Geometric Progression
Problems 31-40

Shiv Shankar Dayal

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Problem 31

31. If the continued product of three numbers in a G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

Solution of Problem 31

Solution: Let the terms be $\frac{a}{r}, a$ and ar, where a be the first term and r be the common ratio of the G.P.
Given product is 216, implies $\frac{a}{r}.a.ar = a^3 = 216 \Rightarrow a = n6$
Sum of the products in pairs is 156. Hence,

$$\frac{a}{r}.a + a.ar + \frac{a}{r}ar = 156$$

$$\Rightarrow a^2 \left(\frac{1}{r} \quad + r + 1 \right) = 156$$

$$\Rightarrow 36 \left(\frac{1 + r^2 + r}{r} \right) = 156$$

$$\Rightarrow 3(1 + r + r^2) = 13r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow r = \frac{1}{3}, 3$$

Thus, required numbers are $18, 6, 2$ or $2, 6, 18$.

Problem 32

32. If a, b, c, d are in G.P., show that $(a+b)^2, (b+c)^2, (c+d)^2$ are in G.P.

Solution of Problem 32

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$b = ar, c = ar^2, d = ar^3$$
$$(a+b)^2 = a^2(1+r)^2$$
$$(b+c)^2 = a^2r^2(1+r)^2$$
$$(c+d)^2 = a^2r^4(a+r)^2$$

It is clear that $(a+b)^2, (b+c)^2, (c+d)^2$ are in G.P. with a common ratio of r^2.

Problem 33

33. If a, b, c, d are in G.P., show that $(a-b)^2, (b-c)^2, (c-d)^2$ are in G.P.

Solution of Problem 33

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$\begin{aligned}
b = ar, c = ar^2, d = ar^3 \\
(a-b)^2 = a^2(1-r)^2 \\
(b-c)^2 = a^2r^2(1-r)^2 \\
(c-d)^2 = a^2r^4(1-r)^2
\end{aligned}$$

It is clear that $(a-b)^2, (b-c)^2, (c-d)^2$ are in G.P. with a common ratio of r^2.

Problem 34

34. If a, b, c, d are in G.P., show that $a^2 + b^2 + c^2, ab + bc + cd, b^2 + c^2 + d^2$ are in G.P.

Solution of Problem 34

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$b = ar, c = ar^2, d = ar^3$$
$$a^2 + b^2 + c^2 = a^2(1 + r^r + r^4)$$
$$ab + bc + cd = a^2 r(1 + r^2 + r^4)$$
$$b^2 + c^2 + d^2 = a^2 r^2(1 + r^2 + r^4)$$

It is clear that $a^2 + b^2 + c^2, ab + bc + cd, b^2 + c^2 + d^2$ are in G.P. with a comon ratio of r.

Problem 35

35. If a, b, c, d are in G.P., show that $\frac{1}{(a+b)^2}, \frac{1}{(b+c)^2}, \frac{1}{(c+d)^2}$ are in G.P.

Solution of Problem 35

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$b = ar, c = ar^2, d = ar^3$$
$$\frac{1}{(a+b)^2} = \frac{1}{a^2(1+r)^2}$$
$$\frac{1}{(b+c)^2} = \frac{1}{a^2r^2(1+r)^2}$$
$$\frac{1}{(c+d)^2} = \frac{1}{a^2r^4(1+r)^2}$$

It is clear that $\frac{1}{(a+b)^2}, \frac{1}{(b+c)^2}, \frac{1}{(c+d)^2}$ are in G.P. with a common ratio of $\frac{1}{r^2}$.

Problem 36

36. If a, b, c, d are in G.P., show that $a(b-c)^3 = d(a-b)^3$

Solution of Problem 36

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$b = ar, c = ar^2, d = ar^3$$
$$a(b-c)^3 = a(ar - ar^2)^3 = a^4r^3(1-r)^3$$
$$d(a-b)^3 = ar^3(a - ar)^3 = a^4r^3(1-r)^3$$

Hence, we have proven the desired equality.

Problem 37

37. If a, b, c, d are in G.P., show that $(a+b+c+d)^2 = (a+b)^2 + (c+d)^2 + 2(b+c)^2$

Solution of Problem 37

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$\begin{aligned}
b = ar, c = ar^2, d = ar^3 \\
L.H.S. = (a+b+c+d)^2 = (a+ar+ar^2+ar^3)^2 \\
= a^2(1+2r+3r^2+4r^3+3r^4+2r^5+r^6) \\
R.H.S. = (a+b)^2+(b+c)^2+2(b+c)^2 \\
= a^2(1+r^2+2r)+a^2(r^4+2r^5+r^6)+a^2(2r^2+2r^4+4r^3) \\
= a^2(1+2r+3r^3+4r^3+3r^4+2r^5+r^6)
\end{aligned}$$

It is evident that L.H.S $=$ R.H.S.

Problem 38

38. If a, b, c are in G.P., show that $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

Solution of Problem 38

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$b = ar, c = ar^2$$

$$L.H.S. = a^2b^2c^2\left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right)$$

$$= \frac{b^2c^2}{a} + \frac{a^2c^2}{b} + \frac{a^2b^2}{c}$$

$$= a^3r^6 + a^3r^3 + a^3 = c^3 + b^3 = a^3$$

Problem 39

39. If a, b, c are in G.P., show that $(a^2 - b^2)(b^2 + c^2) = (b^2 - c^2)(a^2 + b^2)$

Solution of Problem 39

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$
\begin{aligned}
b = ar, c &= ar^2 \\
L.H.S. = (a^2 - b^2)(b^2 + c^2) &= a^2(1 - r^2)a^2r^2(1 + r^2) \\
&= a^2r^2(1 - r^2)a^2(1 + r^2) = (a^2r^2 - a^2r^4)(a^2 + a^2r^2) \\
&= (b^2 - c^2)(a^2 + b^2)
\end{aligned}
$$

Problem 40

40. If a, b, c are in G.P., show that $\log a, \log b, \log c$ are in A.P.

Solution of Problem 40

Solution: Let a be the first term and r be the common ratio of the G.P., then we have

$$b = ar, c = ar^2$$

$$\begin{aligned} \log a &= \log a \\ \log b &= \log ar = \log a + \log r \\ \log c &= \log ar^2 = \log a + 2\log r \end{aligned}$$

Clearly, $\log a, \log b, \log c$ are in A.P. with a common difference of $\log r$.