3.3.7

AI24BTECH11031 - Shivram S

Question:

Write the steps of construction for drawing a ΔABC in which $BC = 8cm$, $\angle B = 45^\circ$ and $\angle C = 30^\circ$.

Solution:

Mark points $\mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix}8\\0\end{pmatrix}$.

We know that the coordinates of $\mathbf{A}$ are

$$\mathbf{A} = \begin{pmatrix}c\cos B\\ c\sin B\end{pmatrix} = \begin{pmatrix}8 - b\cos C\\ b\sin C\end{pmatrix} \tag{0.1}$$

So,

$$\begin{pmatrix}\cos C & \cos B\\ -\sin C & \sin B\end{pmatrix}\begin{pmatrix}b\\c\end{pmatrix} = \begin{pmatrix}8\\0\end{pmatrix} \tag{0.2}$$

By performing row reduction,

$$\left(\begin{array}{cc|c}\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} & 8\\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0\end{array}\right) \longleftrightarrow \left(\begin{array}{cc|c}1 & 0 & \frac{16}{\sqrt{3}+1}\\ 0 & 1 & \frac{8\sqrt{2}}{\sqrt{3}+1}\end{array}\right) \tag{0.3}$$

$$b = \frac{16}{\sqrt{3}+1}, c = \frac{8\sqrt{2}}{\sqrt{3}+1} \tag{0.4}$$

Hence,

$$\mathbf{A} = \begin{pmatrix}c\cos B\\ c\sin B\end{pmatrix} = \begin{pmatrix}\frac{8}{\sqrt{3}+1}\\ \frac{8}{\sqrt{3}+1}\end{pmatrix} \tag{0.5}$$

ΔABC is the required triangle.

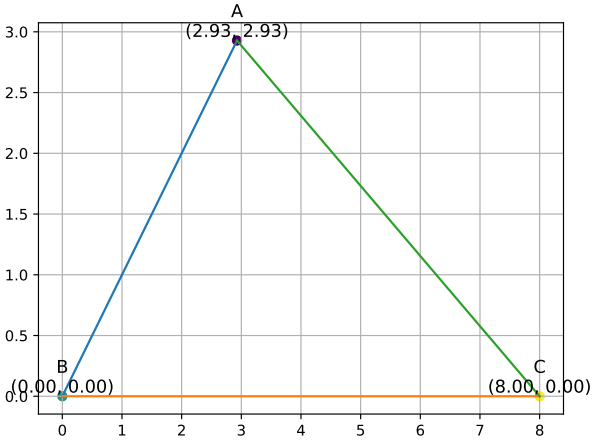

Fig. 0.1: Triangle ABC where $BC = 8cm$, $\angle B = 45°$ and $\angle C = 30°$