Complex Numbers Problems 11-20

Shiv Shankar Dayal

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Problem 11

11. If $z_1 = 2 + 3i$ and $z_2 = 1 + 2i$, then find the value of z_1/z_2.

Solution of Problem 11

Solution: $z_1/z_2 = \frac{2+3i}{1+2i} = \frac{(2+3i)(1-2i)}{(1+2i)(1-2i)}$

$= \frac{2-4i+3i+6}{1^2+2^2} = \frac{8-i}{5}$

Problem 12

12. If $z_1 = 9y^2 - 4 - i10x$ and $z_2 = 8y^2 - 20i$ such that $z_1 = \overline{z_2}$, then find $z = x + iy$.

Solution of Problem 12

Solution: Given $z_1 = \overline{z_2} \Rightarrow 9y^2 - 4 - i10x = 8y^2 + 20i$

$\Rightarrow (y^2 - 4) - i10(x + 2) = 0$

Comparing real and imaginary parts, we get

$y^2 - 4 = 0 \Rightarrow y = \pm 2$ and $x + 2 = 0 \Rightarrow x = -2$

Thus, $z = x + iy = -2 \pm 2i$

Problem 13

13. Find z if $|z+1| = z + 2(1+i)$, where $z \in C$.

Solution of Problem 13

Solution: Let $z = x + iy, \Rightarrow |x + 1 + iy| = (x + 2) + i(y + 2)$

$\Rightarrow \sqrt{(x+1)^2 + y^2} = (x + 2) + i(y + 2)$

Comparing real and imaginary parts, we get

$y + 2 = 0 \Rightarrow y = -2$ and $(x + 1)^2 + y^2 = (x + 2)^2 \Rightarrow x^2 + 2x + 1 + 4 = x^2 + 4x + 4 \Rightarrow x = \frac{1}{2}$

Thus, $z = \frac{1}{2}(1 - 4i)$

Problem 14

14. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

Solution of Problem 14

Solution: $z = \frac{1+2i}{1-3i} = \frac{(1+2i)(1+3i)}{1^2+3^2}$

$\Rightarrow z = \frac{-1+i}{2} \Rightarrow |z| = -\sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$

$arg(z) = \tan^{-1} -1 \Rightarrow arg(z) = \frac{3\pi}{4}$

Problem 15

15. If $\frac{x-3}{3+i}+\frac{y-3}{3-i}=i$, where $x, y \in R$ then find x and y.

Solution of Problem 15

Solution: Given, $\frac{x-3}{3+i} + \frac{y-3}{3-i} = i(3-i)(3+i)$

$\Rightarrow (x-3)(3-i) + (y-3)(3+i) = 10i$

$\Rightarrow 3x - 9 + i(3-x) + (3y-9) + i(y-3) = 10i$

Comp[aring real and imaginary parts, we get

$3x + 3y - 18 = 0$ and $y - x = 10 \Rightarrow x = -2, y = 8$

Problem 16

16. What is the real part of $(1 + i)^{50}$.

Solution of Problem 16

Solution: $(1+i)^2 = 1 + 2i - i = 2i$

$\Rightarrow (1+i)^{50} = (2i)^{25} = 2^{25} i^{4.6+1} = 2^{25} i$

Thus, real part will be 0.

Problem 17

17. If a complex number is z, such that $z + |z| = 2 + 8i$. Find the value of z.

Solution of Problem 17

Solution: Let $z = x + iy$ then $x + iy + \sqrt{x^2 + y^2} = 2 + 8i$

Comparing real and imaginary parts, we get

$y = 8$ and $x + \sqrt{x^2 + y^2} = 2 \Rightarrow \sqrt{x^2 + y^2} = 2 - x$

$x^2 + 64 = 4 - 4x + x^2 \Rightarrow x = -15 \Rightarrow z = -15 + 8i$

Problem 18

18. Find the sum of sequence $S = i + 2i^2 + 3i^3 + \ldots$ up to 100 terms.

Solution of Problem 18

Solution: $S = i + 2i^2 + 3i^3 + ... + 100i^{100}$

$iS = i^2 + 2i^3 + ... + 99i^{100} + 100i^{101}$

$S(1-i) = i + i^2 + ... + i^{100} - 100i^{101} = \frac{1-i^{101}}{1-1} - 100i^{101}$

$S = \frac{1-i^{101}}{(1-i^2)} - \frac{100i^{101}}{1-i}$

Problem 19

19. Find the value of the sum $\frac{1}{1+i}+\frac{1}{1-i}+\frac{1}{-1+i}+\frac{1}{-1-i}+\frac{2}{1+i}+\frac{2}{1-i}+\frac{2}{-1+i}+\frac{2}{-1-i}+\ldots\frac{n}{1+i}+\frac{n}{1-i}+\frac{n}{-1+i}+\frac{n}{-1-i}$

Solution of Problem 19

Solution: Consider $t_1 = \frac{1}{1+i} + \frac{1}{1-i} + \frac{1}{-1+i} + \frac{1}{-1-i}$

$= \frac{1+i+1-i}{1^2-i^2} + \frac{-1+i-1-i}{(-1)^2-i^2} = \frac{2}{2} + \frac{-2}{2} = 0$

$t_2 = 2\left(\frac{1}{1+i} + \frac{1}{1-i} + \frac{1}{-1+i} + \frac{1}{-1-i}\right) = 0$

Similarly all other terms and sum will be zero.

Problem 20

20. Find the product of real parts of the roots of $z^2 - z - 5 + 5i = 0$

Solution of Problem 20

Solution: Given, $z^2 - z - 5 + 5i = 0 \Rightarrow D = (-1)^2 - 4.1.(-5 + 5i) = 21 - 20i$ and we will need $\sqrt{D}$

$$\sqrt{D} = \sqrt{b^2 - 4ac} = \sqrt{21 - 20i} = \pm\left[\sqrt{\frac{x^2+y^2+x}{2}} - i\sqrt{\frac{x^2+y^2-x}{2}}\right] = \pm(5 - 2i)$$

$z = \frac{1+5-2i}{2}$ or $z = \frac{1-5+2i}{2} \Rightarrow z = 3 - i, i - 2$

Thus, product of real parts $= -2 \times 3 = -6$