

Analyzing Preferential Attachment in Peer-to-Peer BITCOIN Networks

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Abstract—Due to the anonymous and remote nature of Electronic Commerce (E-commerce), reviews of products and vendors left by previous customers have emerged as an integral part of most online transactions. The reviews may influence the decision of customers buying the product since E-commerce websites/services do not allow customers to validate and inspect products in-store. In this paper, we analyze data from two BITCOIN marketplaces which include transactions between marketplace users and the ratings of those transactions given by those users. In this analysis we create a synthetic network model with similar topological properties as the networks of the interactions of both marketplaces. The results of our analysis show an interesting phenomenon in which user ratings, which range from -10 to 10, converge to a value of approximately two as the number of a user's transactions increase. Finally, we suggest future work on our synthetic model to improve its agreement with the transaction networks in order to better understand how reviews influence user decisions on transactions.

I. INTRODUCTION AND MOTIVATION

As communication technology has developed, business models have evolved. Nearly all historical business models centered around a store front in which customers must be physically present to conduct business. However, with the expansion of the Internet, a new model has developed centered around E-commerce, in which transactions are conducted electronically via the world wide web. While the traditional model allows the customer to inspect and validate the quality of a product in-store, for most E-commerce businesses it is not possible for customers to validate quality of products before purchase. This means that E-commerce businesses depend largely on trust and merchant reputation, which generally come in the form of previous customers' reviews.

This paper aims to study data from a BITCOIN marketplace which contain interactions and ratings, to analyze the network theoretic properties of the marketplace, and to IEEE/ACM ASONAM 2018, August 28-31, 2018, Barcelona, Spain
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generate a synthetic network model with rating attributes which can be scaled by the number of nodes. The data set from BITCOIN ALPHA was used as a basis for modelling and analysis. We also create a synthetic model for such a network, that we verify against BITCOIN - OTC.

The research community has been interested in introducing realistic synthetic models for networks to aid in numerical experimentation and simulation. Having a model in which several parameters can be varied, allow the creation of network assembly on which evaluation of empirically observed phenomena and their dependency to these parameters can be studied. Such examples can include epidemics spreading, vulnerability to deliberate or targeted attacks, social influence and so on.

II. PROBLEM DEFINITIONS

We set out to investigate the economic phenomenon “rich get richer, poor get poorer” in the BITCOIN ALPHA data set [1]. BITCOIN ALPHA is an anonymous online marketplace through which customers and vendors conduct transactions using the cryptocurrency BITCOIN. Customers rate their transaction with a vendor on a scale from -10 to 10 based on their satisfaction. This results in a continuously evolving directed trust network in which users are represented by nodes and transactions are represented by edges. Potential customers can then use this network to determine if they want to do business with a given vendor.

We examine the BITCOIN ALPHA data generated from November 2010 to January 2016 obtained from the Stanford Network Analysis Project [2]. If the “rich get richer, poor get poorer” phenomenon exists within the BITCOIN alpha marketplace, we would expect the nodes with high average ratings to increase to 10 as time goes on. Similarly, nodes with low average ratings are expected to drop to -10.

Additionally, we expect that a high average rating is the primary metric by which potential customers judge a vendor fit to conduct business with. To analyze the data, we create a

network representing users with nodes and rated transactions between users with weighted, directed edges. We expect that nodes with higher average ratings will have a higher frequency of incoming attachment (i.e. more transactions with new users) within the network.

Finally, using the data from BITCOIN ALPHA, we strive to create a synthetic model with tunable parameters to potentially gain deeper understanding of their networks. We will test our synthetic model with the data from BITCOIN ALPHA and BITCOIN OTC.

III. RELATED WORK

Diffusion in social networks is a well researched topic [3, 4, 5, 6, 7]. Relevant to our work is identifying of a seed set of nodes whose adoption of new innovation would trigger a large cascade of further followers [8].

Particularly, past studies into the economic trends of e-commerce environments have found that “rich get richer” phenomena to exists [9, 10, 11]. These studies find noticeable effects of customer reviews on an online vendor’s business. A study of e-Bay transactions [10], investigated the asymmetric effect of positive and negative user ratings. Standifird et al. find that each positive rating has a small positive effect on a vendor’s ability to command a higher price at auction, but that a single negative review can do significant damage to this ability. The influence of a negative comment vastly outweighs the influence of a positive one. Kondor et al. [9] conduct an in depth analysis of the BITCOIN trading website MtGox, investigating topological properties such as degree distribution and clustering. They further find that growth within this network was governed by preferential attachment, and that the wealth associated with a node, that node’s reputation, and its ability to gain new connections are all fundamentally related.

