Assignment 1

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1 Match The Following

1) $z \neq 0$ is a complex number (1992 - 2 marks)

Column I	**Column II**
(A) Re $z = 0$	(p) Re $z^2 = 0$
(B) Arg $z = \frac{\pi}{4}$	(q) Im $z^2 = 0$
	(r) Re $z^2 =$ Im z^2

2) Match the statements in **Column I** with those in **Column II** (2010)
[Note: here z is a set of points taking values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z. **]**

Column I	**Column II**
(A) The set of points z satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$ is in or equals	(p) an ellipse with eccentricity $\frac{4}{5}$
(B) The set of points z satisfying $\|z + 4\| + \|z - 4\| = 10$ is in or equals	(q) the set of points z satisfying Im $z = 0$
(C) If $\|w\| = 2$, then the set of points $z = w - \frac{1}{w}$ is in or equals	(r) the set of points z satisfying $\|\text{Im } z\| \leq 1$
(D) If $\|w\| = 1$, then the set of points $z = w - \frac{1}{w}$ is in or equals	(s) the set of points z satisfying $\|\text{Re } z\| < 2$
	(t) the set of points z satisfying $\|z\| \leq 3$

3) Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \ldots, 9$. (JEE Adv. 2014)

List I	**List II**
P. For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	1. True
Q. There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution in the set of complex numbers	2. False
R. $\frac{\|1-z_1\|\|1-z_2\|\ldots\|1-z_9\|}{10}$ equals	3. 1
S. $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals	4. 2

	P	**Q**	**R**	**S**		**P**	**Q**	**R**	**S**
(*a*)	1	2	4	3	(*b*)	2	1	3	4
(*c*)	1	2	3	4	(*d*)	2	1	4	3

2 Comprehension Based Questions

2.1 Passage-2

Let $S = S_1 \cap S_2 \cap S_3$ where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\}$$

$$S_2 = \left\{z \in \mathbb{C} : \operatorname{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$$

$$\text{and } S_3 = \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$$

4) Area of S = (JEE Adv. 2013)

a) $\frac{10\pi}{3}$ b) $\frac{20\pi}{3}$ c) $\frac{16\pi}{3}$ d) $\frac{32\pi}{3}$

5) $\min\limits_{z \in S} |1 - 3i - z| =$ (JEE Adv. 2013)

a) $\frac{2-\sqrt{3}}{2}$
b) $\frac{2+\sqrt{3}}{2}$
c) $\frac{3-\sqrt{3}}{2}$
d) $\frac{3+\sqrt{3}}{2}$

3 Integer Value Correct Type

1) If z is any complex number satisfying $|z - 3 - 2i| < 2$, then the minimum value of $|2z - 6 + 5i|$ is (2011)
2) Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that: (2011)

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z$$

Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is

3) For any integer k, let $a_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12} |a_{k+1} - a_k|}{\sum_{k=1}^{3} |a_{4k-1} - a_{4k-2}|}$ is (JEE Adv. 2015)
4) Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\left\{\left|a + b\omega + c\omega^2\right|^2 : a, b, c \text{ distinct non-zero integers}\right\}$ equals ______. (JEE Adv. 2019)