# Logarithm Problem 11-20

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# Problem 11

**11.** Prove that $\log_3 \log_2 \log_{\sqrt{3}} 81 = 1$

# Solution of Problem 11

**Solution:**

$$\begin{aligned} L.H.S. &= \log_3 \log_2 \log_{\sqrt{3}} 81 = \log_3 \log_2 \log_{\sqrt{3}} (\sqrt{3})^8 \\ &= \log_3 \log_2 8 = \log_3 3 = 1 = R.H.S. \end{aligned}$$

# Problem 12

**12.** Prove that $\log_a x \log_b y = \log_b x \log_a y$

## Solution of Problem 12

**Solution:**

$$L.H.S = \log_a x \log_b y = \frac{\log x}{\log a} \cdot \frac{\log y}{\log b}$$

$$= \frac{\log x}{\log b} \cdot \frac{\log y}{\log a} = \log_b x \log_a y = R.H.S.$$

# Problem 13

**13.** Prove that $a^x = 10^{x \log_{10} a}$

# Solution of Problem 13

**Solution:**

$$R.H.S. = 10^x \log_{10} a = z \text{ (say)}$$

Taking $\log$ of both sides with base $10$

$$\log_{10} z = x \log_{10} a = \log_{10} a^x \Rightarrow z = a^x = L.H.S.$$

# Problem 14

**14.** Prove that $\log_2 \log_2 \log_2 16 = 1$

# Solution of Problem 14

**Solution:**

$$\begin{aligned} L.H.S. &= \log_2 \log_2 \log_2 16 = \log_2 \log_2 4 \\ &= \log_2 2 = 1 = R.H.S. \end{aligned}$$

# Problem 15

**15.** Prove that $\log_a x = \log_b x \log_c b \ldots \log_n m \log_a n$

# Solution of Problem 15

**Solution:**

$$R.H.S. = \log_b x \log_c b \ldots \log_n m \log_a n$$

$$= \frac{\log x}{\log b} \cdot \frac{\log b}{\log c} \ldots \frac{\log m}{\log n} \cdot \frac{\log n}{\log a}$$

$$= \frac{\log x}{\log a} = \log_a x = L.H.S.$$

# Problem 16

**16.** If $a^2 + b^2 = 7ab$, prove that $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$

# Solution of Problem 16

**Solution:** Given,

$$a^2 + b^2 = 7ab \Rightarrow a^2 + b^2 + 2ab = 9ab$$
$$\Rightarrow \left(\frac{a+b}{3}\right)^2 = ab$$

Taking $\log$ of both sides, we get

$$2\log\frac{a+b}{3} = \log(ab) \Rightarrow \log\frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$$

# Problem 17

**17.** Prove that $\frac{\log a(\log_b a)}{\log b(\log_a b)} = -\log_b a$

# Solution of Problem 17

**Solution:**

$$L.H.S. = \frac{\log a(\log_b a)}{\log b(\log_a b)} = \frac{\log a \frac{\log a}{\log b}}{\log b \frac{\log a}{\log b}}$$

$$= \frac{\log a \log a - \log a \log b}{\log b \log b - \log b \log a}$$

$$= \frac{\log a(\log a - \log b)}{\log b(\log b - \log a)} = -\log_b a$$

# Problem 18

**18.** Prove that $\log(1 + 2 + 3) = \log 1 + \log 2 + \log 3$

# Solution of Problem 18

**Solution:**

$$L.H.S. = \log(1+2+3) = \log 6 = \log(2.3) = \log 2 + \log 3$$
$$= \log 1 + \log 2 + \log 3 [\because \log 1 = 0]$$

# Problem 19

**19.** Prove that $2\log(1+2+4+7+14) = \log 1 + \log 2 + \log 4 + \log 7 + \log 14$

# Solution of Problem 19

**Solution:**

$$L.H.S. = 2\log(1+2+4+7+14) = 2\log 28$$

$$= \log 28^2 = \log 784 = \log(1.2.4.7.14) = \log 1 + \log 2 + \log 4 + \log 7 + \log 14$$

# Problem 20

**20.** Prove that $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 3 \log \frac{81}{80} = 1$

# Solution of Problem 20

**Solution:**

$$L.H.S. = \log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 3 \log \frac{81}{80}$$

$$= \log 2 + 16[\log 16 - \log 15] + 12[\log 25 - \log 24] + 7[\log 81 - \log 80]$$

$$= \log 2 + 16[\log 2^4 - \log 3 * 5] + 12[\log 5^2 - \log 2^3 * 3] + 7[\log 3^4 - \log 2^4 * 5]$$

$$= \log 2 + 16[4 \log 2 - \log 3 - \log 5] + 12[2 \log 5 - 3 \log 2 - \log 3] + 7[4 \log 3 - 4 \log 2 - \log 5]$$

$$= \log 2[1 + 64 - 36 - 28] + \log 3[28 - 16 - 12] + \log 5[24 - 7 - 16]$$

$$= \log 2 + \log 5 = \log 10 = 1[\text{ default base is } 10 \text{ for common logarithms}]$$