The BITCOIN transaction networks we study are best represented as weighted signed social networks. There has been fairly extensive research into the structure of such networks [12, 13, 14, 15, 16]. Much of this study is grounded in the theory of social balance, a social psychology theory dating back to the 1950’s. Developed by Dr. Fritz Heider, social balance theory has to do with stable and unstable group dynamics [17]. At its most basic, the theory studies the four possible relationships between a group of three people, finding two to be stable (three friends or two friends with a mutual enemy), and two to be unstable (one person friends with a pair of enemies, or three enemies).

This concept is easily extended to graph theory. The three people and their relationships can be modeled as complete signed graph of three nodes (triangles), in which a positive edge represents friendship between the two nodes and a negative edge represents their being enemies. Several of the previous works on weighted signed networks concerned with social balance develop models [13, 14, 15] which generate synthetic networks that reproduce the properties of their

original network, including degree distribution and clustering coefficients, but with a special focus on conserving the number and type of signed triangles.

A fairly unique analysis of weighted signed networks (WSN) is performed by Kumar et al., that propose a method for the prediction of edge weights between the nodes of WSNs [1]. They approach the problem through the development of two novel metrics for WSN nodes, which they called goodness and fairness. Fairness is a measure of how fair a node is when rating other nodes, and goodness is a measure of how positively other nodes in the network think a particular node is. Using these two metrics, a model was created to predict edge weights between nodes of real networks, which was then verified against the initial data. A different approach is proposed by in [16]. Shafaei et al. study information cascades in signed networks, particularly how they can be used to identify communities in these networks.

BITCOIN ALPHA and BITCOIN OTC [1] are online marketplaces in which the cryptocurrency BITCOIN can be exchanged for other currencies and in which other commerce is conducted using BITCOIN. The users in these marketplaces are anonymous, with no official record associating the identity of users to their usernames. However, transactions between users are recorded. Past studies which focus on the BITCOIN ALPHA and BITCOIN OTC data sets include [1, 18]. Kumar et al. perform a comprehensive analysis of BITCOIN OTC and BITCOIN Alpha in [18], in which they attempt to characterize the BITCOIN trust networks through various network properties, such as degree distribution weight distribution, and clustering coefficients, as well as the application of status and balance theories from social psychology. In [1], the BITCOIN data sets are studied primarily for their weighted signed network properties along with several other WSNs, mostly drawn from social networks.

IV. TEMPORAL NETWORK ANALYSIS

We provide a general analysis of the BITCOIN ALPHA transaction network characteristics in Subsection A. In Subsection B we engage in a more detailed analysis of the temporal evolution of the ratings of individual nodes, and in Subsection C we examine the network wide rating trends as well as their evolution with time.

A. Basic Characterization

The network obtained from the data set consists of 3,783 nodes (users) and 24,186 edges (user-to-user ratings) cumulatively. Figure 1 presents the network’s size (normalized for each of edge count and node count) as a function of time, from Nov 2010 ($t = 0$) to Jan 2016 ($t = 1$).

TABLE I: Growth of network in discrete time slices.

Time Slice	Nodes	Edges
τ_1 (1/3)	57.0 %	47.5
τ_2 (2/3)	93.7 %	92.0
τ_3 (3/3)	100 %	100

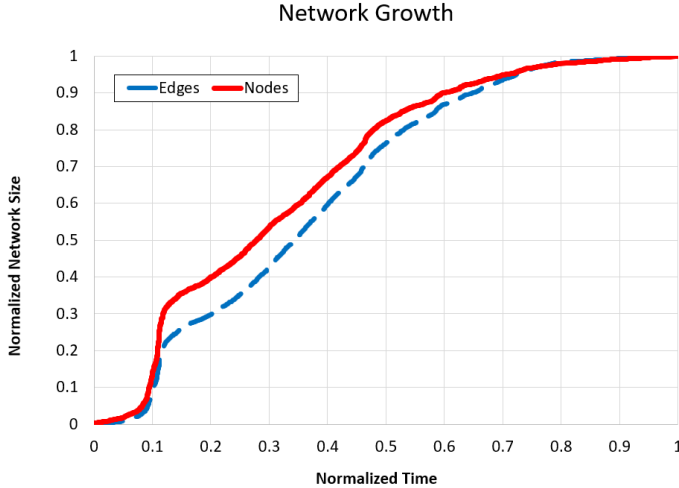


Fig. 1: Normalized network growth for nodes and edges over sampled time frame. The November 2010 to January 2016 period has been represented as normalized time.

