2024-January Session-01-31-2024-shift-1 1-15

AI24BTECH11031 - Shivram S

1) For $0 < c < b < a$, let $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ and $\alpha \neq 1$ be one of its roots. Then, among the two statements
(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c
(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

   a) Both (I) and (II) are true
   b) Neither (I) nor (II) is true
   c) Only (II) is true
   d) Only (I) is true

2) Let a be the sum of all coefficients in the expansion of $\left(1-2x+2x^2\right)^{2023}\left(304x^2+2x^3\right)^{2024}$ and $b = \lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where $c, d, e \in R$, then $d : c : e$ equals

   a) $2:1:4$
   b) $4:1:4$
   c) $1:2:4$
   d) $1:1:4$

3) If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

   a) $7\sqrt{\frac{2}{5}} - \frac{8}{3}$
   b) $14\sqrt{\frac{2}{5}} - \frac{4}{3}$
   c) $14\sqrt{\frac{2}{5}} - \frac{16}{3}$
   d) $7\sqrt{\frac{2}{5}} + \frac{8}{3}$

4) If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle C is

   a) $\sqrt{20}$
   b) 4
   c) 6
   d) $3\sqrt{2}$

5) The area of the region

$$\left\{(x,y) : y^2 \leq 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \neq 3\right\}$$

is

a) $\frac{16}{3}$ b) $\frac{64}{3}$ c) $\frac{8}{3}$ d) $\frac{32}{3}$

6) If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ and $(fof)(x) = g(x)$, where $g : \mathbb{R} - \left\{\frac{2}{3}\right\} \to \mathbb{R} - \left\{\frac{2}{3}\right\}$ then $(gogog)(4)$ is equal to

a) $-\frac{19}{20}$ b) $\frac{19}{20}$ c) -4 d) 4

7) $\lim\limits_{x\to 0} \frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

a) is equal to -1 b) does not exist c) is equal to 1 d) is equal to 2

8) If the system of linear equations

$$x - 2y + z = -4$$
$$2x + \alpha y + 3z = 5$$
$$3x - y + \beta z = 3$$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

a) 60 b) 64 c) 54 d) 58

9) The solution curve of the differential equation $y\frac{dx}{dy} = x(\log_e x - \log_e y + 1), x > 0, y > 0$ passing through the point $(e, 1)$ is

a) $\left|\log_e \frac{y}{x}\right| = x$ b) $\left|\log_e \frac{y}{x}\right| = y^2$ c) $\left|\log_e \frac{x}{y}\right| = y$ d) $\left|\log_e \frac{x}{y}\right| = y + 1$

10) Let $\alpha,\beta,\gamma,\delta \in Z$ and let $A(\alpha,\beta)$, $B(1,0)$, $C(\gamma,\delta)$ and $D(1,2)$ be the vertices of a parallelogram $ABCD$. If $AB = 10$ and the points A and C lie on the line $3y = 2x+1$, then $2(\alpha+\beta+\gamma+\delta)$ is equal to

a) 10 b) 5 c) 12 d) 8

11) Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{(\tan x) + y}{\sin x(\sec x - \sin x \tan x)}, x \in \left(0, \frac{\pi}{2}\right)$$

satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then $y\left(\frac{\pi}{3}\right)$ is

a) $\sqrt{3}\left(2 + \log_e \sqrt{3}\right)$ c) $\sqrt{3}(1 + 2\log_e 3)$
b) $\frac{\sqrt{3}}{2}(2 + \log_e 3)$ d) $\sqrt{3}(2 + \log_e 3)$

12) Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vector $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot \left(\hat{i} - \hat{j} - \hat{k}\right)$ is equal to

a) 24 b) 36 c) 28 d) 32

13) The sum of the series $\frac{1}{1-3\cdot 1^2+1^4}+\frac{2}{1-3\cdot 2^2+2^4}+\frac{3}{1-3\cdot 3^2+3^4}+\ldots$ upto 10 terms is

a) $\frac{45}{109}$ b) $-\frac{45}{109}$ c) $\frac{55}{109}$ d) $-\frac{55}{109}$

14) The distance of the point $Q(0,2,-2)$ from the line passing through the point $P(5,-4,3)$ and perpendicular to the lines $\vec{r}=\left(-3\hat{i}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+5\hat{k}\right), \lambda \in \mathbb{R}$ and $\vec{r}=\left(\hat{i}-2\hat{j}+\hat{k}\right)+\mu\left(-\hat{i}+3\hat{j}+2\hat{k}\right), \mu \in \mathbb{R}$ is

a) $\sqrt{86}$ b) $\sqrt{20}$ c) $\sqrt{54}$ d) $\sqrt{74}$

15) For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\pi$ and $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta)=3\alpha\beta$ then γ is equal to

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ d) $\sqrt{3}$