

1.11.14

AI24BTECH11031 - Shivram S

Question:

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$

Solution:

Let the two unit vectors be \mathbf{a} and \mathbf{b}

$$\|\mathbf{a} + \mathbf{b}\|^2 = 1 \quad (0.1)$$

$$(\mathbf{a} + \mathbf{b})^\top (\mathbf{a} + \mathbf{b}) = 1 \quad (0.2)$$

$$\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + 2\mathbf{a}^\top \mathbf{b} = 1 \quad (0.3)$$

$$\mathbf{a}^\top \mathbf{b} = \frac{-1}{2} \quad (0.4)$$

Hence,

$$\|\mathbf{a} - \mathbf{b}\| = \sqrt{(\mathbf{a} - \mathbf{b})^\top (\mathbf{a} - \mathbf{b})} \quad (0.5)$$

$$= \sqrt{\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} - 2\mathbf{a}^\top \mathbf{b}} \quad (0.6)$$

$$= \sqrt{1 + 1 - 2 \cdot \frac{-1}{2}} \quad (0.7)$$

$$= \sqrt{3} \quad (0.8)$$