Notice the sharp increase shortly after $t = 0$, and the plateau towards the end, $t = 1$. We thus partition the data into three times slices of equal duration. Table I then shows that the first 15% of the network’s life shows about 33% of the network growth, and nearly all network data is known at time slice two ($\tau_2 = 67\%$).

B. Individual Trajectories

Complementary to the analysis of the overall network, we analyze the network temporally by tracking a few users through time. Growth of large-scale parameters do not answer important questions such as whether a user prefers to conduct business with better-rated users.

Many statistics can attempt to capture the concept of preferential attachment based on rates, as seen in [18], which focus on weighted in-degree and average rating. These may be basic, but also mimic human ability to quickly determine from potentially hundreds of numeric-only ratings on whether to proceed with a BITCOIN transaction. To this end, the overall network progression is less important than the individual stories of a specific user’s rating evolution.

Since it can be difficult to extract meaning from over 3,500 trajectories, Figure 2 shows a few examples of how the network data can provide rating evolution trajectories for specific users. The x-axis shows normalized time of the dataset and the y-axis shows the evolution of prior average rating at the time of a new edge. Red lines show two users that lost favor over a significant number of ratings

and the green line shows an example user with consistent performance. The blue dots are data points covered by these overall trendlines.

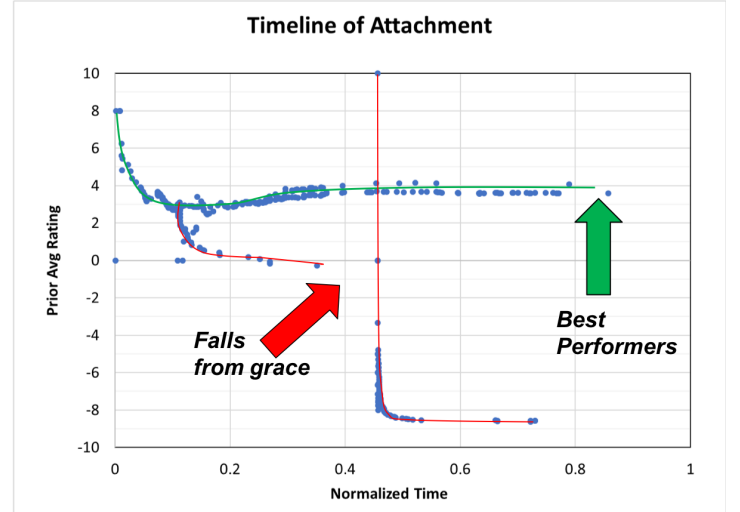


Fig. 2: A few example trajectories of nodes in the network.

Often a drop in reputation can quickly turn negative in what appears to be a collective effort by the marketplace community to identify and isolate bad actors. See the red lines in Figure 2. Ratings tend to be highest among small networks of presumably real-world friendships.

User contacts outside of these groups of friends tend to rated lower. Even users that maintain a solid average rating trend downward as their network of associations expanded. They might have an intrinsic value which indicates a reason to attract future business. See the green line in Figure 2. Thus, average rating appears to be a reasonable measure for the intrinsic value of a user.

Next, the average ratings of users with the top 15 in-degrees (most ratings) at the final time are tracked over the discrete time slices (τ_i). This was done using waterfall contour plots in Figures 3 and 4.

