Arithmetic Progression
Problems 51 to 60

Shiv Shankar Dayal

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Problem 51

51. In an A.P. if $S_n = t_1 + t_2 + \ldots + t_n$ (n odd), $S_2 = t_2 + t_4 + \ldots + t_{n-1}$, then find the value of S_1/S_2 in terms of n.

Solution: S_1 is an A.P. of n terms but S_2 is an A.P. of $\frac{n-1}{2}$ terms with a common difference of $2d$

$$S_1 = \frac{n}{2}[t_1 + t_n]$$

$$S_2 = \frac{n-1}{2.2}[t_2 + t_{n-1}] = \frac{n-1}{4}[t_1 + t_n]$$

$$\therefore \frac{S_1}{S_2} = \frac{2n}{n-1}$$

Problem 52

52. Find the degree of the equation $(1+x)(1+x^6)(1+x^{11})\dots(1+x^{101})$

Solution of problem 52

Solution: The degree of the equation is $1 + 6 + 11 + \ldots + 101$
The above progression is an A.P. with first term being 1 and common difference being 5

$$101 = 1 + (n-1)5 = 5n - 4 \Rightarrow 5n = 105 \Rightarrow n = 21$$

$$S = \frac{n}{2}[2a + (n-1)d] = \frac{21}{2}[2.1 + (21-1)5] = 21.51 = 1071$$

Problem 53

53. Prove that a sequence is an A.P. if the sum of its terms is of the form $An^2 + Bn$, where A, B are constants.

Solution of problem 53

Solution: Let sum S_n denotes sum for n terms of a sequence $t_1 + t_2 + \ldots + t_n$

$$S_n = An^2 + Bn$$

$$S_{n-1} = A(n-1)^2 + B(n-1)$$

$$t_n = An^2 + Bn - [A(n-1)^2 + B(n-1)]$$

$$t_n = 2An + (B - A)$$

$$t_n - t_{n-1} = 2An + (B - A) - 2A(n-1) - (B - A)$$

$$d = 2A$$

Since commond difference is a constant the given series is an A.P.

Problem 54

54. If the sequence $a_1, a_2, \ldots, a_n$ form an A.P., then prove that
$a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2 = \frac{n}{2n-1}(a_1^2 - a_{2n}^2)$

Solution of problem 54

Solution:

$$a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2 = (a_1 + a_2)(a_1 - a_2) + (a_3 + a_4)(a_3 - a_4) + \ldots + (a_{2n-1} + a_{2n})(a_{2n-1} - a_{2n})$$

$$= -d(a_1 + a_2 + a_3 + a_4 \ldots + a_{2n-1} + a_{2n})$$

$$= \frac{-d.2n}{2}[a_1 + a_{2n}]$$

$$= -nd\frac{(a_1 - a_{2n})^2}{a_1 - a_{2n}}$$

$$= -nd\frac{(a_1 - a_{2n})^2}{-(2n-1)d}$$

$$= \frac{n}{2n-1}(a_1 - a_{2n})^2$$

Problem 55

55. Find the sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots, a_{24}$, if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

Solution of problem 55

Solution: We know that in an A.P. the sum of term equidistant from the beginning and end is always same and is equal to the sum of first and last term, i.e.

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \ldots = 2a + (n-1)d$$

So if an A.P. consists of 24 terms, then

$$a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$$

$$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$$

$$3(a_1 + a_{24}) = 225$$

$$a_1 + a_{24} = 75$$

$$\therefore S_{24} = \frac{n}{2}[a_1 + a_{24}] = 12.75 = 900$$

Problem 56

56. If the arithmetic progression whose common difference is non-zero, the sum of first $3n$ terms is equal to next n terms. Then, find the ratio of sum of first $2n$ terms to the sum of next $2n$ terms.

