

Assignment 1

AI24BTECH11031 - Shivram S

I. MATCH THE FOLLOWING

- 1) $z \neq 0$ is a complex number (1992 – 2 marks)

Column I

- (A) $\operatorname{Re} z = 0$
(B) $\operatorname{Arg} z = \frac{\pi}{4}$

Column II

- (p) $\operatorname{Re} z^2 = 0$
(q) $\operatorname{Im} z^2 = 0$
(r) $\operatorname{Re} z^2 = \operatorname{Im} z^2$

- 2) Match the statements in **Column I** with those in **Column II** (2010)

[Note: here z is a set of points taking values in the complex plane and $\operatorname{Im} z$ and $\operatorname{Re} z$ denote, respectively, the imaginary part and the real part of z .]

Column I

- (A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is in or equals
(B) The set of points z satisfying $|z + 4| + |z - 4| = 10$ is in or equals
(C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is in or equals
(D) If $|w| = 1$, then the set of points $z = w - \frac{1}{w}$ is in or equals

Column II

- (p) an ellipse with eccentricity $\frac{4}{5}$
(q) the set of points z satisfying $\operatorname{Im} z = 0$
(r) the set of points z satisfying $|\operatorname{Im} z| \leq 1$
(s) the set of points z satisfying $|\operatorname{Re} z| < 2$
(t) the set of points z satisfying $|z| \leq 3$
3) Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i \sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \dots, 9$. (JEE Adv. 2014)

List I

- P. For each z_k there exists a z_j such that $z_k \cdot z_j = 1$
Q. There exists a $k \in \{1, 2, \dots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution in the set of complex numbers

- R. $\frac{|1-z_1||1-z_2|\dots|1-z_9|}{10}$ equals

- S. $1 - \sum_{k=1}^9 \cos\left(\frac{2k\pi}{10}\right)$ equals

List II

1. True

2. False

3. 1

4. 2

	P	Q	R	S		P	Q	R	S
(a)	1	2	4	3	(b)	2	1	3	4
(c)	1	2	3	4	(d)	2	1	4	3

II. COMPREHENSION BASED QUESTIONS

A. Passage-2

Let $S = S_1 \cap S_2 \cap S_3$ where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\}$$

$$S_2 = \left\{z \in \mathbb{C} : \operatorname{Im} \left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i} \right] > 0 \right\}$$

$$\text{and } S_3 = \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$$

- 4) Area of $S =$ (JEE Adv. 2013)

- a) $\frac{10\pi}{3}$ b) $\frac{20\pi}{3}$ c) $\frac{16\pi}{3}$ d) $\frac{32\pi}{3}$

- 5) $\min_{z \in S} |1 - 3i - z| =$ (JEE Adv. 2013)

- a) $\frac{2-\sqrt{3}}{2}$ c) $\frac{3-\sqrt{3}}{2}$
b) $\frac{2+\sqrt{3}}{2}$ d) $\frac{3+\sqrt{3}}{2}$

III. INTEGER VALUE CORRECT TYPE

- 1) If z is any complex number satisfying $|z - 3 - 2i| < 2$, then the minimum value of $|2z - 6 + 5i|$ is (2011)
2) Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that: (2011)

$$a + b + c = x$$

$$a + b\omega + c\omega^2 = y$$

$$a + b\omega^2 + c\omega = z$$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

- 3) For any integer k , let $a_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12} |a_{k+1} - a_k|}{\sum_{k=1}^3 |a_{4k-1} - a_{4k-2}|}$ is (JEE Adv. 2015)
4) Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$\left\{ \left| a + b\omega + c\omega^2 \right|^2 : a, b, c \text{ distinct non-zero integers} \right\}$
equals _____. (JEE Adv. 2019)