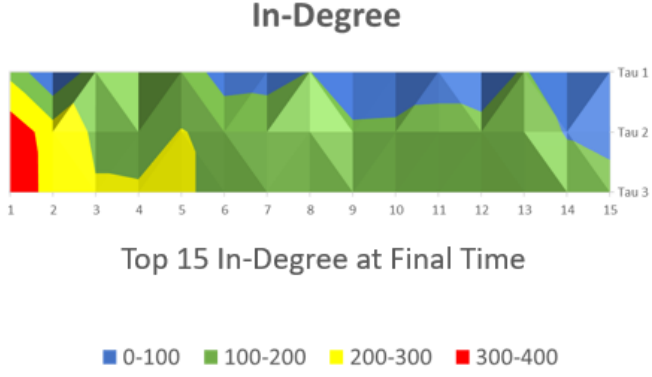


Fig. 3: Evolution of the in-degree of the most-rated nodes in the network.

In these plots, each of the top 15 users are represented by a numbered vertical line, which has three values corresponding to either in-degree or average rating at the end of a given τ_i .

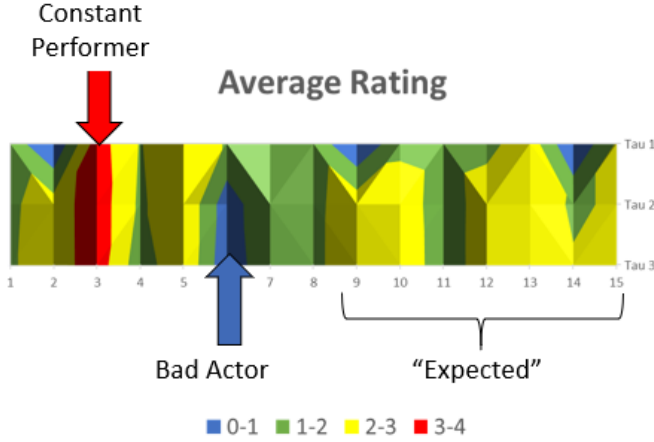


Fig. 4: Evolution of the average rating of the most-rated nodes in the network.

As seen in Figure 2, average rating can reflect stark contrasts between consistent performance and bad actors. This contrast is also seen in Figure 4. T

he third highest in-degree node can be seen to consistently receive ratings between 3 and 4. This user was well-established in the network at τ_{u1} , but others were relatively unknown as seen in Figure 3. Specifically, the second-, ninth-, and fourteenth-most rated users joined after τ_{u1} . Regardless, these users went on to achieve an expected rise in average rating, which reflects a version of the “rich-get-richer” phenomenon.

Where this general trend of early adopters remaining well-rated is epitomized in the sixth-most rated user, which appears to be identified as a bad actor over the course of the

network snapshot. Despite the outliers (both good and bad), the majority of the most-rated users exhibit the “expected” rich-get-richer phenomenon.

C. Network Trajectories

The previously shown trajectories are unique and do not necessarily imply overall trends in the network. We therefore study the rating distribution of the network over time.

Figures 5 and 6 introduce two snapshots of the overall distribution of average rating in the network in comparison to the number of ratings received (i.e. number of in-edges). The top five in-degree nodes are colored red and labeled accordingly. The sixth-most rated user is also shown to see the previously discussed decent in average rating.

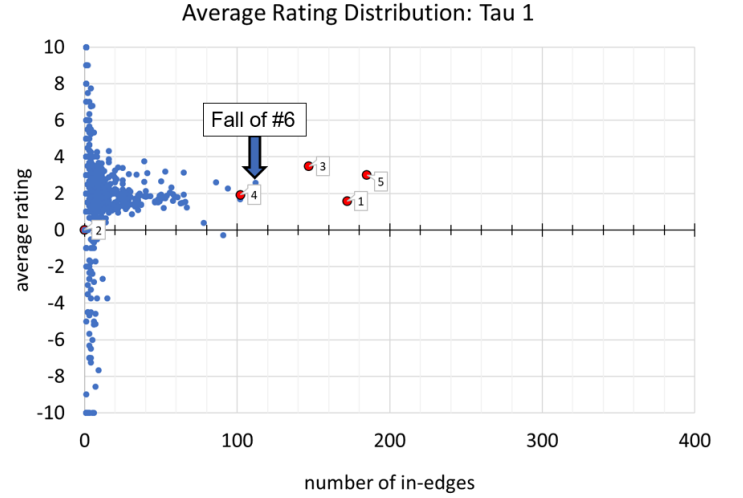


Fig. 5: Network average rating distribution at τ_1 .

