1.11.14

AI24BTECH11031 - Shivram S

Question:

If the sum of two unit vectors is a unit vector, peove that the magnitude of their difference is $\sqrt{3}$

Solution:

Let the two unit vectors be $\mathbf{a}$ and $\mathbf{b}$

$$\begin{aligned} \|\mathbf{a}+\mathbf{b}\|^2 &= 1 \\ (\mathbf{a}+\mathbf{b})^\top(\mathbf{a}+\mathbf{b}) &= 1 \\ \mathbf{a}^\top\mathbf{a}+\mathbf{b}^\top\mathbf{b}+2\mathbf{a}^\top\mathbf{b} &= 1 \\ \mathbf{a}^\top\mathbf{b} &= \frac{-1}{2} \end{aligned}$$

Hence,

$$\begin{aligned} \|\mathbf{a}+\mathbf{b}\| &= \sqrt{(\mathbf{a}-\mathbf{b})^\top(\mathbf{a}-\mathbf{b})} \\ &= \sqrt{\mathbf{a}^\top\mathbf{a}+\mathbf{b}^\top\mathbf{b}-2\mathbf{a}^\top\mathbf{b}} \\ &= \sqrt{1+1-2\cdot\frac{-1}{2}} \\ &= \sqrt{3} \end{aligned}$$