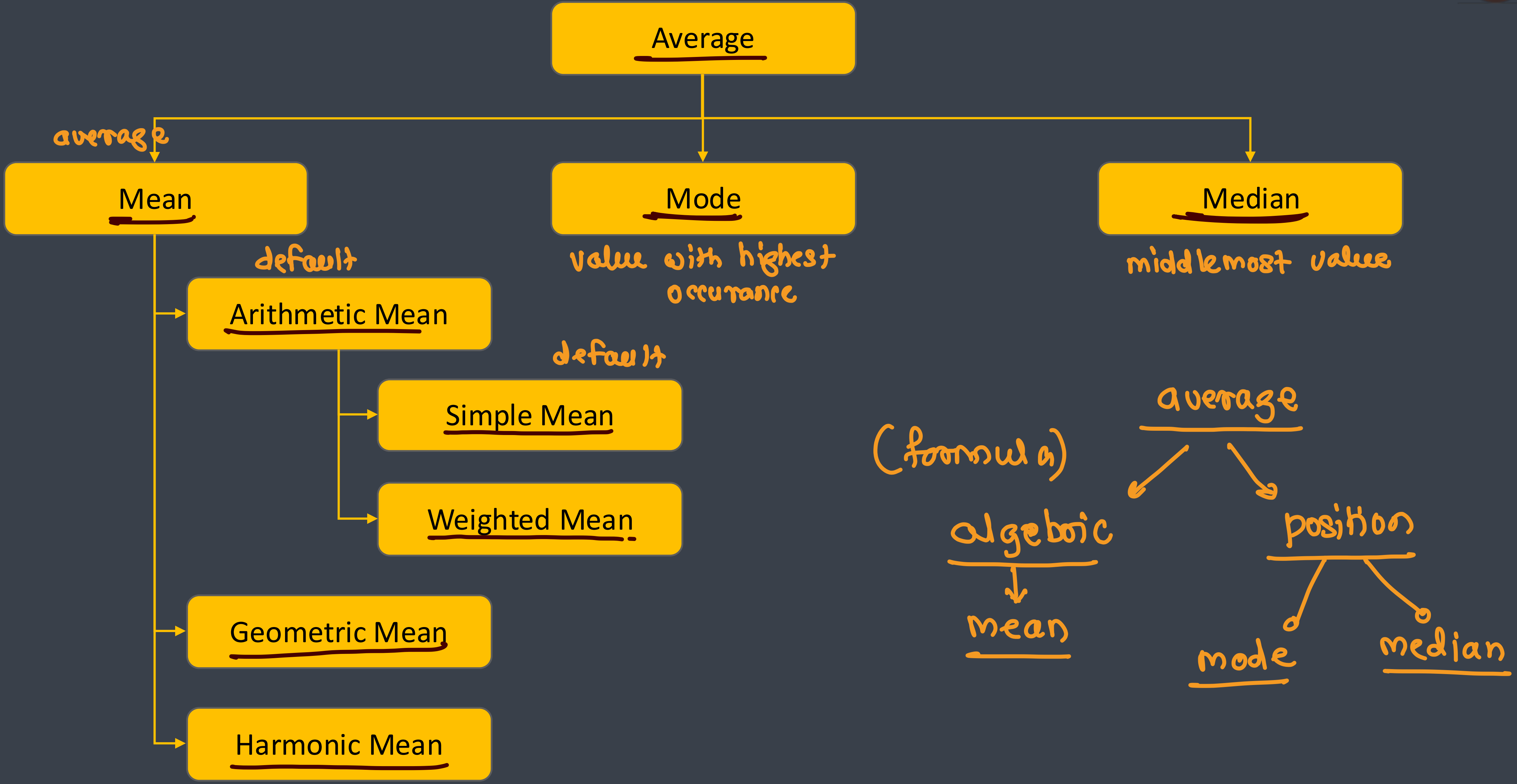
central

- One of the important objectives of statistical analysis is to get one single value that describes the characteristic of entire mass of selected data
- Such value is called as "Central Value" or "Average" or expected value of the variable
- Average
  - Average is an attempt to find one single figure to describe the whole of figures
  - Average is a single value selected from a group of values to represent them in some way
  - Average is sometimes described as a number which is typical of the whole group
- Objectives of averaging
  - To get single value that describes the characteristics of the entire group
  - To facilitate comparison