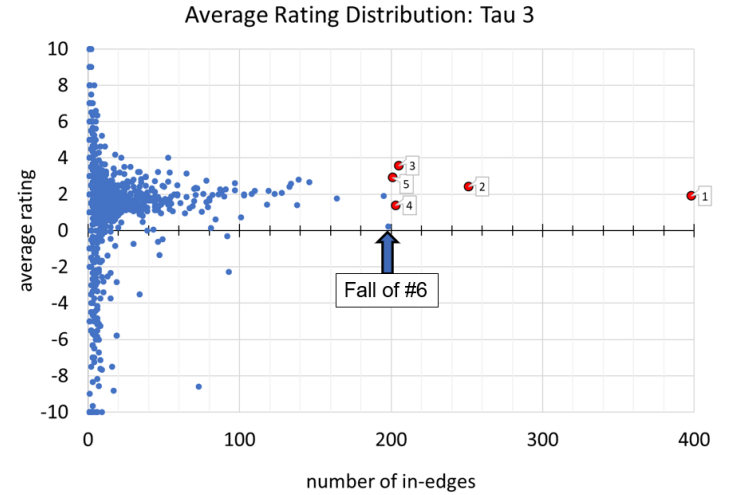


Fig. 6: Network average rating distribution at τ_3 .

Overall, it becomes clear from Figures 5 and 6 that the extreme ratings (i.e. near -10 or +10) are populated by users with only a few ratings. As the user begins to have more

transactions outside their tight-knit group, ratings move toward an asymptote of an average rating of approximately 2. This slightly positive rating can be seen as “rich” in terms of the rating guidelines given for these networks.

These findings become clearer through a histogram analysis in an effort to correlate prior average rating with future growth (number of future in-edges) and future rating. Figure 7 shows a correspondence of number of ratings given for any user based on the average rating prior to the rating. Values are shown using a logarithmic scale.

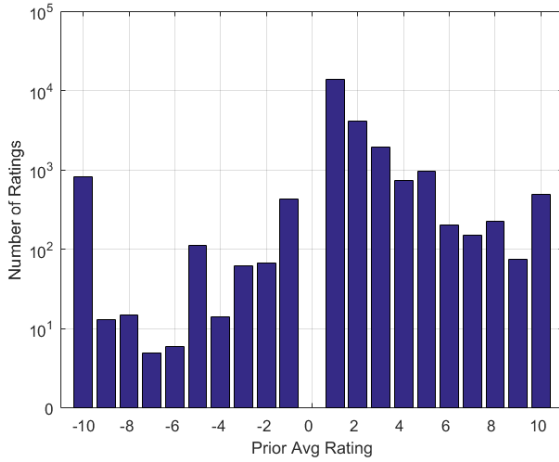


Fig. 7: Histogram analysis of number of attachments for any user with a given average rating prior to the new rating.

Thus, we can observe that positively-rated users collect more future ratings. This preferential attachment does not, on its own, mean that future ratings do not tend to sink all high-flyers. That fact can be seen specifically for outliers in Figure 6 and Figure 2.

A bivariate histogram analysis, however, does demonstrate this preferential attachment. In Figure 8, the number of ratings are grouped by both the prior average rating of the user being rated and the new rating they received.

The number of ratings in this category (represented by squares in Figure 8) are colored according to a logarithmic scale. In the figure red lines delineate quadrants as well as a diagonal, which represents an increase in rating (above the diagonal) or a decrease in rating (below the diagonal).

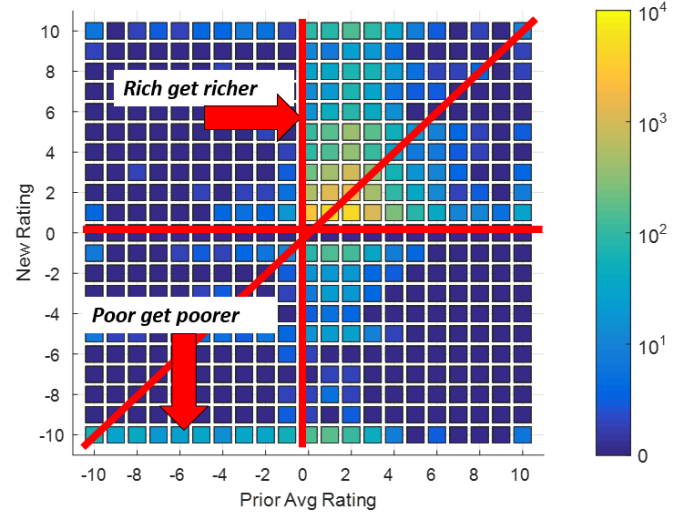


Fig. 8: Bivariate histogram analysis of future rating for any user with a given average rating prior to that new rating.

