Matrices in Geometry: Q. 8.4.24

AI24BTECH11031 - Shivram S

Question

The altitude of a right angled triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Variables Used

Variable	**Description**	**Value**
BC	Hypotenuse of the triangle	13 cm
AB	Base of the triangle	x cm
AC	Altitude of the triangle	$x - 7$ cm

Table: Variables Used

Conic Form

Let the length of the base be x cm. The altitude of the triangle is 7 cm less than its base, i.e., $x - 7$ cm. By Pythagoras' Theorem

$$AB^2 + AC^2 = BC^2 \tag{1}$$

$$x^2 + (x-7)^2 = 13^2 \tag{2}$$

$$2x^2 - 14x - 120 = 0 \tag{3}$$

$$x^2 - 7x - 60 = 0 \tag{4}$$

The equation $y = x^2 - 7x - 60$ can be expressed as a conic

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{5}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{7}{2} \\ -\frac{1}{2} \end{pmatrix}, f = -60 \tag{6}$$

Intersection with x-axis

To find the roots of the equation, we find the points of intersection of the conic with the x-axis

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{7}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

The values of k are given by

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g\left(\mathbf{h} \right) \left(\mathbf{m}^\top \mathbf{V} \mathbf{m} \right)} \right) \tag{9}$$

$$= \frac{1}{1} \left(\frac{7}{2} \pm \sqrt{\left(\frac{7}{2} \right)^2 + 60} \right) \tag{10}$$

$$k_1 = -5, k_1 = 12 \tag{11}$$

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 12 \\ 0 \end{pmatrix} \tag{12}$$

Hence the solutions of the equation are $x = -5$ and $x = 12$. We reject $x = -5$ as the length of the side can't be negative. Hence, the lengths of the sides are

$$AB = 12\ cm \tag{13}$$

$$AC = 7\ cm \tag{14}$$

$$BC = 13\ cm \tag{15}$$

Figure I

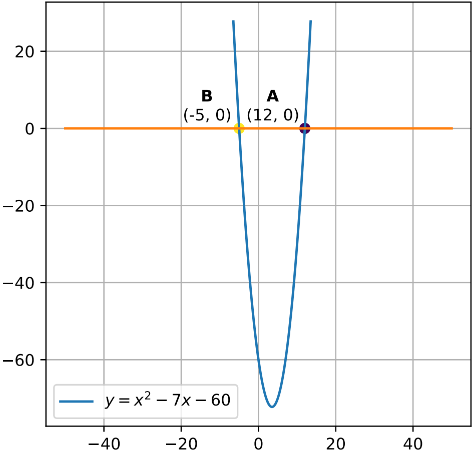

Figure: Points of intersection of $y = x^2 - 7x - 60$ with x-axis

Python Code I

```
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA

import sys
sys.path.insert(0, 'CoordGeo')

#local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import *

#setting up plot
fig = plt.figure()
ax = fig.add_subplot(111, aspect='equal')
len = 100
y = np.linspace(-10,10,len)
```

Python Code II

```
#conic parameters
V = np.array(([1,0],[0,0]))
u = -7/2*e1 - 1/2*e2
f = -60

n,c,F,O,lam,P,e = conic_param(V,u,f)
flen = parab_param(lam,P,u)
x = parab_gen(y,flen)
xStandard = np.block([[x],[y]])

#Affine conic generation
Of = O.flatten()
xActual = P@xStandard + Of[:,np.newaxis]

n = np.array([0, 1]).reshape(-1, 1)
c = 0
m,h = param_norm(n, c)

q = chord(V,u,f,m,h)
```

Python Code III

```
A = q[:, 0]
B = q[:, 1]

xAxis = line_norm(n, c, -50, 50)
plt.plot(xAxis[0,:], xAxis[1,:])

#plotting
plt.plot(xActual[0,:],xActual[1,:],label='$y = x^2 - 7x - 60$')

colors = np.arange(1,3)
#Labeling the coordinates
tri_coords = q
plt.scatter(tri_coords[0,:], tri_coords[1,:], c=colors)
vert_labels = ['$\\mathbf{A}$','$\\mathbf{B}$']
for i, txt in enumerate(vert_labels):
# plt.annotate(txt, # this is the text
    plt.annotate(f'{txt}\n({tri_coords[0,i]:.0f}, {tri_coords[1,i]:.0f})',
```

Python Code IV

```
                    (tri_coords[0,i], tri_coords[1,i]), # this
                        is the point to label
                    textcoords="offset␣points", # how to
                        position the text
                    xytext=(-20,5), # distance from text to
                        points (x,y)
                    ha='center') # horizontal alignment can be
                        left, right or center

plt.legend()
plt.grid() # minor

plt.savefig('../figs/parabola.pdf')
```

Figure II

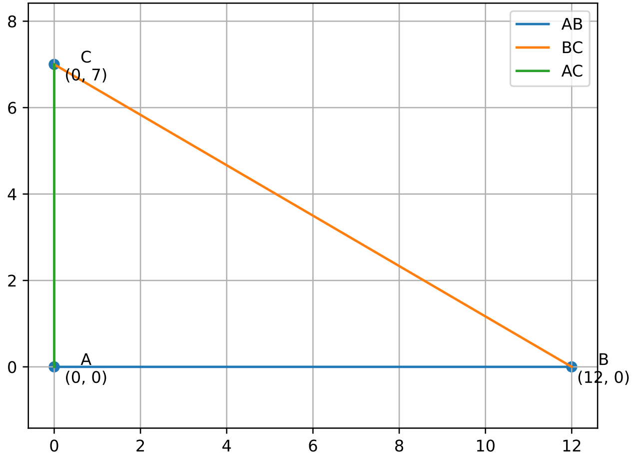

Figure: Triangle with sides $AB = 12$ cm, $AC = 7$ cm, and $BC = 13$ cm

Python Code I

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import numpy as np
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import sys
sys.path.insert(0, 'CoordGeo')

#local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import *

#setting up plot
fig = plt.figure()
ax = fig.add_subplot(111, aspect='equal')

#vertices
A = np.array([0, 0]).reshape(-1, 1)
```

Python Code II

```python
B = np.array([12, 0]).reshape(-1, 1)
C = np.array([0, 7]).reshape(-1, 1)

xAB = line_gen(A, B)
plt.plot(xAB[0,:], xAB[1,:], label="AB")
xBC = line_gen(B, C)
plt.plot(xBC[0,:], xBC[1,:], label="BC")
xAC = line_gen(A, C)
plt.plot(xAC[0,:], xAC[1,:], label="AC")

points = np.hstack([A, B, C])
plt.scatter(points[0,:], points[1,:])
verts = [A, B, C]
labels = ['A', 'B', 'C']
for i in range(len(verts)):
    x, y = verts[i][0, 0], verts[i][1, 0]
    plt.annotate(f"{labels[i]}\n({x}, {y})",
                 (x, y),
                 textcoords="offset points",
```

Python Code III

```
                xytext=(20, -10),
                ha="center")

plt.legend()
plt.axis('equal')
plt.grid()
plt.savefig('../figs/triangle.pdf')
```