Logarithm Problem 21-30

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Problem 21

21. Simplify $\frac{\log_9 11}{\log_5 13} \div \frac{\log_3 11}{\log_{\sqrt{5}} 13}$

Solution of Problem 21

Solution: Given

$$\frac{\log_9 11}{\log_5 13} \div \frac{\log_3 11}{\log_{\sqrt{5}} 13}$$

$$= \frac{\log_{3^2} 11}{\log_5 13} \cdot \frac{\log_{5^{\frac{1}{2}}} 13}{\log_3 11}$$

$$= \frac{\frac{1}{2}\log_3 11}{\log_5 13} \cdot \frac{2\log_5 13}{\log_3 11} = 1$$

Problem 22

22. Simplify $3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$

Solution of Problem 22

Solution: Taking $\log$ with base 10, we get

$$= \sqrt{\log_3 2}\log 3 - \sqrt{\log_2 3}\log 2$$

$$= \sqrt{\frac{\log 2}{\log 3}(\log 3)^2} - \sqrt{\frac{\log 3}{\log 2}(\log 2)^2}$$

$$= \sqrt{\log 2 \log 3} - \sqrt{\log 3 \log 2} = 0$$

Problem 23

23. Find the least integer n such that $7^n > 10^5$, given that $\log_{10} 343 = 2.5353$

Solution of Problem 23

Solution:

$$\log_{10} 343 = 2.5353 \Rightarrow \log_{10} 7^3 = 2.5353 \Rightarrow \log_{10} 7 = 0.8451$$

$$7^n > 10^5 \Rightarrow n \log_{10} 7 > 5 \Rightarrow n > \frac{5}{0.8451}$$

Thus, least value of such integer is 6.

Problem 24

24. If a, b, c are in G.P. then prove that $\log_a x, \log_b x, \log_c x$ are in H.P.

Solution of Problem 24

Solution: Since a, b, c are in G.P. therefore we can write $b^{=}ac$

Taking log, on both sides, we get $2\log b = \log a + \log c$. Thus, $\log a, \log b, \log c$ are in A.P.

$\therefore \frac{1}{\log a}, \frac{1}{\log b}, \frac{1}{\log c}$ are in H.P.

$\therefore \frac{\log x}{\log a}, \frac{\log x}{\log b}, \frac{\log x}{\log c}$ are in H.P.

$\therefore \log_x a, \log_x b, \log_x c$ are in H.P.