The strong concentration above the diagonal in the upper-right quadrant in Figure 8 indicates that given a positive prior average rating, a user’s future in-degree tends to increase slightly until a drop off at around 5. This corresponds to a discrete change in the meaning of rating guidelines above this value.

Further, the near void of the upper-left quadrant demonstrates that few poorly-rated users receive much future positive attention, though they do tend to receive some strongly negative reviews. Though previously positively rated users do receive some negative reviews, these are greatly outnumbered by new positive reviews. This plot at once shows that the rich get richer and poor get poorer phenomenon is present, but also that there is no inherent protection from decreases in average rating.

Our analysis therefore suggests, these temporal results all tend to point to preferential attachment in the sense that there are very few poorly-rated users with high in-degrees. The rich get richer phenomenon is most likely less pronounced due to guidelines for ratings, the character of an anonymous peer-to-peer transaction network, and a network still in its infancy in terms of the presence of lots of tight-knit communities.

V. SYNTHETIC NETWORK METHODOLOGY

Algorithmically generated synthetic models are powerful tools in the analysis of a complex networks or processes/dynamics on complex networks. Often, these synthetic models are designed based upon the original network and they include tunable parameters to control the network properties.

Studying how the model changes as the parameters are adjusted often allows for a deeper understanding of a process or parameter, as the network grows, for example. Also, networks which are prohibitively large or studying the temporal

evolution within a network could be better studied through the models created from observed data or the understanding of the process that created the network.

We thus create a synthetic model based on the BITCOIN Alpha data set which is able to maintain certain network properties while its number of nodes is adjusted. We then compare the model against the original data from BITCOIN Alpha, as well as data from BITCOIN OTC.

Notice that BITCOIN OTC was not used as input to our analysis or synthetic generation, thus we would like to see the similarity between it and the synthetically generated network. This comparison with a similar but different data set is useful in testing the robustness of the model.

A. Step 1: Determining the Probability Density Function (PDF) of Degree Distribution

- 1) Plotting the degree distribution from BITCOIN-ALPHA, we find that the PDF of the degree distribution to be:
- 2) $PDF = \frac{k}{x+1}$
- 3) k is a constant;
- 4) x is degree;
- 5) Integrating the PDF, we obtain an equation for Cumulative Distribution Function (CDF). Normalizing the CDF to 1, we find k to be 1 and the equation of the degree sequence to be:
- 6) Degree sequence = $\{x\}$, where $x = \frac{1}{1-u} - 1$
- 7) x values lies within 0 to $n-1$;
- 8) n = maximum number of nodes;
- 9) u = random uniform generator from 0 to 1;
- 10) Under this configuration, we can scale the synthetic model based on the number of nodes.

B. Step 2: Creating Network Connectivity

With the degree sequence generated, we input the degree sequence into the NetworkX configuration model [19] in Python and plot the digraph. Note that the NetworkX configuration model will result in self-loop degree. In addition, this method can generate an invalid network in which there is an odd number of odd degree vertices. In this case, the model can be run again.

C. Step 3: Assign Node Ratings

We derive the rating of each targeted node by finding the probability of having each rating (-10 to 10) from the actual graph. A random rating is assigned to each targeted node based on the probability of the actual graph. Probability of each rating are shown in Table II. Note that a null rating of 0 (missing from Table II) had an exact probability of 0.

TABLE II: Probability of Ratings

Ratings	Probability	Ratings	Probability
-10	0.03357314	1	0.56892417
-9	0.0005375	2	0.17005706
-8	0.00062019	3	0.07992227
-7	0.00020673	4	0.0307616
-6	0.00024808	5	0.03956834
-5	0.00463078	6	0.00831059
-4	0.00057885	7	0.00616059
-3	0.00256347	8	0.00926156
-2	0.00281154	9	0.00310097
-1	0.01773753	10	0.02042504

VI. ANALYSIS AND SYNTHETIC NETWORK RESULTS

Running synthetic Graph 100 times and averaging the result yields the graph properties as shown in Table III.