Requisites of good average

- Easy to understand
- Simple to compute
- Based on all the items
- Not be unduly affected by extreme observations
- Rigidly defined → formula
- Capable of further algebraic treatment → computed
- Sampling stability

Types of Averages



Average

average

Mean

Mode

value with highest occurance

Median

middlemost value

default

Arithmetic Mean

default

Simple Mean

Weighted Mean

Geometric Mean

Harmonic Mean

average

(formula)

algebric

mean

position

mode

median

Mean

Simple Arithmetic Mean – Individual Series

- Direct method
- Steps
  - Add all the observations together and obtain the total $\sum X$
  - Divide the total by number of observations

10, 20, 30, 40, 35, 38, 29, 41,
50, 60, 65, 55, 53

$$\text{mean} = \frac{\text{sum}(\ldots)}{13}$$

$$\text{mean} = 40.46$$

$$\bar{X} = \frac{X1 + X2 + X3 \; \ldots + Xn}{N}$$

OR

$$\bar{X} = \frac{\sum X}{N}$$

Simple Arithmetic Mean – Individual Series

- Shortcut method (Using Assumed Mean)
- Steps
  - Take an assumed mean and denote it as A
  - Take the deviations of items from assumed mean and denote them by d
  - Obtain the sum of these deviations i.e. $\sum d$
  - Apply the formula

$d = x_i - A$

$$\bar{X} = \underline{A} + \frac{\sum d}{N}$$

Simple Arithmetic Mean – Individual Series

- Following are the monthly income of 10 employees in an office
  - 14780, 15760, 26690, 27750, 24840, 24920, 16100, 17810, 27050, 16950
- Calculate arithmetic mean of income

using direct method

$$\text{mean} = \frac{\Sigma x}{N}$$

$$\text{mean} = \frac{14780 + 15760 + \dots + 16950}{10}$$

$$\text{mean} = \frac{212650}{10} = \boxed{21265}$$

using assumed mean

X	X - A
14780	4780
15760	5760
26690	16690
27750	17750
24840	14840
24920	14920
16100	6100
17810	7810
27050	17050
16950	6950

$\Sigma d = 112650$

$A = 10000$

$$\text{mean} = A + \frac{\Sigma d}{N}$$

$$= 10000 + \frac{112650}{10}$$

$$\boxed{\text{mean} = 21265}$$

Simple Arithmetic Mean – Discrete Series

- Direct method
- Steps
  - Multiply the frequency of each row with the variable and obtain the total $\sum fX$
  - Divide the total by number of observation that is the total frequency

$$\bar{X} = \frac{\sum fX}{N}$$

$N = \Sigma f$

- Where
  - f = frequency
  - X = observations
  - N = total frequency

Simple Arithmetic Mean – Discrete Series

- Shortcut method - Using Assumed mean
- Steps
  - Take an assumed mean and denote it by A
  - Take the deviations of the variable X from the assumed mean and denote the deviations by d
  - Multiply this deviation by respective frequency and take the total $\sum fd$
  - Apply the formula

$d = x_i - A$, $N = \Sigma f$

$$\boxed{\bar{X} = A + \frac{\sum fd}{N}}$$

$N = \Sigma f$

- Where
  - f = frequency
  - d = deviation from Assumed mean
  - A = assumed mean
  - N = total frequency

Simple Arithmetic Mean – Discrete Series

- From the following data of marks obtained by students, calculate arithmetic mean

Marks	20	30	40	50	60	70
# students	8	12	20	10	6	4

marks (x)	# students (f)	$f \cdot x$
20	8	160
30	12	360
40	20	800
50	10	500
60	6	360
70	4	280
N =	60	2460

$$\text{mean} = \frac{\Sigma f \cdot x}{N}$$

$$N = 60 \ (\Sigma f)$$

$$\text{mean} = \frac{2460}{60} = 41$$

$$\boxed{\text{mean} = 41}$$

Simple Arithmetic Mean – Continuous Series

- Direct method
- Steps

$$\frac{\text{lower + upper}}{2}$$

  - Obtain the mid point of each class and denote it by m
  - Multiply these mid points by the respective frequency of each class and obtain $\sum fm$
  - Divide the total obtained by the sum of frequency (N)

$$\boxed{\bar{X} = \frac{\sum fm}{N}}$$

$$N = \Sigma f$$

- Where
  - f = frequency
  - m = mid point of each class
  - N = total frequency

