

Assignment 2

AI24BTECH11031 - Shivram S

I. FILL IN THE BLANKS

1. Let $p\lambda^2 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & -2\lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$ be an identity in λ where p, q, r, s and t are constants. Then the value of t is _____. (1981 – 2 Marks)

2. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is _____. (1981 – 2 Marks)

3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive is _____. (1982 – 2 Marks)

4. Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$ the other two roots are _____ and _____. (1983 – 2 Marks)

5. The system of equations

$$\begin{aligned} \lambda x + y + z &= 0 \\ -x + \lambda y + z &= 0 \\ -x - y + \lambda x &= 0 \end{aligned}$$

Will have a non-zero solution if real values of λ are given by _____. (1984 – 2 Marks)

6. The value of the determinant $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ is _____. (1988 – 2 Marks)

7. For positive numbers x, y and z , the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is _____. (1993 – 2 Marks)

II. TRUE / FALSE

1. The determinants $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal. (1983 – 1 Mark)

2. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$ then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, and $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ must be congruent. (1985 – 1 Mark)

III. MCQs WITH ONE CORRECT ANSWER

1. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1. Then (1981 – 2 Marks)

- (a) C is empty
(b) B has as many elements as C
(c) $A = B \cup C$
(d) B has twice as many elements as C

2. If $\omega (\neq 1)$ is a cube root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$ (1995S)

- (a) 0 (b) 1 (c) i (d) ω

3. Let a, b, c be the real numbers. Then following system of equations in x, y and z (1995S)
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

- (a) no solution (c) infinitely many solutions
(b) unique solution (d) finitely many solutions

4. If A and B are square matrices of equal degree, then which one is correct among the followings? (1995S)

- (a) $A + B = B + A$ (c) $A - B = B - A$
(b) $A + B = A - B$ (d) $AB = BA$

5. The parameter on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is (1997 – 2 Marks)

- (a) a (b) p (c) d (d) x

6. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to (1999 – 2 Marks)

- (a) 0 (b) 1 (c) 100 (d) -100