Logarithm Problem 61-70

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Problem 61

61. Solve $\sqrt{\log_2 x^4} + 4\log_4 \sqrt{\frac{2}{x}} = 2$

Solution of Problem 61

Solution:

$$\text{Given, } \sqrt{\log_2 x^4} + 4\log_4 \sqrt{\frac{2}{x}} = 2$$

$$\Rightarrow \sqrt{4\log_2 x} + 2\log_2 \sqrt{\frac{2}{x}} = 2$$

$$\Rightarrow 2\sqrt{\log_2 x} + \log_2 \frac{2}{x} = 2$$

$$\Rightarrow 2\sqrt{\log_2 x} + 1 - \log_2 x = 2$$

$$\Rightarrow 2\sqrt{\log_2 x} = 1 + \log_2 x$$

$$\Rightarrow 4\log_2 x = 1 + 2\log_2 x + (\log_2 x)^2$$

$$\Rightarrow (\log_2 x)^2 - 2\log_2 x + 1 = 0$$

$$\Rightarrow \log_2 x = 1 \Rightarrow x = 2$$

Problem 62

62. Solve $2\log_{10} x - \log_x 0.001 = 5$

Solution of Problem 62

Solution:

$$\text{Given, } 2\log_{10} x - \log_x 0.001 = 5$$
$$\Rightarrow 2\log_{10} x - \log_x (10)^{-2} = 5$$
$$\Rightarrow 2\log_{10} x + 2\log_x 10 = 5$$
$$\Rightarrow 2\log_{10} x + \frac{2}{\log_{10} x} = 5$$
$$\Rightarrow 2(\log_{10} x)^2 + 2 = 5\log_{10} x$$
$$\Rightarrow \log_{10} x = 2, \frac{1}{2}$$
$$\Rightarrow x = 100, \sqrt{10}$$

Problem 63

63. Solve $\log_{\sin x} 2 \log_{\cos x} 2 + \log_{\sin x} 2 + \log_{\cos x} 2 = 0$

Solution of Problem 63

Solution:

$$\text{Given, } \log_{\sin x} 2 \log_{\cos x} 2 + \log_{\sin x} 2 + \log_{\cos x} 2 = 0$$

$$\Rightarrow \frac{\log 2}{\log \sin x} \cdot \frac{\log 2}{\log \cos x} + \frac{\log 2}{\log \sin x} + \frac{2}{\log \cos x} = 0$$

$$\Rightarrow \log 2 + \log \sin x + \log \cos x = 0$$

$$\Rightarrow \log \sin 2x = 0 \Rightarrow \sin 2x = 1$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{4}, \ \forall n \in I$$

Problem 64

64. Solve $2^{x+3} + 2^{x+2} + 2^{x+1} = 7^x + 7^{x-1}$

Solution of Problem 64

Solution:

$$2^{x+3} + 2^{x+2} + 2^{x+1} = 7^x + 7^{x-1}$$

$$\Rightarrow 2^{x+1}(2^2 + 2 + 1) = 7^{x-1}(7 + 1)$$

$$\Rightarrow 2^{x+1}.7 = 7^{x-1}.2^3$$

Taking $\log$ of both sides, we get

$$(x+1)\log 2 + \log 7 = (x-1)\log 7 + 3\log 2$$

$$\Rightarrow (x-2)(\log 7 - \log 2) = 0$$

$$\Rightarrow x = 2$$

Problem 65

65. Solve $\log_{\sqrt{2}\sin x}(1+\cos x) = 2$

Solution of Problem 65

Solution:

$$\text{Given, } \log_{\sqrt{2}\sin x}(1+\cos x) = 2$$

$$\Rightarrow 1 + cosx = (\sqrt{2}\sin x)^2 = 2\sin^2 x$$

$$\Rightarrow 1 + \cos x = 2 - 2\cos^2 x$$

$$\Rightarrow 2\cos^2 x + \cos x - 1 = 0$$

$$\Rightarrow \cos x = -1, \frac{1}{2}$$

$$\Rightarrow x = 2n\pi, 2n\pi + \frac{\pi}{3}, n \in I$$

Problem 66

66. Solve $\log_{10}[98 + \sqrt{x^3 - x^2 - 12x + 36}] = 2$

Solution of Problem 66

Solution:

