# Problems 11 to 20

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# Problem 11

**11.** Show that the seuquence $9, 12, 15, 18, \ldots$ is an A.P. Find its $16^{th}$ term and the general term.

## Solution of problem 11

**Solution:** Since $12 - 9 = 15 - 12 = 18 - 15 = 3$, which is a constant, therefore given sequence is an A.P. having common difference as 3 and first term as 9.
$t_{16} = a + (16 - 1)d = 9 + 15.3 = 54$
The general term or $n^{th}$ term is given by $t_n = a + (n - 1)d = 9 + (n - 1)3$
$t_n = 3n + 6$

## Problem 12

**12.** Show that the sequence $\log a, \log(ab), \log(ab^2), \log(ab^3), \ldots$ is an A.P. Find its $n^{th}$ term.

# Solution of problem 12

**Solution:** We have,

$\log(ab) - \log a = \log\left(\frac{ab}{a}\right) = \log b$

$\log(ab^2) - \log(ab) = \log\left(\frac{ab^2}{ab}\right) = \log b$

$\log(ab^3) - \log(ab^2) = \log\left(\frac{ab^3}{ab^2}\right) = \log b$

Since the difference of a term and the preceding term is alwyas same, therefore the given sequence is an A.P.

Now,

$t_n = a + (n-1)d = \log a + (n-1)\log b = \log(ab^{n-1})$

# Problem 13

**13.** Find the sum to $n$ terms of the sequence $\langle t_n \rangle$, where $t_n = 5 - 6n, n \in N$

# Solution of problem 13

**Solution:** $t_{n+1} - t_n = 5 - 6(n+1) - 5 + 6n = -6 \forall n \in N$ which is constant, therefore the given sequence is an A.P.
Putting $n = 1$, we get $t_1 = 5 - 6.1 = -1$, so the sum $S_n$ to $n$ term is given by
$S_n = \frac{n}{2}[t_1 + t_n] = \frac{n}{2}[-1 + 5 - 6n] = n(2 - 3n)$

# Problem 14

**14.** How many terms are there in the A.P. $3, 7, 11, \ldots, 407$?

# Solution of problem 14

**Solution:** From first three terms we have $a = 3, d = 7 - 3 = 11 - 7 = 4$
Formula for general term is $t_n = a + (n-1)d \Rightarrow 407 = 3 + (n-1)4 \Rightarrow n = 102$

# Problem 15

**15.** If $a, b, c, d, e$ are in A.P. find the value of $a - 4b + 6c - 4d + e$.

# Solution of problem 15

**Solution:** Let $p$ be the first term and $q$ be the common difference. Then we have,
$p = a, p + q = b, p + 2q = c, p + 3q = d, p + 4q = e$
Thus, we see that $a + e = 2c, b + d = 2c$
Now, $a - 4b + 6c - 4d + e = (a + e) - 4(b + d) + 6c = 2c - 8c + 6c = 0$

# Problem 16

**16.** In a certain A.P. 5 times the $5^{th}$ term is equal to 8 times the $8^{th}$ term, then prove that $13^{th}$ term is zero.

# Solution of problem 16

**Solution:** Given $5.t_5 = 8.t_8$
$5(a+4d) = 8(a+7d)$, where $a$ is the first term and $d$ is the common difference.
$\Rightarrow a + 12d = 0 \Rightarrow t_{13} = 0$

## Problem 17

**17.** Find the term of the series $25, 22\frac{3}{4}, 20\frac{1}{2}, 18\frac{1}{4}, \ldots$ which is numerically smallest positive number.

# Solution of problem 17

**Solution:** The given series is an A.P. with $a = 25, d = -9/4$
$t_n = 25 + (n-1)\frac{-9}{4} = \left(25 + \frac{9}{4}\right) - \frac{9}{4}n$
$\Rightarrow t_n = \frac{109}{4} - \frac{9}{4}n$
Now $t_n$ will be negative if $\frac{109}{4} - \frac{9}{4}n < 0$ or $n > 12\frac{1}{9}$
Hence, $t_{12}$ will be smallest positive number. $t_{12} = \frac{1}{4}$

# Problem 18

**18.** A person was appointed in the pay scale of Rs. $700 - 40 - 1500$. Find in how many years he will reach the maximum of the scale.

## Solution of problem 18

**Solution:** Given, $t_n = 1500, a = 700, d = 40$
$\because t_n = a + (n-1)d$
$\therefore 1500 = 700 + (n-1)40 \Rightarrow n = 21$
Hence, he will reach the maximum scale in 20 years because his pay in 20 years will be $a + 20d = 21^{st}$ term 1500

# Problem 19

**19.** Find the A.P. whose $7^{th}$ and $13^{th}$ terms are respectively 34 and 64.

# Solution of problem 19

**Solution:** Let $a$ be the first term and $d$ be the c.d.
$t_7 = a + 6d = 34$
$t_{13} = a + 12d = 64$
Subtracting we get, $6d = 30 \Rightarrow d = 5$
Substituting for $t_7$ we get, $a + 30 = 34 \Rightarrow a = 4$
Thus, our A.P.is $4, 9, 14, 19, \ldots$

# Problem 20

**20.** Is 55 a term of the sequence $1, 3, 5, 7, \ldots$? If yes, find which term it is.

# Solution of problem 20

**Solution:** Clearly, $a = 1$ and $d = 3 - 1 = 5 - 3 = 7 - 5 = 2$
Let 55 be the $n^{th}$ term of the series. Then we have $55 = 1 + (n - 1)2$
$\Rightarrow 55 = 1 + 2n - 2 = 2n - 1$
$\Rightarrow 56 = 2n \Rightarrow n = 28$
Since $n$ is an integer 55 is a member of A.P. and it is $28^{th}$ term of the A.P.