Geometric Progression
Problems 41-50

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September 10, 2021

Problem 41

41. Find $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ to n terms.

Solution to Problem 41

Solution: Here given sequence is a G.P. with $a = 1, r = \frac{1}{2}, n = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{1\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = 2\left(1-\frac{1}{2^n}\right)$$

Problem 42

42. Find $1 + 2 + 4 + 8 + \ldots$ to 12 terms.

Solution to Problem 42

Solution: Here given sequence is a G.P. with $a = 1, r = 2, n = 12$

$$S_{12} = \frac{a(r^n - 1)}{r - 1} = \frac{1(2^{12} - 1)}{2 - 1} = 4095$$

Problem 43

43. Find $1 - 3 + 9 - 27 + \ldots$ to 9 terms.

Solution to Problem 43

solution: Here given sequence is a G.P. with $a = 1, r = -3, n = 9$

$$S_9 = \frac{a(r^n - 1)}{r - 1} = \frac{1(-3^9 - 1))}{-3 - 1} = 4921$$

Problem 44

44. Find $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \ldots$ to n terms.

Solution to Problem 44

Solution: Here given seuqence is a G.P. with $a = 1, r = \frac{1}{3}, n = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{1\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}} = \frac{3}{2}\left(1-\frac{1}{3^n}\right)$$

Problem 45

45. Find the sum of n terms of the series $(a + b) + (a^2 + 2b) + (a^3 + 3b) + \ldots$ to n terms.

Solution of Problem 45

Solution: $(a+b)+(a^2+2b)+(a^3+3b)+\ldots$ to n terms

$$= (a+a^2+a^3+\ldots) \text{ to } n \text{ terms} + b(1+2+3+\ldots) \text{ to } n \text{ terms}$$

$$= \frac{a(1-a^n)}{1-a} + b.\frac{n(n+1)}{2}$$

Problem 46

46. A man agrees to work at the rate of one dollar the first day, two dollars the second day, four dollars the third day, eight dollars the fourth day and so on. How much would he get at the end of 120 days.

Solution of Problem 46

Solution: Total amount rececived at the end of 120 days

$$= 1 + 2 + 4 + 8 \text{ to } 120 \text{ terms}$$
$$= \frac{1.(2^{120} - 1)}{2 - 1} = 2^{120} - 1$$
$$= 1329227995784915872903807060280344575$$

Problem 47

47. Find the sum to n terms of the series $8 + 88 + 888 + \ldots$

Solution of Problem 47

Solution: Let $S_n = 8 + 88 + 888 + \ldots$ to n terms

$$= 8[1 + 11 + 111 + \ldots]$$

$$= \frac{8}{9}[9 + 99 + 999 + \ldots]$$

$$= \frac{8}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots]$$

$$= \frac{8}{9}[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$$

$$= \frac{8}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$

$$= \frac{8}{81}(10^{n+1} - 10 - 9n)$$

Problem 48

48. Find the sum to n terms of the series $6 + 66 + 666 + \ldots$

Solution of Problem 48

Solution: Let $S_n = 6 + 66 + 666 + \ldots$ to n terms

$$
\begin{aligned}
&= 6[1 + 11 + 111 + \ldots] \\
&= \frac{6}{9}[9 + 99 + 999 + \ldots] \\
&= \frac{2}{3}[(10-1) + (100-1) + (1000-1) + \ldots] \\
&= \frac{2}{3}[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)] \\
&= \frac{2}{3}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right] \\
&= \frac{2}{27}(10^{n+1} - 10 - 9n)
\end{aligned}
$$

Problem 49

49. Find the sum to n terms of the series $4 + 44 + 444 + \dots$

Solution of Problem 49

Solution: Let $S_n = 4 + 44 + 444 + \ldots$ to n terms

$$= 4[1 + 11 + 111 + \ldots]$$

$$= \frac{4}{9}[9 + 99 + 999 + \ldots]$$

$$= \frac{4}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots]$$

$$= \frac{4}{9}[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$$

$$= \frac{4}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$

$$= \frac{4}{81}(10^{n+1} - 10 - 9n)$$

Problem 50

50. Find the sum to n terms of the series $.5 + .55 + .555 + \dots$

Solution of Problem 50

Solution: Let $S_n = .5 + .55 + .555 + \ldots$ to n terms

$$= 5[.1 + .11 + .111 + \ldots]$$

$$= \frac{5}{9}[.9 + .99 + .999 + \ldots]$$

$$= \frac{5}{9}[(1 - .1) + (1 - .01) + (1 - .001) + \ldots]$$

$$= \frac{5}{9}[(1 + 1 + 1 + \ldots) - (.1 + .01 + .001)]$$

$$= \frac{5}{9}\left[n - \frac{1}{10}\frac{\frac{1}{1-10^n}}{1 - \frac{1}{9}}\right]$$