Matrices in Geometry
Question 3.3.7

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November 5, 2024

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Problem Statement

Write the steps of construction for drawing a ΔABC in which $BC = 8$ cm, $\angle B = 45^\circ$ and $\angle C = 30^\circ$.

Information Table

Variable	Description	Formula
BC	Length of the side BC	$BC = 8$ cm
B	Measure of the angle at vertex **B**	$B = 45^\circ$
C	Measure of the angle at vertex **C**	$C = 30^\circ$

Linear Equations

Let us assume that $\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$.

We know that

$$\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC} \implies \begin{pmatrix} c\cos B \\ c\sin B \end{pmatrix} + \begin{pmatrix} b\cos C \\ -b\sin C \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{3.1}$$

$$\tag{3.2}$$

This can be written as a pair of linear equations in b and c.

$$\begin{pmatrix} \cos C & \cos B \\ -\sin C & \sin B \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{3.3}$$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{3.4}$$

Row Reduction

By performing row reduction,

$$\left(\begin{array}{cc|c} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} & 8 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|c} \frac{\sqrt{3}+1}{2} & 0 & 8 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \end{array}\right) \tag{3.5}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{2R_1}{\sqrt{3}+1}} \left(\begin{array}{cc|c} 1 & 0 & \frac{16}{\sqrt{3}+1} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \end{array}\right) \tag{3.6}$$

$$\xleftrightarrow{R_2 \leftarrow 2R_2 + R_1} \left(\begin{array}{cc|c} 1 & 0 & \frac{16}{\sqrt{3}+1} \\ 0 & \sqrt{2} & \frac{16}{\sqrt{3}+1} \end{array}\right) \tag{3.7}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{\sqrt{2}}} \left(\begin{array}{cc|c} 1 & 0 & \frac{16}{\sqrt{3}+1} \\ 0 & 1 & \frac{8\sqrt{2}}{\sqrt{3}+1} \end{array}\right) \tag{3.8}$$

Solving for A

Thus,

$$b = \frac{16}{\sqrt{3}+1}, c = \frac{8\sqrt{2}}{\sqrt{3}+1} \tag{3.9}$$

We can now substitute values from (3.9) to obtain

$$\mathbf{A} = \begin{pmatrix} c\cos B \\ c\sin B \end{pmatrix} = \begin{pmatrix} \frac{8}{\sqrt{3}+1} \\ \frac{8}{\sqrt{3}+1} \end{pmatrix} \tag{3.10}$$

C Code I

The following C program was used to calculate the value of **A**:

```c
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
#include "libs/matfun.h"

int main(void) {
    FILE *f = fopen("output.dat", "w");

    // Given values
    double a = 8;
    double B = 45 * M_PI / 180;
    double C = 30 * M_PI / 180;

    double ratio = sin(B) / sin(C); // b / c
    double c = a / (cos(B) + ratio * cos(C));

    // Vertex A
```

C Code II

```
double **vA = createMat(2, 1);
vA[0][0] = c * sin(B);
vA[1][0] = c * cos(B);

// Vertex B
double **vB = createMat(2, 1);
vB[0][0] = 0;
vB[1][0] = 0;

// Vertex C
double **vC = createMat(2, 1);
vC[0][0] = a;
vC[1][0] = 0;

// Write the coordinates to a file
for (int i = 0; i < 2; i++) {
    fprintf(f, "%lf\n", vA[i][0]);
}
```

C Code III

```
    for (int i = 0; i < 2; i++) {
        fprintf(f, "%lf\n", vB[i][0]);
    }
    for (int i = 0; i < 2; i++) {
        fprintf(f, "%lf\n", vC[i][0]);
    }

    return 0;
}
```

Python Code I

The following Python program was used to plot the triangle:

```
import matplotlib.pyplot as plt
import numpy as np

# Generates a line from point A to point B
def line_gen(A,B):
  len =10
  dim = A.shape[0]
  x_AB = np.zeros((dim,len))
  lam_1 = np.linspace(0,1,len)
  for i in range(len):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB


# Points A, B and C
A, B, C = np.loadtxt("output.dat").reshape(-1, 2, 1)
```

