Problems 41 to 50

Shiv Shankar Dayal

September 1, 2019

Problem 41

41. If $(b+c-a)/a, (c+a-b)/b, (a+b-c)/c$ are in A.P. then prove that $1/a, 1/b, 1/c$ are also in A.P.

Solution of problem 41

Solution: $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.
Adding 2 to each term
$\frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2$ are in A.P.
$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.
Dividing each term by $a+b+c$
$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Problem 42

42. If $a, b, c \in R+$ form an A.P., then prove that $a + 1/bc, b + 1/ca, c + 1/ab$ are also in A.P.

Solution of problem 42

Solution: a, b, c are in A.P.
Dividing each term by abc
$1/bc, 1/ca, 1/ab$ are in A.P.
Adding the two A.Ps.
$a + 1/bc, b + 1/ca, c + 1/ab$ are also in A.P.

Problem 43

43. If a, b, c are in A. P., then prove that $a^2(b+c), b^2(c+a), c^2(a+b)$ are also in A.P.

Solution of problem 43

Solution: a, b, c are in A.P.
$\Rightarrow b - a = c - b$
$\Rightarrow (b - a)(ab + bc + ca) = (c - b)(ab + bc + ca)$
$\Rightarrow b^2(c + a) - a^2(b + c) = c^2(a + b) - b^2(c + a)$
$a^2(b + c), b^2(c + a), c^2(a + b)$ are also in A.P.

Problem 44

44. If a, b, c are in A.P., then prove that $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are also in A.P.

Solution of problem 44

Solution: a, b, c are in A.P.
$\Rightarrow b - a = c - b$
$\Rightarrow \frac{\sqrt{b}-\sqrt{a}}{\sqrt{b}+\sqrt{c}} = \frac{\sqrt{c}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$
$\Rightarrow \frac{\sqrt{b}-\sqrt{a}}{(\sqrt{c}+\sqrt{a})(\sqrt{b}+\sqrt{c})} = \frac{\sqrt{c}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{a})}$
$\Rightarrow \frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}$
$\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

Problem 45

45. If a, b, c are in A.P., then prove that $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are also in A.P.

Solution of problem 45

Solution: a, b, c are in A.P.
Dividing each term by abc
$\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.
Multiplying each term by $ab + bc + ca$
$\frac{ab+bc+ca}{bc}, \frac{ab+bc+ca}{ca}, \frac{ab+bc+ca}{ab}$ are in A.P.
Subtracting 1 from each term
$\frac{ab+ca}{bc}, \frac{ab+bc}{ca}, \frac{bc+ca}{ab}$ are in A.P.
$a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{c} + \frac{1}{a}\right), c\left(\frac{1}{a} + \frac{1}{b}\right)$ are also in A.P.

Problem 46

46. If $(b-c)^2, (c-a)^2, (a-b)^2$ are in A.P. then prove that $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are also in A.P.

Solution of problem 46

Solution: $(b-c)^2, (c-a)^2, (a-b)^2$ are in A.P.
Adding $ab + bc + ca - a^2 - b^2 - c^2$ to each term
$ab + ca - bc - a^2, ab + bc - ca - b^2, bc + ca - ab - c^2$ are in A.P.
$(c-a)(a-b), (a-b)(b-c), (c-a)(b-c)$ are in A.P.
Dividing each term by $(a-b)(b-c)(c-a)$
$\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are also in A.P.

Problem 47

47. If a, b, c are in A.P. then prove that $b + c, c + a, a + b$ are also in A.P.

Solution of problem 47

Solution: a, b, c are in A.P.
Subtracting $a + b + c$ from each term
$a - (a + b + c), b - (a + b + c), c - (a + b + c)$ are in A.P.
$-(b + c), -(c + a), -(a + b)$ are in A.P.
$b + c, c + a, a + b$ are in A.P.

Problem 48

48. If a^2, b^2, c^2 are in A.P. then prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Solution of problem 48

Solution: a^2, b^2, c^2 are in A.P.
$\Rightarrow b^2 - a^2 = c^2 - b^2$
$\Rightarrow (b+a)(b-a) = (c+b)(c-b)$
$\Rightarrow \frac{b-a}{b+c} = \frac{c-b}{a+b}$
$\Rightarrow \frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+a)}$
$\Rightarrow \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$
$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Problem 49

49. If a, b, c are in A.P., show that $2(a-b) = a-c = 2(b-c)$

Solution: Let d be the common difference, then, $b = a + d, c = a + 2d$
$2(a - b) = 2(a - a - d) - 2d$
$a - c = a - a - 2d = -2d$
$2(b - c) = 2(a + d - a - 2d) = -2d$
Hence, $2(a - b) = a - c = 2(b - c)$

Problem 50

50. If a, b, c are in A.P., then prove that $(a - c)^2 = 4(b^2 - ac)$

Solution of problem 50

Solution: Let d be the common difference, then $b = a + d, c = a + 2d$
$(a - c)^2 = (a - a - 2d)^2 = 4d^2$
$4(b^2 - ac) = 4[(a + d)^2 - a(a + 2d)] = 4(a^2 + d^2 + 2ad - a^2 - 2ad) = 4d^2$
Hence, $(a - c)^2 = 4(b^2 - ac)$