

# Assignment 1

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## I. MATCH THE FOLLOWING

- 1)  $z \neq 0$  is a complex number (1992 – 2 marks)

### Column I

- (A)  $\operatorname{Re} z = 0$   
(B)  $\operatorname{Arg} z = \frac{\pi}{4}$

### Column II

- (p)  $\operatorname{Re} z^2 = 0$   
(q)  $\operatorname{Im} z^2 = 0$   
(r)  $\operatorname{Re} z^2 = \operatorname{Im} z^2$

- 2) Match the statements in **Column I** with those in **Column II** (2010)

[Note: here  $z$  is a set of points taking values in the complex plane and  $\operatorname{Im} z$  and  $\operatorname{Re} z$  denote, respectively, the imaginary part and the real part of  $z$ .]

### Column I

- (A) The set of points  $z$  satisfying  $|z - i|z|| = |z + i|z||$  is in or equals  
(B) The set of points  $z$  satisfying  $|z + 4| + |z - 4| = 10$  is in or equals  
(C) If  $|w| = 2$ , then the set of points  $z = w - \frac{1}{w}$  is in or equals  
(D) If  $|w| = 1$ , then the set of points  $z = w - \frac{1}{w}$  is in or equals

### Column II

- (p) an ellipse with eccentricity  $\frac{4}{5}$   
(q) the set of points  $z$  satisfying  $\operatorname{Im} z = 0$   
(r) the set of points  $z$  satisfying  $|\operatorname{Im} z| \leq 1$   
(s) the set of points  $z$  satisfying  $|\operatorname{Re} z| < 2$   
(t) the set of points  $z$  satisfying  $|z| \leq 3$   
3) Let  $z_k = \cos\left(\frac{2k\pi}{10}\right) + i \sin\left(\frac{2k\pi}{10}\right)$ ;  $k = 1, 2, \dots, 9$ . (JEE Adv. 2014)

### List I

- P. For each  $z_k$  there exists a  $z_j$  such that  $z_k \cdot z_j = 1$   
Q. There exists a  $k \in \{1, 2, \dots, 9\}$  such that  $z_1 \cdot z = z_k$  has no solution in the set of complex numbers

- R.  $\frac{|1-z_1||1-z_2|\dots|1-z_9|}{10}$  equals

- S.  $1 - \sum_{k=1}^9 \cos\left(\frac{2k\pi}{10}\right)$  equals

### List II

1. True

2. False

3. 1

4. 2

	P	Q	R	S		P	Q	R	S
(a)	1	2	4	3	(b)	2	1	3	4
(c)	1	2	3	4	(d)	2	1	4	3

## II. COMPREHENSION BASED QUESTIONS

### A. Passage-2

Let  $S = S_1 \cap S_2 \cap S_3$  where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\}$$

$$S_2 = \left\{z \in \mathbb{C} : \operatorname{Im} \left[ \frac{z-1+\sqrt{3}i}{1-\sqrt{3}i} \right] > 0 \right\}$$

$$\text{and } S_3 = \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$$

- 4) Area of  $S =$  (JEE Adv. 2013)

- a)  $\frac{10\pi}{3}$       b)  $\frac{20\pi}{3}$       c)  $\frac{16\pi}{3}$       d)  $\frac{32\pi}{3}$

- 5)  $\min_{z \in S} |1 - 3i - z| =$  (JEE Adv. 2013)

- a)  $\frac{2-\sqrt{3}}{2}$       c)  $\frac{3-\sqrt{3}}{2}$   
b)  $\frac{2+\sqrt{3}}{2}$       d)  $\frac{3+\sqrt{3}}{2}$

## III. INTEGER VALUE CORRECT TYPE

- 1) If  $z$  is any complex number satisfying  $|z - 3 - 2i| < 2$ , then the minimum value of  $|2z - 6 + 5i|$  is (2011)  
2) Let  $\omega = e^{\frac{i\pi}{3}}$ , and  $a, b, c, x, y, z$  be non-zero complex numbers such that: (2011)

$$a + b + c = x$$

$$a + b\omega + c\omega^2 = y$$

$$a + b\omega^2 + c\omega = z$$

Then the value of  $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$  is

- 3) For any integer  $k$ , let  $a_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$ , where  $i = \sqrt{-1}$ . The value of the expression  $\frac{\sum_{k=1}^{12} |a_{k+1} - a_k|}{\sum_{k=1}^3 |a_{4k-1} - a_{4k-2}|}$  is (JEE Adv. 2015)  
4) Let  $\omega \neq 1$  be a cube root of unity. Then the minimum of the set

$\left\{ \left| a + b\omega + c\omega^2 \right|^2 : a, b, c \text{ distinct non-zero integers} \right\}$   
equals \_\_\_\_\_. (JEE Adv. 2019)