$$\text{Given, } \log_{10}[98+\sqrt{x^3-x^2-12x+36}]=2$$

$$\Rightarrow 98+\sqrt{x^3-x^2-12x+36}=100$$

$$\Rightarrow x^3-x^2-12x+36=0$$

Only one root, -4, is appropriate solution.

Problem 67

67. If $\log 2 = 0.30103$ and $\log 3 = 0.47712$, solve the equation $2^x 3^{2x} - 100 = 0$

Solution of Problem 67

Solution:

$$\text{Given, } 2^x 3^{2x} - 100 = 0$$

$$\Rightarrow x \log_{10} 2 + 2x \log_{10} 3 = 2$$

$$0.30103x + 0.95424x = 2$$

$$x = 1.593$$

Problem 68

68. Solve $\log_x 3 \log_{\frac{x}{3}} 3 + \log_{\frac{x}{81}} 3 = 0$

Solution of Problem 68

Solution:

$$\text{Given, } \log_x 3 \log_{\frac{x}{3}} 3 + \log_{\frac{x}{81}} 3 = 0$$

$$\Rightarrow \frac{1}{\log_3 x} \cdot \frac{1}{\log_3 \frac{x}{3}} + \frac{1}{\log_3 \frac{x}{81}} = 0$$

$$\Rightarrow \frac{1}{\log_3 x} \cdot \frac{1}{\log_3 x - \log_3 3} + \frac{1}{\log_3 x - \log_3 81} = 0$$

$$\text{Let } z = \log_3 x$$

$$\frac{1}{z} \cdot \frac{1}{z-1} + \frac{1}{z-4} = 0$$

$$\Rightarrow z^2 - 4 = 0 \Rightarrow z = \pm 2$$

$$\Rightarrow x = 9, \frac{1}{9}$$

Problem 69

69. Solve $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$

Solution of Problem 69

Solution:

Given, $\log_{(2x+3)}(6x^2+23x+21) = 4 - \log_{(3x+7)}(4x^2+12x+9)$

$$\Rightarrow \log_{(2x+3)}(2x+3)(3x+7) = 4 - \log_{(3x+7)}(2x+3)^2$$

$$\Rightarrow 1 + \log_{(2x+3)}(3x+7) = 4 - 2\log_{(3x+7)}(2x+3)$$

$\Rightarrow$ Let $z = \log_{(2x+3)}(3x+7)$

$$1 + z = 4 - \frac{2}{z} \Rightarrow z = 1, 2$$

When $z = 1, 2x+3 = 3x+7 \Rightarrow x = -4$

When $z = 2, (2x+3)^2 = 3x+7, \Rightarrow x = -4, -2$

For logarithm to be deffined $2x+3 > 0, 2x+3 \neq 1$ and $3x+7 > 0, 3x+7 \neq 1$. Thus, $x = -\frac{1}{4}$ is the only valid solution.

Problem 70

70. Solve $\log_2(x^2 - 1) = \log_{\frac{1}{2}}(x - 1)$

Solution of Problem 70

Solution:

$$\text{Given, } \log_2(x^2-1) = \log_{\frac{1}{2}}(x-1)$$

$$\Rightarrow \log_2(x^2-1) = \log_{2^{-1}}(x-1) = -\log_2(x-1) = \log_2 \frac{1}{x-1}$$

$$\Rightarrow x^2-1 = \frac{1}{x-1}$$

$$x = 0, x^2-x-1 = 0 \Rightarrow x = 0, \frac{1 \pm \sqrt{5}}{2}$$

For logarithm to be defined $x^2-1>0$ and $x-1>0$, which implies that $x = \frac{1+\sqrt{5}}{2}$ is the only acceptable solution.