Geometric Progression
Problems 71-80

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71. After striking the floor a certain ball rebound to $\frac{4}{5}$th of the height from which it has fallen. Find the total distance it travels before coming to rest if it is gently dropped from a height of 120 meters.

Solution: Distance covered before first bounce = 120 meters.

After striking the floor the ball will go up for $120\frac{4}{5}$ meters and then fall the same distance so distance covered = $2.120.\frac{4}{5}$ meteres.

For the next bounce distance covered would be $2.120.\frac{4^2}{5^2}$ meters.

This will keep happening till ball comes to rest.

Thus, total distance covered would be = $120 + 240.\frac{4}{5} + 240.\frac{4^2}{5^2} + \ldots$ to ∞

$$= 120 + 240.\frac{4}{5}\left[1 + \frac{4}{5} + \frac{4^2}{5^2} + \ldots \text{ to } \infty\right]$$

$$= 120 + 240.\frac{4}{5}.5 = 1080$$

meters.

72. If a be the first term and b be the nth term and p be the product of n terms of a G.P., show that $p^2 = (ab)^n$

Solution: Let r be the common ratio. $b = ar^{n-1} \Rightarrow ab = a^2 r^{n-1} \Rightarrow (ab)^n = a^{2n} r^{n(n-1)}$

$$p = a.ar.ar^2.\ldots.ar^{n-1} = a^n r^{\frac{n(n-1)}{2}} \Rightarrow p^2 = a^{2n} r^{n(n-1)}$$

Thus, $p^2 = (ab)^n$

73. Show that the ratio of sum of n terms of two G.P.'s having the same common ratio is equal to the ratio of their nth terms.

Solution: Let a and b be first terms and r be the common ratio of two G.P.

Ratio of sums $= \dfrac{\frac{a(r^n-1)}{r-1}}{\frac{b(r^n-1)}{r-1}} = \underline{a/b}$

Ratio of nth terms $= \dfrac{ar^{n-1}}{br^{n-1}} = \dfrac{a}{b}$

Hence, proved.

74. If S_1, S_2, S_3 be the sum of $m, 2n, 3n$ terms respectively of a G.P. show that $(S_2 - S_1)^2 = S_1(S_3 - S_2)$

Solution: Let a be the first term and r be the common ratio of the G.P. Then,

$$S_1 = \frac{a(r^n - 1)}{r - 1}, S_2 = \frac{a(r^{2n} - 1)}{r - 1}, S_3 = \frac{a(r^{3n} - 1)}{r - 1}$$

$$S_2 - S_1 = \frac{ar^n(r^n - 1)}{r - 1}$$

$$S_3 - S_2 = \frac{ar^{2n}(r^n - 1)}{r - 1}$$

$$(S_2 - S_1)^2 = \frac{a^2 r^{2n}(r^n - 1)^2}{(r - 1)^2}$$

$$S_1(S_3 - S_2) = \frac{a(r^n - 1)}{r - 1} \left(\frac{a(r^{3n} - 1)}{r - 1} - \frac{a(r^{2n} - 1)}{r - 1} \right)$$

$$= \frac{a(r^n - 1)}{(r - 1)^2} [a(r^{3n} - r^{2n})] = \frac{a^2 r^{2n}(r^n - 1)^2}{(r - 1)^2}$$

Hence, proved.

75. If S_n denotes the sum of n terms of a G.P.,whose first term is a and common ratio is r, find $S_1 + S_2 + \ldots + S_{2n-1}$

Solution:

$$S_1 = a = \frac{a(r-1)}{r-1}$$

$$S_2 = a + ar = \frac{a(r^2-1)}{r-1}$$

$$\ldots$$

$$S_{2n-1} = a + ar + \ldots + ar^{2n-2} = \frac{a(r^{2n-1}-1)}{r-1}$$

$$S_1 + S_2 + \ldots + S_{2n-1} = \frac{a}{1-r}\left[(r-1) + (r^2-1) + \ldots + (r^{2n-1}-1)\right]$$

$$= \frac{a}{r-1}\left[\frac{r(r^{2n-1-1})}{r-1} - 2n + 1\right]$$