Assignment 1

AI24BTECH11031 - Shivram S

I. Match The Following

1) $z \neq 0$ is a complex number $(1992 - 2\ marks)$

Column I	Column II
(A) Re $z = 0$	(p) Re $z^2 = 0$
(B) Arg $z = \frac{\pi}{4}$	(q) Im $z^2 = 0$
	(r) Re $z^2 = $ Im z^2

2) Match the statements in **Column I** with those in **Column II** (2010)
[Note: here z is a set of points taking values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z. **]**
Column I
(A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is in or equals
(B) The set of points z satisfying $|z + 4| + |z - 4| = 10$ is in or equals
(C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is in or equals
(D) If $|w| = 1$, then the set of points $z = w - \frac{1}{w}$ is in or equals
Column II
(p) an ellipse with eccentricity $\frac{4}{5}$
(q) the set of points z satisfying Im $z = 0$
(r) the set of points z satisfying $|\text{Im } z| \leq 1$
(s) the set of points z satisfying $|\text{Re } z| < 2$
(t) the set of points z satisfying $|z| \leq 3$

3) Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i \sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \ldots, 9$. $(JEE\ Adv.\ 2014)$
List I
P. For each z_k there exists a z_j such that $z_k \cdot z_j = 1$
Q. There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution in the set of complex numbers
R. $\frac{|1-z_1||1-z_2|\ldots|1-z_9|}{10}$ equals
S. $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals
List II
1. True
2. False
3. 1
4. 2

	P	Q	R	S		P	Q	R	S
(a)	1	2	4	3	(b)	2	1	3	4
(c)	1	2	3	4	(d)	2	1	4	3

II. Comprehension Based Questions

A. *Passage-2*

Let $S = S_1 \cap S_2 \cap S_3$ where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\}$$

$$S_2 = \left\{ z \in \mathbb{C} : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0 \right\}$$

and $S_3 = \{z \in \mathbb{C} : \text{Re } z > 0\}$

4) Area of S = $(JEE\ Adv.\ 2013)$

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$ (c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

5) $\min_{z \in S} |1 - 3i - z| =$ $(JEE\ Adv.\ 2013)$

(a) $\frac{2-\sqrt{3}}{2}$ (c) $\frac{3-\sqrt{3}}{2}$
(b) $\frac{2+\sqrt{3}}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

III. Integer Value Correct Type

1) If z is any complex number satisfying $|z - 3 - 2i| < 2$, then the minimum value of $|2z - 6 + 5i|$ is (2011)
2) Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that: (2011)

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z$$

Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is
3) For any integer k, let $a_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12} |a_{k+1} - a_k|}{\sum_{k=1}^{3} |a_{4k-1} - a_{4k-2}|}$ is $(JEE\ Adv.\ 2015)$
4) Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$\left\{ \left| a + b\omega + c\omega^2 \right|^2 : a, b, c \text{ distinct non-zero integers} \right\}$ equals _____. (*JEE Adv.* 2019)

IV. Fill in The Blanks

1. Let $p\lambda^2 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$ be an identity in λ where p, q, r, s and t are constants. Then the value of t is _____. (1981 – 2 *Marks*)
2. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is _____. (1981 – 2 *Marks*)
3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive is _____. (1982 – 2 *Marks*)
4. Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$ the other two roots are _____ and _____. (1983 – 2 *Marks*)
5. The system of equations

$$\lambda x + y + z = 0$$
$$-x + \lambda y + z = 0$$
$$-x - y + \lambda x = 0$$

Will have a non-zero solution if real values of λ are given by _____. (1984 – 2 *Marks*)

6. The value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is _____. (1988 – 2 *Marks*)
7. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is _____. (1993 – 2 *Marks*)

V. True / False

1. The determinants $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal. (1983 – 1 *Mark*)
2. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$ then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, and $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ must be congruent. (1985 – 1 *Mark*)

VI. MCQs with One Corrrect Answer

1. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1. Then (1981 – 2 *Marks*)
   (a) C is empty
   (b) B has as many elements as C
   (c) $A = B \cup C$
   (d) B has twice as many elements as C
2. If $\omega (\neq 1)$ is a cube root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$ (1995*S*)
   (a) 0 (b) 1 (c) i (d) ω
3. Let a, b, c be the real numbers. Then following system of equations in x, y and z (1995*S*)
   $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has
   (a) no solution
   (b) unique solution
   (c) infinitely many solutions
   (d) finitely many solutions
4. If A and B are square matrices of equal degree, then which one is correct among the followings? (1995*S*)
   (a) $A + B = B + A$
   (b) $A + B = A - B$
   (c) $A - B = B - A$
   (d) $AB = BA$
5. The parameter on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is (1997 – 2 *Marks*)
   (a) a (b) p (c) d (d) x
6. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to (1999 – 2 *Marks*)

(a) 0 (b) 1 (c) 100 (d) -100