Assignment 1

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I. Match The Following

Question 1: $z \neq 0$ is a complex number (1992 – 2 *marks*)

	Column I		Column II
(A)	Re $z = 0$	(p)	Re $z^2 = 0$
(B)	Arg $z = \frac{\pi}{4}$	(q)	Im $z^2 = 0$
		(r)	Re $z^2 =$ Im z^2

Question 2: Match the statements in **Column I** with those in **Column II** (2010)
[Note: here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z. **]**

Column I

(A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is contained in or equal to
(B) The set of points z satisfying $|z+4|+|z-4| = 10$ is contained in or equal to
(C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to
(D) If $|w| = 1$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to

Column II

(p) an ellipse with eccentricity $\frac{4}{5}$
(q) the set of points z satisfying Im $z = 0$
(r) the set of points z satisfying $|\text{Im } z| \leq 1$
(s) the set of points z satisfying $|\text{Re } z| < 2$
(t) the set of points z satisfying $|z| \leq 3$

Question 3: Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \ldots, 9$. (*JEE Adv.* 2014)

List I

P. For each z_k there exists a z_j such that $z_k \cdot z_j = 1$
Q. There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution in the set of complex numbers
R. $\frac{|1-z_1||1-z_2|\ldots|1-z_9|}{10}$ equals
S. $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals

List II

1. True
2. False
3. 1
4. 2

	P	Q	R	S		P	Q	R	S
(a)	1	2	4	3	(b)	2	1	3	4
(c(	1	2	3	4	(d)	2	1	4	3

II. Comprehension Based Questions

A. *Passage-2*

Let $S = S_1 \cap S_2 \cap S_3$ where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\}$$
$$S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$$
$$\text{and } S_3 = \{z \in \mathbb{C} : \text{Re } z > 0\}$$

Question 4: Area of S = (*JEE Adv.* 2013)

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$ (c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

Question 5: $\min_{z \in S} |1 - 3i - z| =$ (*JEE Adv.* 2013)

(a) $\frac{2-\sqrt{3}}{2}$ (b) $\frac{2+\sqrt{3}}{2}$
(b) $\frac{3-\sqrt{3}}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

III. Integer Value Correct Type

Question 1: If z is any complex number satisfying $|z - 3 - 2i| < 2$, then the minimum value of $|2z - 6 + 5i|$ is (2011)

Question 2: Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that: (2011)

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z$$

Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is

Question 3: For any integer k, let $a_k = \cos(\frac{k\pi}{7}) + i\sin(\frac{k\pi}{7})$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12} |a_{k+1}-a_k|}{\sum_{k=1}^{3} |a_{4k-1}-a_{4k-2}|}$ is (*JEE Adv.* 2015)

Question 4: Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\left\{|a + b\omega + c\omega^2|^2 : a, b, c \text{ distinct non-zero integers}\right\}$ equals ______. (*JEE Adv.* 2019)