Logarithm Problem 91-100

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Problem 91

91. Prove that $\log_2 17 \log_{\frac{1}{5}} 2 \log_3 \frac{1}{5} > 2$

Solution of Problem 91

Solution:

$$L.H.S. = \log_2 17 \log_{\frac{1}{5}} 2 \log_3 \frac{1}{5}$$

$$= \log_2 17 \log_3 2 = \log_3 17$$

$$\because 17 > 3^2 \Rightarrow \log_3 17 > 2$$

Problem 92

92. Show that $\log_{49} 3$ lies between $\frac{1}{3}$ and $\frac{1}{4}$.

Solution of Problem 92

Solution:

$$
\begin{aligned}
&3^3 < 49 < 3^4 \\
&\Rightarrow 3\log_3 3 < \log_3 49 < 4\log_3 3 \\
&\Rightarrow 3 < \log_3 49 < 4 \\
&\Rightarrow \frac{1}{3} > \frac{1}{\log_3 49} > \frac{1}{4} \\
&\Rightarrow \frac{1}{3} > \log_{49} 3 > \frac{1}{4}
\end{aligned}
$$

Problem 93

93. Show that $\log_{20} 3$ lies between $\frac{1}{2}$ and $\frac{1}{3}$.

Solution of Problem 93

Solution:

$$
\begin{aligned}
&3^2 < 20 < 3^3 \\
&\Rightarrow 2\log_3 3 < \log_3 20 < 3\log_3 3 \\
&\Rightarrow 2 < \log_3 20 < 3 \\
&\Rightarrow \frac{1}{2} > \frac{1}{\log_3 20} > \frac{1}{3} \\
&\Rightarrow \frac{1}{2} > \log_{20} 3 > \frac{1}{3}
\end{aligned}
$$

Problem 94

94. Show that $\log_{10} 2$ lies between $\frac{1}{4}$ and $\frac{1}{3}$.

Solution of Problem 94

Solution:

$$
\begin{aligned}
& 2^3 < 10 < 2^4 \\
\Rightarrow\ & 3\log_2 2 < \log_2 10 < 4\log_2 2 \\
\Rightarrow\ & 3 < \log_2 10 < 4 \\
\Rightarrow\ & \frac{1}{3} > \frac{1}{\log_2 10} > \frac{1}{4} \\
\Rightarrow\ & \frac{1}{3} > \log_{10} 2 > \frac{1}{4}
\end{aligned}
$$

Problem 95

95. Solve $\log_{0.1}(4x^2 - 1) > \log_{0.1} 3x$

Solution of Problem 95

Solution:

$$\text{Given, } \log_{0.1}(4x^2 - 1) > \log_{0.1} 3x$$

$$\Rightarrow 4x^2 - 3x - 1 < 0 \Rightarrow (4x + 1)(x - 1) < 0$$

Thus, $\frac{-1}{4} < x < 1$ is the initial solution.
However, $x > 0$ from R.H.S. From L.H.S. $4x^2 - 1 > 0 \Rightarrow x < \frac{-1}{2}, x > \frac{1}{2}$
Thus, $\frac{1}{2<x<1}$ is what we have combining all the solutions.

Problem 96

96. Solve $\log_2(x^2 - 24) > \log_2 5x$

Solution of Problem 96

Solution:

$$\text{Given, } \log_2(x^2 - 24) > \log_2 5x$$
$$\Rightarrow x^2 - 24 > 5x$$
$$\Rightarrow (x - 8)(x + 3) > 0$$
$$\Rightarrow x < -3, x > 8$$

But $x^2 - 24 > 0$ and also $x > 0$ for logarithm function to be defined. $\therefore x > 8$ is the solution.

Problem 97

97. Show that $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > 2$

Solution of Probelm 97

Solution:

$$\begin{aligned}
&\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > 2 \\
&\Rightarrow \log_\pi 3 + \log_\pi 4 > 2 \\
&\Rightarrow \log_\pi 12 > 3 \\
&\Rightarrow 12 > \pi^2
\end{aligned}$$

which is true.

Problem 98

98. Without actual computation find greater among $(0.01)^{\frac{1}{3}}$ and $(0.001)^{\frac{1}{5}}$

Solution of Problem 98

Solution: Taking log of both with base 10 we get $\frac{1}{3}\log 0.01$ and $\frac{1}{4}\log 0.001$ i.e. $-\frac{2}{3}$ and $-\frac{3}{5}$
Since .$\frac{3}{5}$ is graeter so $(0.001)^{\frac{1}{5}}$ is graeter.

Problem 99

99. Without actual computation find greater among $\log_2 3$ and $\log_3 11$

Solution of Problem 99

Solution:

$$\log_2 3 < \log_2 4 = 2$$
$$\log_3 11 > \log_3 9 = 2$$

So $\log_3 11$ is greater.

Problem 100

100. Solve, $\log_3(x^2+10) > \log_3 7x$

Solution of Problem 100

Solution:

$$\text{Given, } \log_3(x^2+10) > \log_3 7x$$

$$\Rightarrow x^2 - 7x + 10 > 0 \Rightarrow (x-2)(x-5) > 0$$

$$\Rightarrow x < 2, x > 5$$

Howeever, for logarithm to be defined $x > 0$ and $x^2 + 10 > 0$. Thus, range is $(0, 2)$ and $(5, \infty]$