Simple Arithmetic Mean – Continuous Series

- Shortcut method - Using Assumed mean
- Steps
  - Take an assumed mean and denote it by A
  - From the mid point of each class deduct the assumed mean
  - Multiply the respective frequencies of each class by the deviations and obtain $\sum fd$
  - Apply formula

$d = X'_i - A$, $N = \Sigma F$

$$\bar{X} = A + \frac{\sum fd}{N}$$

- Where
  - f = frequency
  - d = deviation of class mid point from assumed mean
  - A = assumed mean
  - N = total frequency

Simple Arithmetic Mean – Continuous Series

- From the following data of marks obtained by students, calculate arithmetic mean

Marks	0-10	10-20	20-30	30-40	40-50	50-60
# students	5	10	25	30	20	10

marks (x)	# students	m	fm
0-10	5	5	25
10-20	10	15	150
20-30	25	25	625
30-40	30	35	1050
40-50	20	45	900
50-60	10	55	550
	100		3300

$$\text{mean} = \frac{\Sigma fm}{N}$$

$N = 100$, $\Sigma fm = 3300$

$$\text{Mean} = \frac{3300}{100} = 33$$

$$\boxed{\text{Mean} = 33}$$

Mathematical Properties of Arithmetic Mean

- Sum of the deviations of the items from the arithmetic mean (taking sign into account) is always zero
- Sum of the squared deviations of the items from arithmetic mean is minimum, that is, less than the sum of squared deviations of the items from any other value

→ Replace all NA (missing) values by mean

- Including the mean value in the series multiple times wont change the mean
- If we have arithmetic mean and number of items of two or more than two related groups, we can compute combined mean of these groups using formula

$$\overline{X_{12}} = \frac{N_1\overline{X_1} + N_2\overline{X_2}}{N_1 + N_2}$$

2, 3, 4, 5, 6 mean = 4

x	x-mean	$(x\text{-mean})^2$
2	-2	4
3	-1	1
4	0	0
5	1	1
6	2	4
	0	10

2, 3, 4, 5, 6, 4, 4

m = 4 m = 4

x	x - 2	$(x-2)^2$
2	0	0
3	1	1
4	2	4
5	3	9
6	4	16
		8

$s1 = 2, 3, 4, 5, 6$, $m1 = 4$

$s2 = \underline{1, 7, 8}$ $m2 = 5.3$

$s3 = 1, 2, 3, 4, 5, 6, 7, 8 = \boxed{m3 = \underline{4.5}}$

$N_1 = 5$, $N2 = 3$

$$m3 = \frac{N_1\bar{x}_1 + N2\bar{x}_2}{N_1 + N2} = \frac{5\times 4 + 3\times 5.3}{5+3}$$

$$= \frac{20 + 15.3}{8} = \boxed{\underline{9.487}}$$

Merits

- It is simplest average to understand and easiest to compute
- It is affected by value of every item in the series
- It is defined by rigid mathematical formula with the result that everyone who computes the average gets the same answer
- It lends itself to subsequent algebraic treatment better than median or mode
- The mean is typical in the sense that it is the center of gravity, balancing the values on the either sides of it
- It is calculated values and not based on the positions

Geometric Mean

- Steps
  - Multiply all the values and get the result
  - Get the square root to the Nth power to find the geometric mean

$$\bar{X} = \sqrt[N]{x_1 * x_2 * \ldots.* x_n}$$

Harmonic Mean

- Steps
  - Get reciprocal of each number and add together
  - Divide the number of values by the total calculated eariler

$$\bar{X} = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Weighted Mean

- Steps
  - Multiply every value with corresponding weight
  - Add the values together
  - Divide the total by sum of all the weights

$$\bar{X} = \frac{\sum WiXi}{W1 + W2 \ldots . + Wn}$$

outlier

$\boxed{-20}, 2, 3, 4, 5, 6, \boxed{100}$ → outlier → Exteremely high or Exteremely low values

mean = 4

mean_new = 20

Median

Qualitative or

Quantitative

Median – middlemost value

series (data)

