Assignment 1

AI24BTECH11031 - Shivram S

I. Match The Following

1) $z \neq 0$ is a complex number (1992 – 2 *marks*)

Column I	Column II
(A) Re $z = 0$	(p) Re $z^2 = 0$
(B) Arg $z = \frac{\pi}{4}$	(q) Im $z^2 = 0$
	(r) Re $z^2 =$ Im z^2

2) Match the statements in **Column I** with those in **Column II** (2010)
[Note: here z is a set of points taking values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z. **]**

Column I

(A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is in or equals
(B) The set of points z satisfying $|z + 4| + |z - 4| = 10$ is in or equals
(C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is in or equals
(D) If $|w| = 1$, then the set of points $z = w - \frac{1}{w}$ is in or equals

Column II

(p) an ellipse with eccentricity $\frac{4}{5}$
(q) the set of points z satisfying Im $z = 0$
(r) the set of points z satisfying $|\text{Im } z| \leq 1$
(s) the set of points z satisfying $|\text{Re } z| < 2$
(t) the set of points z satisfying $|z| \leq 3$

3) Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \ldots, 9$. (*JEE Adv*. 2014)

List I

P. For each z_k there exists a z_j such that $z_k \cdot z_j = 1$
Q. There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution in the set of complex numbers
R. $\frac{|1-z_1||1-z_2|\ldots|1-z_9|}{10}$ equals
S. $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals

List II

1. True
2. False
3. 1
4. 2

	P	Q	R	S		P	Q	R	S
(a)	1	2	4	3	(b)	2	1	3	4
(c)	1	2	3	4	(d)	2	1	4	3

II. Comprehension Based Questions

A. *Passage-2*

Let $S = S_1 \cap S_2 \cap S_3$ where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\}$$

$$S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$$

and $S_3 = \{z \in \mathbb{C} : \text{Re } z > 0\}$

4) Area of S = (*JEE Adv*. 2013)

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$ (c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

5) $\min_{z \in S} |1 - 3i - z| =$ (*JEE Adv*. 2013)

(a) $\frac{2-\sqrt{3}}{2}$
(b) $\frac{2+\sqrt{3}}{2}$
(c) $\frac{3-\sqrt{3}}{2}$
(d) $\frac{3+\sqrt{3}}{2}$

III. Integer Value Correct Type

1) If z is any complex number satisfying $|z - 3 - 2i| < 2$, then the minimum value of $|2z - 6 + 5i|$ is (2011)
2) Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that: (2011)

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z$$

Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is

3) For any integer k, let $a_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12}|a_{k+1}-a_k|}{\sum_{k=1}^{3}|a_{4k-1}-a_{4k-2}|}$ is (*JEE Adv*. 2015)
4) Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$\left\{\left|a + b\omega + c\omega^2\right|^2 : a, b, c \text{ distinct non-zero integers}\right\}$
equals ______. (*JEE Adv.* 2019)