

Qn. 1. Answer.

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$h_1 = \frac{1}{1+e^{-w_1x_1-w_2x_2}}$$

$$L(y, \hat{y}) = \|\hat{y} - y\|^2$$

$$x_1 = 0.7$$

$$x_2 = 1.2$$

$$x_3 = 1.1$$

$$x_4 = 1.2$$

$$y = 0.5$$

$$\begin{aligned} S_1 &= x_1 w_1 + x_2 w_2 \\ &= (0.7)(-1.7) + (1.2)(0.1) \\ &= (-1.19) + (0.12) \\ &= -1.07 \end{aligned}$$

$$\Rightarrow S_1 = -1.07$$

$$\begin{aligned} S_2 &= x_3 w_3 + x_4 w_4 \\ &= (1.1)(-0.6) + (1.2)(-1.8) \\ &= (-0.66) + (-3.6) \\ &= -4.26 \end{aligned}$$

$$h_1 = \frac{1}{1+e^{1.07}}$$

$$\Rightarrow h_1 = 0.255$$

$$\begin{aligned} S_3 &= h_1 w_5 + h_2 w_6 \\ &= (0.255)(-0.2) + (0.0139)(0.5) \\ &= -0.0441 \end{aligned}$$

$$\hat{y} = \frac{1}{1+e^{-S_3}} = \frac{1}{1+e^{0.0441}} = 0.4889$$

Back Propagation

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial s_3} \cdot \frac{\partial s_3}{\partial h_1} \cdot \frac{\partial h_1}{\partial s_1} \cdot \frac{\partial s_1}{\partial w_1}$$

$$= 2 \|\hat{y} - y\| \cdot \sigma'(s_3) \cdot w_5 \cdot \sigma'(s_1) \cdot x_1$$

$$= 2 \times \|0.4889 - 0.5\| \times \sigma'(-0.441) \cdot (-0.2)$$

$$\sigma'(s_3) = \frac{1}{1 + e^{-(-0.044)}} = 0.4884$$

$$\sigma'(s_1) = \frac{1}{1 + e^{-(-1.07)}} = 0.2554$$

$$\Rightarrow \frac{\partial E}{\partial w_1} = -0.0014$$

$$\frac{\partial s_3}{\partial h_1} = w_5$$

$$\frac{\partial s_1}{\partial w_1} = x_1$$

$$2.0 \times (0.2554) \times (-0.2) = -0.10216$$

$$-0.10216 \times 0.4884 = -0.0500$$