Solution of problem 56

Solution: Let S_{3n} denote the sum of first $3n$ terms. Let a be first term and d be the common difference. Then,

$$S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$$

$$t_{3n+1} = a + 3nd$$

and

$$t_{4n} = a + (4n-1)d$$

Let S'_n denote the sum of next n terms. Then,

$$S'_n = \frac{n}{2}[t_{3n+1} + t_{4n}]$$

$$S'_n = \frac{n}{2}[2a + (7n-1)d]$$

Given that $S_{3n} = S'_n$

$$\frac{3n}{2}[2a + (3n-1)d] = \frac{n}{2}[2a + (7n-1)d]$$

$$6a + 3(3n-1)d = 2a + (7n-1)d$$

$$4a = (-2n+2)d \Rightarrow 2a = (1-n)d$$

Let S_{2n} be sum of first $2n$ terms and S'_{2n} be sum of next $2n$ terms. Then,

$$S_{2n} = n[2a + (2n-1)d]$$

$$S_{2n} = n[(1-n)d + (2n-1)d] = n^2 d$$

$$S'_{n2} = \frac{2n}{2}[t_{2n+1} + t_{4n}]$$

$$S'_{2n} = n[2a + (6n-1)d] = n[(1-n)d + (6n-1)d] = 5n^2 d$$

$$\therefore \frac{S_{2n}}{S'_{2n}} = \frac{1}{5}$$

Problem 57

57. If the sum of n terms of a series be $5n^2 + 3n$, find its nth term. Are the terms of this series in A.P.?

Solution of problem 57

Solution:

$$t_n = S_n - S_{n-1}$$

$$t_n = 5n^2 + 3n - 5(n-1)^2 - 3(n-1) = 10n - 2$$

Let d be the common difference.

$$d = t_n - t_{n-1}$$

$$d = 10n - 2 - 10(n-1) + 2$$

$$d = 10$$

Since d is constant therefore given series is in A.P.

Problem 58

58. Find the sum of the series $(a+b)^2 + (a^2+b^2) + (a-b)^2 + \ldots$ to n terms

Solution of problem 58

Solution:

$$t_1 = (a+b)^2$$

Let d be the common difference, then

$$d = t_2 - t_1 = a^2 + b^2 - (a+b)^2 = -2ab$$

$$S_n = \frac{n}{2}[2(a+b)^2 + (n-1)(-2ab)]$$

$$S_n = n(a^2+b^2) + nab(3-n)$$

Problem 59

59. Find $1 - 3 + 5 - 7 + 9 - 11 + \ldots$ to n terms.

Solution of problem 59

Solution: Case I When n is even

$$1 - 3 + 5 - 7 + 9 - 11 + \dots \text{ to } n \text{ terms}$$
$$= -2 - 2 - 2 + \dots \text{ to} \frac{n}{2} \text{ terms}$$
$$= -n$$

Case II When n is odd, let $n = 2m + 1$ where $m \in N$

$$1 - 3 + 5 - 7 + 9 - 11 + \dots \text{ to } n \text{ terms}$$
$$= 1 - 3 + 5 - 7 + 9 - 11 + \dots \text{ to } (2m+1) \text{ terms}$$
$$= [1 + 5 + 9 + \dots \text{ to } (m+1) \text{ terms}] - [3 + 7 + 11 + \dots \text{ to } m \text{ terms}]$$
$$= \frac{m+1}{2}[2.1 + 4m] - \frac{m}{2}[2.3 + (m-1)4]$$
$$= 2m + 1 = n$$

Problem 60

60. The interior angles of a polygon are in A.P. The smallest angle is $120^\circ$and the commnon difference is 5°. Find the number of sides of the polygon.

Solution of problem 60

Solution: Let n be the number of sides of polygon. From geometry, sum of interior angles of a polygon of n sides $= (n-2).180°$
Given, $a = 120°$, $d = 5°$, $n = n$
Sum of interior sides of a polygon,

$$\frac{n}{2}[2.120 + (n-1)5] = (n-2).180$$

$$5n^2 - 125n + 720 = 0$$

$$n = 9, 16$$

But if $n = 16$, greatest angle of polygon $= 120 + 15.5 = 195°$which is not possible as all interior angles of a polygon are less that $180°$.
Hence, $n = 9$