The synthetic model generates a network showing mostly similar network properties when compared with BITCOIN-ALPHA. The largest difference lies between the number of strongly and weakly connected components.

TABLE III: Graph Properties Comparison between Model and BITCOIN-ALPHA

Properties	BITCOIN-ALPHA	Synthetic Model	Difference(%)
Number of Nodes	3783	3783	0
Number of Edges	24186	21059.92	12.9
Average Degree	4.812	5.432	12.9
Network Diameter	10	8.33	16.9
Average Path Length (Largest Component)	3.679	3.25	11.6
Modularity ¹	0.46	0.35	23.9
Strongly Connected Components	540	67.8	87.4
Weakly Connected Components	5	67.8	92.6

It is observed that NetworkX configuration model in python generates equal number of strongly and weakly components. One possible reason was due to random matching of edges of configuration model instead of other types of matching such as preferential attachments that may be found in BITCOIN-ALPHA network.

The rating distribution for the actual graph and synthetic model are shown in Figure 9.

¹Modularity is run only in Gephi as the NetworkX community detection algorithm does not allow directed graph

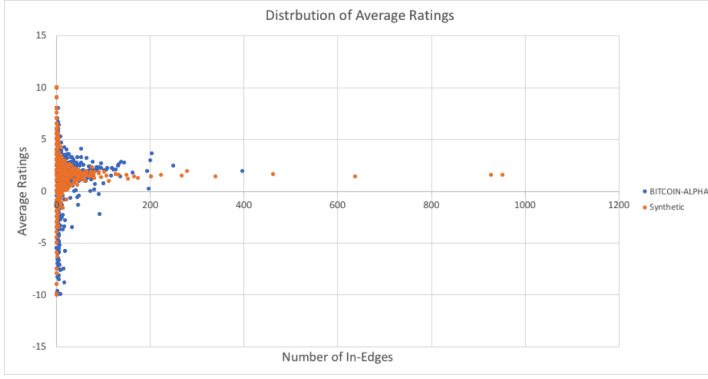


Fig. 9: Ratings Distribution for BITCOIN-ALPHA and Synthetic Model

Both distributions show a wide range of average ratings for nodes with low in-degrees and converge to approximately 2 as in-degree increases. A reasonable explanation for this phenomenon is the provision of rating guideline given by BITCOIN-ALPHA, stating users should give a rating of 1 or 2 for a good transaction for the first few transactions with another user, and to gradually increase their rating after more transactions. Many business relationships consist of only a few transactions, especially in vendors who do business with a large number of other users.

We then compare our network model with another similar BITCOIN transaction platform, BITCOIN-OTC, which also includes transaction ratings from -10 to 10. This data set from BITCOIN-OTC is larger, so the synthetic network model was scaled to the same number of nodes. The results are shown in Table IV:

TABLE IV: Graph Properties of BITCOIN-OTC and Model

Properties	BITCOIN OTC	Synthetic Model	Difference(%)
Number of Nodes	5881	5881	0
Number of Edges	35592	34052	4.3
Average Degree	6.052	5.922	2.1
Network Diameter	11	8.90	1.9
Average Path Length	3.719	3.30	11.2
Modularity ²	0.49	0.348	29.0
Strongly Connected Components	1144	96.6	91.6
Weakly Connected Components	4	96.6	95.9

The degree distribution of the synthetic model derived from BITCOIN-ALPHA is compared to the degree distribution of the data from BITCOIN-OTC. Our model achieves a similar degree sequence as shown in Figure 10.