Python Code II

```
# Plot AB, BC, and CA
x_AB = line_gen(A, B)
x_BC = line_gen(B, C)
x_CA = line_gen(C, A)
plt.plot(x_AB[0,:], x_AB[1,:], label="AB")
plt.plot(x_BC[0,:], x_BC[1,:], label="BC")
plt.plot(x_CA[0,:], x_CA[1,:], label="CA")

# Plot the points
colors = np.arange(1, 4)
p = np.block([A, B, C])
plt.scatter(p[0, :], p[1, :], c=colors)

# Labels and their coordinates
points = {
    'A': A,
    'B': B,
```

Python Code III

```
    'C': C,
}

# Label the points
for label, point in points.items():
    plt.text(
        point[0, 0], point[1, 0],
        f"{label}\n({point[0,0]:.2f}, {point[1,0]:.2f})",
        fontsize=12, ha="center", va="bottom"
    )

# Enable grid
plt.grid()

# Save the figure
plt.savefig('../figs/fig.pdf')
```

Plot

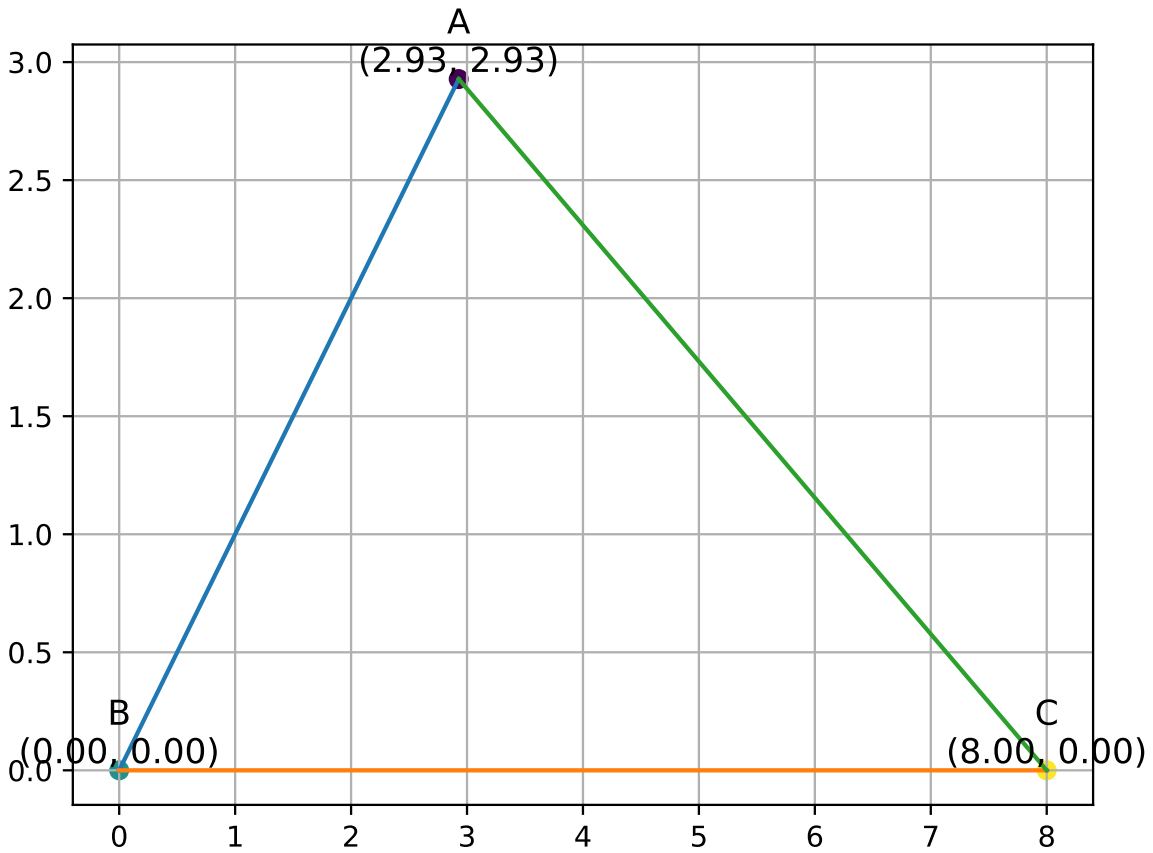

Figure: Triangle ABC where $BC = 8cm$, $\angle B = 45°$ and $\angle C = 30°$