## **Question:**

If the sum of two unit vectors is a unit vector, peove that the magnitude of their difference is  $\sqrt{3}$ 

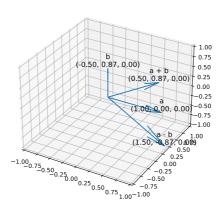
## **Solution:**

Let the two unit vectors be a and b

$$\|\mathbf{a} + \mathbf{b}\|^2 = 1$$
$$(\mathbf{a} + \mathbf{b})^{\mathsf{T}} (\mathbf{a} + \mathbf{b}) = 1$$
$$\mathbf{a}^{\mathsf{T}} \mathbf{a} + \mathbf{b}^{\mathsf{T}} \mathbf{b} + 2\mathbf{a}^{\mathsf{T}} \mathbf{b} = 1$$
$$\mathbf{a}^{\mathsf{T}} \mathbf{b} = \frac{-1}{2}$$

Hence,

$$||\mathbf{a} + \mathbf{b}|| = \sqrt{(\mathbf{a} - \mathbf{b})^{\top} (\mathbf{a} - \mathbf{b})}$$
$$= \sqrt{\mathbf{a}^{\top} \mathbf{a} + \mathbf{b}^{\top} \mathbf{b} - 2\mathbf{a}^{\top} \mathbf{b}}$$
$$= \sqrt{1 + 1 - 2 \cdot \frac{-1}{2}}$$
$$= \sqrt{3}$$



1