- By definition, it refers to the middle value in a distribution [arranged in an order]
- The median is just 50th percentile value below which 50% of the values in the sample fall
- It splits the observations into two halves
- Unlike the mean, median is calculated by position (which refers to the place of the value in the series)

middlemost

$$\underbrace{2 \quad 3}_{50\%} \quad \underset{\substack{\uparrow \\ \text{median}}}{\boxed{4}} \quad \underbrace{5 \quad 6}_{50\%}$$

observation = record

row

individual

Median – Individual Series

- Steps
  - Arrange the data in the ascending or descending order of magnitude
  - In a group composed of an odd number of values such as 7, add 1 to the total number of values and divide it by 2. Thus 7 + 1 would be 8 which divided by 2 gives 4 – the position used to calculate the mean
  - In a group composed of even number of values such as 10, use the average of middle two values. Thus 10 / 2 gives 5 – which will produce a median by taking average of 5th and 6th position values

$$median = \left(\frac{N+1}{2}\right)^{th} value$$

median

odd no of values

even no of values

$$median = \left(\frac{N+1}{2}\right)^{th} value$$

Qualitative + Quantitative

median

$$= \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N+1}{2}\right)^{th}}{2}$$

Quantitative

Median – Individual Series

- E.g. 1:
  - 1 2 3 4 5 6 7
  - find median of 14100, 14150, 16080, 17120, 15200, 16160, 17400
  - Arrange them in ascending order
    - 14100, 14150, 15200, $\boxed{16080,}$ 16160, 17120, 17400
    - 1 2 3 4 5 6 7
  - Median = (N + 1) / 2th item
  - Median = 7 + 1 / 2 = 4th item => 16080

- E.g. 2:
  - Find median of 19, 28, 40, 10, 29, 50, 37, 89, 90, 60
  - Arrange them in ascending order
    - 1 2 3 4 5 6 7 8 9 10
    - 10, 19, 28, 29, $\boxed{37, 40,}$ 50, 60, 89, 90
  - Median = (N + 1)/ 2 the item
  - Median = average of 5th and 6th items => Average(37, 40) =>38.50

red green white

green red white

φ

Median – Discrete Series

$N = \Sigma f$

- Steps
  - Arrange the data in ascending or descending order of magnitude
  - Find out cumulative frequencies
  - Apply the formula (N + 1) / 2 the item
  - Now look at the cumulative frequency and find the total which is either equal to (N + 1) /2 or next higher to that and determine the value of variable corresponding to it
  - This gives the value of median

Median – Discrete Series

values

Marks	20	30	40	50	60	70
# students	8	12	20	10	6	4

frequency

Marks	#students	Cumulative frequency ✓		
20	8	8	≥ 30.5	✗
30	12	20	≥ 30.5	✗
40	20	40	≥ 30.5	✓
50	10	50		
60	6	56		
70	4	60		

median → 40

$N = 60$

$\frac{N+1}{2} = \frac{61}{2} = 30.5$

$N = \Sigma F$

- Median is (N + 1) / 2 th item => (60 + 1) / 2 = 30.5 th item
- Since the value at 30.5th (or just higher than it) is 40
- Median = 40

Median – Continuous Series

- Steps
  - Determine the particular class in which the value of median lies, consider this as median class
  - Calculate the cumulative frequencies
  - Use N/2 as the rank of the median
  - Use the formula

$$\boxed{median = \underline{L + \frac{\frac{N}{2} - cf}{f} * i}}$$

- Where
  - L = Lower limit of the median class (the class in which middle item of the distribution lies)
  - cf = cumulative frequency of the class preceding the median class
  - f = frequency of the median class
  - i = class interval of the median class

i = upper – lower

Median – Continuous Series

Marks	0-10	10-20	20-30	30-40	40-50	50-60
# students	5	10	25	30	20	10

- The median class is => 100 /2 => 50 lies in (30-40)
- Median = 30 + ((100/2 – 40) / 30) * 10
- Median = 30 + (10/30) * 10 = 33.33

