8.4.24

AI24BTECH11031 - Shivram S

Question: The altitude of a right-angled triangle is 7 cm less than its base. If the hypotenuse if 13 cm, find the other two sides.

Solution:

Variable	Description	Value
BC	Hypotenuse of the triangle	13 cm
AB	Base of the triangle	x cm
AC	Altitude of the triangle	$x-7$ cm

TABLE 0: Variables Used

Let the length of the base be x cm. The altitude of the triangle is 7 cm less than its base, i.e., $x-7$ cm. By Pythagoras' Theorem

$$AB^2 + AC^2 = BC^2 \tag{1.1}$$
$$x^2 + (x-7)^2 = 13^2 \tag{1.2}$$
$$2x^2 - 14x - 120 = 0 \tag{1.3}$$
$$x^2 - 7x - 60 = 0 \tag{1.4}$$
$$\tag{1.5}$$

The equation $y = x^2 - 7x - 60$ can be expressed as a conic

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1.6}$$
$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{7}{2} \\ -\frac{1}{2} \end{pmatrix}, f = -60 \tag{1.7}$$

To find the roots of the equation, we find the points of intersection of the conic with the x-axis

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{1.8}$$
$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.9}$$

The values of k are given by

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{1.10}$$

$$= \frac{1}{1} \left(\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 + 60} \right) \tag{1.11}$$

$$k_1 = -5, k_1 = 12 \tag{1.12}$$

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 12 \\ 0 \end{pmatrix} \tag{1.13}$$

Hence the solutions of the equation are $x = -5$ and $x = 12$. We reject $x = -5$ as the length of the side can't be negative. Hence, the lengths of the sides are

$$AB = 12 \ cm \tag{1.14}$$

$$AC = 7 \ cm \tag{1.15}$$

$$BC = 13 \ cm \tag{1.16}$$

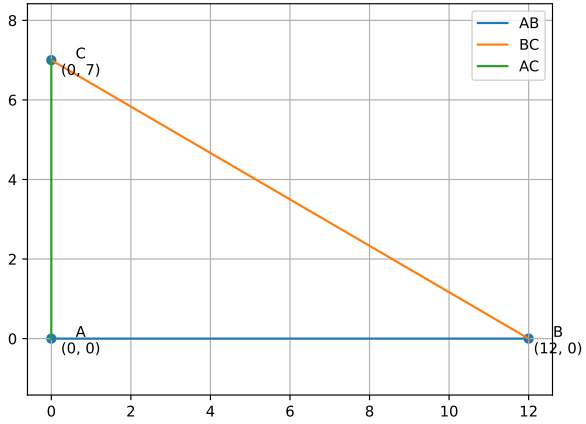

Fig. 1.1: Triangle with sides $AB = 12$ cm, $AC = 7$ cm, and $BC = 13$ cm