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Session-01-31-2024-shift-1 1-15

AI24BTECH11031 - Shivram S

- 1) For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its roots. Then, among the two statements
 (I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c
 (II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c
- a) Both (I) and (II) are true c) Only (II) is true
 b) Neither (I) nor (II) is true d) Only (I) is true
- 2) Let a be the sum of all coefficients in the expansion of $(1 - 2x + 2x^2)^{2023}(304x^2 + 2x^3)^{2024}$ and $b = \lim_{x \rightarrow 0} \left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1} dt}{x^2} \right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where $c, d, e \in \mathbb{R}$, then $d : c : e$ equals
- a) $2 : 1 : 4$ b) $4 : 1 : 4$ c) $1 : 2 : 4$ d) $1 : 1 : 4$
- 3) If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}})$ on the hyperbola, is equal to
- a) $7\sqrt{\frac{2}{5}} - \frac{8}{3}$ b) $14\sqrt{\frac{2}{5}} - \frac{4}{3}$ c) $14\sqrt{\frac{2}{5}} - \frac{16}{3}$ d) $7\sqrt{\frac{2}{5}} + \frac{8}{3}$
- 4) If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C , whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle C is
- a) $\sqrt{20}$ b) 4 c) 6 d) $3\sqrt{2}$
- 5) The area of the region
- $$\left\{ (x, y) : y^2 \leq 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \neq 3 \right\}$$

is

a) $\frac{16}{3}$

b) $\frac{64}{3}$

c) $\frac{8}{3}$

d) $\frac{32}{3}$

6) If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ and $(f \circ f)(x) = g(x)$, where $g : \mathbb{R} - \left\{\frac{2}{3}\right\} \rightarrow \mathbb{R} - \left\{\frac{2}{3}\right\}$ then $(g \circ g)(4)$ is equal to

a) $-\frac{19}{20}$

b) $\frac{19}{20}$

c) -4

d) 4

7) $\lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$

a) is equal to -1

b) does not exist

c) is equal to 1

d) is equal to 2

8) If the system of linear equations

$$x - 2y + z = -4$$

$$2x + \alpha y + 3z = 5$$

$$3x - y + \beta z = 3$$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

a) 60

b) 64

c) 54

d) 58

9) The solution curve of the differential equation $y \frac{dx}{dy} = x(\log_e x - \log_e y + 1)$, $x > 0, y > 0$ passing through the point $(e, 1)$ is

a) $\left| \log_e \frac{y}{x} \right| = x$

b) $\left| \log_e \frac{y}{x} \right| = y^2$

c) $\left| \log_e \frac{x}{y} \right| = y$

d) $\left| \log_e \frac{x}{y} \right| = y + 1$

10) Let $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ and let $A(\alpha, \beta)$, $B(1, 0)$, $C(\gamma, \delta)$ and $D(1, 2)$ be the vertices of a parallelogram $ABCD$. If $AB = 10$ and the points A and C lie on the line $3y = 2x + 1$, then $2(\alpha + \beta + \gamma + \delta)$ is equal to

a) 10

b) 5

c) 12

d) 8

11) Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{(\tan x) + y}{\sin x (\sec x - \sin x \tan x)}, x \in \left(0, \frac{\pi}{2}\right)$$

satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then $y\left(\frac{\pi}{3}\right)$ is

a) $\sqrt{3}(2 + \log_e \sqrt{3})$

b) $\frac{\sqrt{3}}{2}(2 + \log_e 3)$

c) $\sqrt{3}(1 + 2 \log_e 3)$

d) $\sqrt{3}(2 + \log_e 3)$

12) Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vector \vec{p} satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to

a) 24

b) 36

c) 28

d) 32

13) The sum of the series $\frac{1}{1-3 \cdot 1^2+1^4} + \frac{2}{1-3 \cdot 2^2+2^4} + \frac{3}{1-3 \cdot 3^2+3^4} + \dots$ upto 10 terms is

a) $\frac{45}{109}$

b) $-\frac{45}{109}$

c) $\frac{55}{109}$

d) $-\frac{55}{109}$

14) The distance of the point $Q(0, 2, -2)$ from the line passing through the point $P(5, -4, 3)$ and perpendicular to the lines $\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k}), \lambda \in \mathbb{R}$ and $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in \mathbb{R}$ is

a) $\sqrt{86}$

b) $\sqrt{20}$

c) $\sqrt{54}$

d) $\sqrt{74}$

15) For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1} \alpha + \sin^{-1} \beta + \sin^{-1} \gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$ then γ is equal to

a) $\frac{\sqrt{3}}{2}$

b) $\frac{1}{\sqrt{2}}$

c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

d) $\sqrt{3}$