Marks	#students	cf
0-10	5	5 ≥50 ✗
10-20	10	15 ≥50 ✗
20-30	25	40 ≥50 ✗
30-40	30	70 ≥50 ✓
40-50	20	90
50-60	10	100 — N
	100	

median class

$$\text{median} = L + \left(\frac{(N/2) - c.f.}{f} \times i\right)$$

$$= 30 + \left(\frac{(100/2) - 40}{30} \times 10\right) = 30 + 3.33 = \underline{\underline{33.33}}$$

$$\left(\frac{N}{2}\right) = \frac{100}{2} = \underline{\underline{50}}$$

Merits

- It is useful in case of open-end classes since only the position and not the values of the items must be known
- Median is recommended if the distribution has unequal classes
- Extreme values do not affect the median as strongly as they do the mean
- It is most appropriate average in dealing with qualitative data
- Value of median can be calculated graphically
- It represents clear-cut the middle value in the distribution

20	2
30	2
40	3

⇒

20 20
30 30
40 40 40

mean is affected by outlier, but median is not as much as mean

2 3 4 5 6

mean = $\boxed{4}$

median = $\underline{\underline{4}}$

2 3 $\underline{\underline{4 \quad 5}}$ 6 100 — $\underline{\text{outlier}}$

mean = $\boxed{20}$

median = $\frac{4+5}{2} = \underline{\underline{4.5}}$

Limitations

- For calculating median, it is necessary to arrange the data in a specific order
- Since it is a middle value, its value is not determined by each and every observation
- It is not capable of algebraic treatment
- The value of median is affected more by fluctuations than the value of the arithmetic mean
- It is erratic if the number of observations is very small

Qualitative Quantitative

Mode

value having highest frequency

Mode

- The mode or modal value is that value in a series which occurs most frequently
- That is the mode always will have the highest frequency in the data
- There are many situations where mean and median fails to reveal the true middle value, in such scenarios mode is used to find the central value

Mode – Individual Series

- Steps
  - Count the number of times the various values repeate themselves and the value occurring maximum number of times is the modal value

- E.g. 10, 28, 39, 40, 10, 20, 40, 50, 10 => mode = [10] → single mode
- E.g. 10, 20, 40, 50, 10, 20, 30, 40, 50 => mode = [~~10, 20, 50~~] [10, 20, 40, 50] = multi-mode
- E.g. 10, 20, 30, 40, 50, 60, 70, 80, 90 => mode = [] → No. mode

Mode – Discrete Series

- Steps
  - Mode can be determined just be inspection
  - i.e. by looking to that value of the variable around which the items are most heavily concentrated

- E.g.

mode

Marks	20	30	40	50	60	70
# students	8	12	20	10	6	4

- The mode here is 40

Mode – Continuous Series

- Steps
  - Find the modal class by finding the largest value
  - Determine the value of mode by applying the following formula

$$mode = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} * i$$

- Where
  - L = Lower limit of modal class
  - Δ_1 = difference between the frequency of modal class and frequency of pre-modal class
  - Δ_2 = difference between the frequency of modal class and frequency of post-modal class

i = class interval of modal class

Mode – Continuous Series

Marks	0-10	10-20	20-30	30-40	40-50	50-60
# students	5	10	25	30	20	10

- Modal class here is: 30-40
- Using the formula
  - Mode = 30 + ((30-25) / ((30-25) + (30-20))) * 10
  - Mode = 30 + (5 / (5 + 10)) * 10
  - Mode = 30 + 3.33 = 33.33

modal class = $\underline{30-40}$

$L = 30$

$\Delta 1 = 30-25 = 5$

$\Delta 2 = 30-20 = 10$

$$\text{mode} = L + \frac{\Delta 1}{\Delta 1 + \Delta 2} \times i$$

$$= 30 + \frac{5}{5+10} \times 10$$

$$= 30 + 3.3 = \underline{33.33}$$

Merits

- Mode is the most typical or representative value of the distribution
- Like median, mode is not unduly affected by extreme values
- It can be used to describe the qualitative phenomenon
- The value of mode can be calculated graphically

Limitations

- The value of mode can not always be determined
- It is not capable of algebraic manipulation
- The value of mode is not based on each and every value of distribution

avg.	# car
10	2
20	5
12	9
15	12
18	3

marks	#st
10-20	8
20-30	2
30-40	5
40-50	3

① 20, 30, 40, 21, 38, 58, 69, 72, 89

② 30, 89, 50, 52, 86, 89, 89, 72