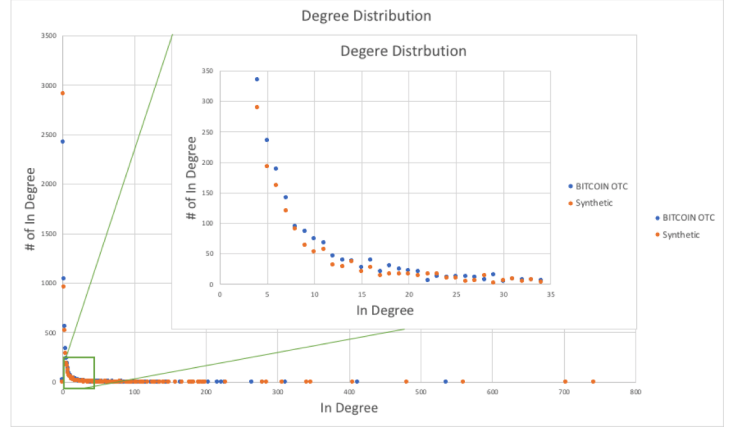


Fig. 10: Degree Distribution of BITCOIN-OTC and Model

The rating distribution for the BITCOIN - OTC and our Synthetic Model are shown in Figure 11. The result from the rating distribution displays much similarity to Figure 9. The ratings from OTC also converge to a value of approximately 2, but with higher in-degree distributions.

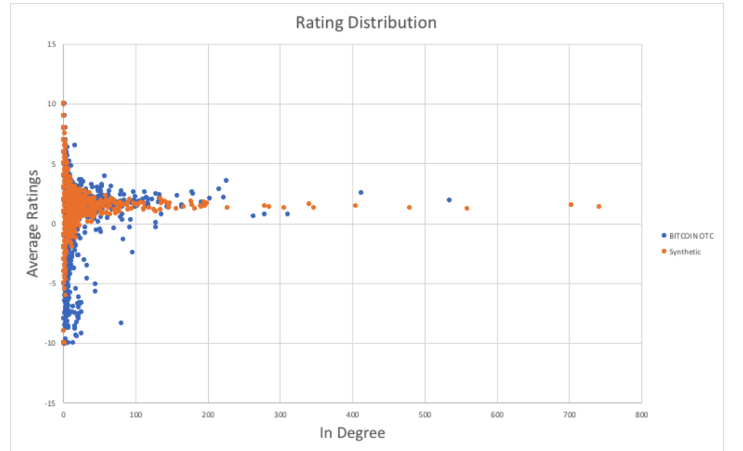


Fig. 11: Rating Distribution of BITCOIN-OTC and Model

Using the data from BITCOIN-ALPHA, we create a model that can be scaled by its number of nodes. The model generates a very similar distribution of ratings when compared with data from both BITCOIN-ALPHA and BITCOIN-OTC. The rating distribution shows that users with higher frequency of ratings converge to an average value of approximately 2, while users with fewer ratings tend to have a wide range of ratings.

These trends do not agree with those from a study of Amazon ratings [11] where, the “rich get richer” phenomenon in ratings is also present. We believe that the reason for the rating distribution is that both BITCOIN platforms have guidelines on how users should rate each transaction, thus framing user’s perception of rating score.

VII. CONCLUSION

We set out to investigate whether weighted signed BITCOIN networks such as BITCOIN-ALPHA and BITCOIN-OTC, follow the “rich get richer, poor get poorer” phenomenon, and if existing ratings have an effect on frequency of future attachment in such networks.

Our work also looked at creating a synthetic model which represents these data sets well, and can be scaled by the cardinality of its node set.

Our results show that both BITCOIN-ALPHA and BITCOIN-OTC follow “the rich get richer, poor get poorer” phenomenon in their own unique ways. While positive ratings tend to result in higher in-degrees, the ratings do not converge to extremes (-10 and 10) but to approximately 2.

Our synthetic model was able to scale based on number of nodes, with random rating attributes based on probability of ratings from BITCOIN-ALPHA. The Comparison of the synthetic model with BITCOIN-ALPHA and BITCOIN-OTC show similarity in many network properties. However, there are some distinct limitations to our synthetic model.

Configuration model in python’s NetworkX uses random attachment, which may not be reflective of real world scenario in which other type of attachments are more prevalent, such as preferential attachment. Our model may also generate self edges, and gave little insight into the temporal evolution of complex networks.

Our project provides insight into how ratings are distributed in a weighted sign BITCOIN network. Future work includes community analysis on the temporal aspect for directed networks, and analysis on how positive and negative weighted degree affects existing network analysis algorithms.

Future work on synthetic models, of this type might include mechanisms to capture preferential attachment by including probability of an edge with a node based on node ratings. Future models might model temporal aspect to reflect how network grow in time, and also model in-degree and out-degree separately to reflect that only nodes with positive rating able to rate other nodes.

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