Complex Numbers Problems 1-10

Shiv Shankar Dayal

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Problem 1

1. Evaluate i. i^5 ii. i^{67} iii. i^{-49} iv. i^{2014}

Solution of Problem 1

Solution: i. $i^5 = i^4.i = 1.i = i$

ii. $i^{67} = i^{64}.i^3 = i^{4.16}.i^3 = 1^{16}. - i = -i$

iii. $i^{-49} = \frac{1}{i^{49}} = \frac{1}{i^{48}.i} = \frac{1}{i^{4.12}.i} = \frac{1}{1^{12}.i} = \frac{1}{i} = -i$

iv. $i^{2014} = i^{2^{1}}007 = (-1)^{1007} = -1$

Problem 2

2. If $a < 0, b > 0$, then prove that $\sqrt{ab}$ is equal to $\sqrt{|a|b}i$

Solution of Problem 2

Solution: $\because a < 0 \Rightarrow a = -|a|$

$\therefore \sqrt{ab} = \sqrt{-|a|b} = \sqrt{|a|b}i$

Problem 3

3. Prove that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$

Solution of Problem 3

Solution: $i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n(1 + i + i^2 + i^3) = i^n(1 = i - 1 - i) = 0$

Problem 4

4. Find the value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$

Solution of Problem 4

Solution: Sum of any four consecutive powers of i is zero. Thus,

$\sum_{n=1}^{13}(i^n + i^{n+1}) = (i + i^2 + i^3 + ... + i^{13}) + (i^2 = i^3 + ... + i^{14})$

$= i - 1$

Problem 5

5. Simplify and find the value of $\frac{2^n}{(1+i)^2 n} + \frac{(1+i)^2 n}{2^n}$

Solution of Problem 5

Solution: Given $\frac{2^n}{(1+i)^2n} + \frac{(1+i)^2n}{2^n}$

$= \frac{2^n}{(1+2i+i^2)^n} + \frac{(1+2i+i^2)^n}{2^n}$

$= \frac{2^n}{2^n i^{2n}} + \frac{2^n i^{2n}}{2^n}$

$= \frac{1}{(-i)^n} + (-1)^n$

Problem 6

6. Find different values of $i^n + i^{-n}$, $\forall\, n \in I$

Solution of Problem 6

Solution: Let $S = i^n + i^{-n} = \frac{i^{2n}+1}{i^n}$

For $n = 1, S = \frac{i^2+1}{i} = 0$

For $n = 2, S = \frac{i^4+1}{i^2} = -2$

For $n = 3, S = \frac{i^6+1}{i^3} = 0$

For $n = 4, S = \frac{i^8+1}{i^4} = 2$

Thus, we find three different values for the given expression.

Problem 7

7. If $4x + (3x - y)i = 3 - 6i$, then find the value of x and y.

Solution of Problem 7

Solution: Comparing real and imaginary parts, we get

$4x = 3$ and $3x - y = -6$

$\Rightarrow x = \frac{3}{4}, y = \frac{33}{4}$

Problem 8

8. Find the value of $\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right) - \left(-\frac{4}{3} + i\right)$

Solution of Problem 8

Solution: Given, $\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)-\left(-\frac{4}{3}+i\right)$

$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$

$=\frac{17}{3}+i\frac{5}{3}$

Problem 9

9. Find the real values of x and y if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i.$

Solution of Problem 9

Solution: Given, $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

$\Rightarrow (1+i)(3-i)x - 2i(3-i) + (2-3i)(3+i)y + (3+i)i = i(3+i)(3-i)$

$\Rightarrow (4x+9y-3) + i(2x-7y-3) = 10i$

Comparing real and imaginary parts, we get

$4x+9y-3=0$ and $2x-7y-3=10$

$\Rightarrow x=3, y=-1$

Problem 10

10. Find the multiplicative inverse of $4 - 3i$.

Solution of Problem 10

Solution: Let $z = 4 - 3i$ then multiplicative inverse would be $\frac{1}{z}$

$\frac{1}{z} = \frac{1}{4-3i} = \frac{4+3i}{(4-3i)(4+3i)} = \frac{4+3i}{25}$