Assignment 2

AI24BTECH11031 - Shivram S

I. Fill in The Blanks

1) Let $p\lambda^2 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$ be an identity in λ where p, q, r, s and t are constants. Then the value of t is _____. $(1981-2\ Marks)$
2) The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is _____. $(1981-2\ Marks)$
3) A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive is _____. $(1982-2\ Marks)$
4) Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$ the other two roots are _____ and _____. $(1983-2\ Marks)$
5) The system of equations

$$\lambda x + y + z = 0$$
$$-x + \lambda y + z = 0$$
$$-x - y + \lambda x = 0$$

Will have a non-zero solution if real values of λ are given by _____. $(1984-2\ Marks)$
6) The value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is _____. $(1988-2\ Marks)$
7) For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is _____. $(1993-2\ Marks)$

II. True / False

1) The determinants $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal. $(1983-1\ Mark)$
2) If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$ then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, and $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ must be congruent. $(1985-1\ Mark)$

III. MCQs with One Corrrect Answer

1) Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1. Then $(1981-2\ Marks)$
   a) C is empty
   b) B has as many elements as C
   c) $A = B \cup C$
   d) B has twice as many elements as C
2) If $\omega(\neq 1)$ is a cube root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$ $(1995S)$

   a) 0 b) 1 c) i d) ω
3) Let a, b, c be the real numbers. Then following system of equations in x, y and z $(1995S)$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

   a) no solution
   b) unique solution
   c) infinitely many solutions
   d) finitely many solutions
4) If A and B are square matrices of equal degree, then which one is correct among the followings? $(1995S)$

   a) $A + B = B + A$
   b) $A + B = A - B$
   c) $A - B = B - A$
   d) $AB = BA$

5) The parameter on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is (1997 – 2 *Marks*)

a) a b) p c) d d) x

6) If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to (1999 – 2 *Marks*)

a) 0 b) 1 c) 100 d) -100