

3.2.23

AI24BTECH11031 - Shivram S

Question:

A triangle ABC can be constructed in which $AB = 5\text{cm}$, $\angle A = 45^\circ$ and $BC + AC = 5\text{cm}$.

Solution:

Let $a + b = K$. Using the cosine formula in ΔABC ,

$$a^2 = b^2 + c^2 - 2bc \cos A \quad (0.1)$$

$$\implies (K - b)^2 = b^2 + c^2 - 2bc \cos A \quad (0.2)$$

$$\implies b = \frac{K^2 - c^2}{2(K - c \cos A)} \quad (0.3)$$

The coordinates of ΔABC can be expressed as

$$\mathbf{A} = \mathbf{0}, \mathbf{B} = \begin{pmatrix} c \\ 0 \end{pmatrix}, \mathbf{C} = b \begin{pmatrix} \cos A \\ \sin A \end{pmatrix} \quad (0.4)$$

By substituting values, we get

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.5)$$

But since $\mathbf{A} = \mathbf{C}$, such a triangle can not be constructed.