# Logarithm Problem 1-10

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**1.** Find the value of $x$ where $\log_{\sqrt{8}} x = \frac{10}{3}$

# Solution of Problem 1

**Solution:**

$$\log_{\sqrt{8}} x = \frac{10}{3} \Rightarrow \log_{2^{\frac{3}{2}}} x = \frac{10}{4}$$

$$\Rightarrow \frac{2}{3} \log_2 x = \frac{10}{3} \Rightarrow \log_2 x = 5$$

$$\Rightarrow x = 5^2 = 25$$

# Problem 2

**2.** Prove that $\log_b a . \log_c b \log_a c = 1$

# Solution of Problem 2

**Solution:**
$$L.H.S. = \log_b a . \log_c b \log_a c = \frac{\log a}{\log b} \frac{\log b}{\log c} \frac{\log c}{\log a} = 1 = R.H.S.$$

# Problem 3

**3.** Prove that $\log_3 \log_2 \log_{\sqrt{5}}(625) = 1$

## Solution of Problem 3

**Solution:**

$$\log_3 \log_2 \log_{\sqrt{5}}(625) = \log_3 \log_2 \log_{\sqrt{5}} 5^4$$
$$= \log_3 \log_2 8 = \log_3 3 = 1$$

# Problem 4

**4.** If $a^2 + b^2 = 23ab$, then prove that $\log \frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$

# Solution of Problem 4

**Solution:**

$$a^2 + b^2 = 23ab \Rightarrow (a+b)^2 = 25ab \Rightarrow \left(\frac{a+b}{5}\right)^2 = ab$$

Taking $\log$ of both sides

$$2\log\frac{a+b}{5} = \log(ab) \Rightarrow \log\frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$$

# Problem 5

**5.** Prove that $7 \log \frac{16}{15} + 5 \log \frac{25}{24} + 3 \log \frac{81}{80} = \log 2$

# Solution of Problem 5

**Solution:**

$$L.H.S. = 7[\log 2^4 - \log(3.5)] + 5[\log 5^2 - \log(8.3)] + 3[\log 3^4 - \log 16.5]$$

$$= 7[4\log 2 - \log 3 - \log 5] + 5[2\log 5 - 3\log 2 - \log 3] + 3[4\log 3 - 4\log 2 - \log 5]$$

$$= \log 2 = R.H.S.$$

## Problem 6

**6.** Find the value of $\log \tan 1° + \log \tan 2° + ... + \log \tan 89°$

# Solution of Problem 6

**Solution:**

$$L.H.S. = \log \tan 1° + \log \tan 2° + ... + \log \tan 89°$$

$$= \log(\tan 1°. \tan 2°. ... \tan 89°)$$

$$= \log(\tan 1°. \tan 89°)(\tan 2°. \tan 88°) ... \tan 45°$$

$$= \log(\tan 1°. \cot 1°)(\tan 2° \cot 2°) ... \tan 45°$$

$$= \log(1.1.1. ... 1) = \log 1 = 0$$

## Problem 7

**7.** Evaluate $\log_9 \tan \frac{\pi}{6}$

# Solution of Problem 7

**Solution:**

$$\log_9 \tan \frac{\pi}{6} = \log_{3^2} \frac{1}{\sqrt{3}}$$

$$= \frac{1}{2} \log_3 3^{-\frac{1}{2}} = -\frac{1}{4}$$

# Problem 8

**8.** Evaluate $\frac{\log_{a^2} b}{\log_{\sqrt{a}} b^2}$

**Solution:**

$$\frac{\log_{a^2} b}{\log_{\sqrt{a}} b^2} = \frac{2\log_a b}{2.2\log_a b}$$

$$= \frac{1}{8}$$

**9.** Evaluate $\log_{\sqrt{5}} .008$

**Solution:**

$$\log_{\sqrt{5}} .008 = \log_{\sqrt{5}} \frac{8}{1000}$$

$$= \log_{\sqrt{5}} 8 - \log_{\sqrt{5}} 1.125 = \log_{\sqrt{5}} 8 - \log_{\sqrt{5}} 8 - \log_{\sqrt{5}} 125 = -\log_{\sqrt{5}} 5^3 = -6$$

**10.** Evaluate $\log_{2\sqrt{3}} 144$

# Solution of Problem 10

**Solution:**

$$\log_{2\sqrt{3}} 144 = \log_{12^{\frac{1}{2}}} 12^2 = 4$$