Arithmetic Progression

Shiv Shankar Dayal

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Sequence and Series

Sequence: A succession of numbers $t_1, t_2, t_3, \ldots, t_n, \ldots$ formed according to some definite rule is called a sequence. $t_1, t_2, t_3, \ldots, t_n$ are called first, second, third, ..., nth term respectively.
Alternatively, a sequence is a function whose domain is the set of natural numbers N or a subset of N and range is a set of real numbers.
Finite and Infinite Sequences: A sequnece is called a finite sequence if it has finite number of elements and is called an infinite sequence if it has infinite number of elements.
Series: By adding or subtracting the terms of a sequence, we get an expression which is called a series. If $a_1, a_2, a_3, \ldots, a_n$' is a sequence then $a_1 + a_2 + a_3 + \ldots + a_n$ is a series.
Progression: It is not mandatory for terms of a sequence to follow a pattern or formula for its nth term but when it does it is called a progression.
Arithmetic Progression: It is a progression where consecutive terms differ by a constant known as common difference.
Examples:
$1 + 2 + 3 + 4 + \ldots + 10$
$20 + 18 + 16 + 14 \ldots + 2$

nth term of an Arithmetic Progression

Let a be the first term and d be the common difference of an A. P., then

First term	$t_1 = a = a + (1-1)d$
Second term	$t_2 = a + d = a + (2-1)d$
Third term	$t_3 = a + 2d = a + (3-1)d$
....	
nth term	$t_n = a + (n-1)d$

To find the sum of n terms of an A.P.

Let a be the first term, d the c.d., t_n the nth term and S_n the sum of n terms of an A. P.

$$S_n = a + (a+d) + (a+2d) + \ldots + (t_n - 2d) + (t_n - d) + t_n$$
$$S_n = t_n + (t_n - d) + (t_n - 2d) + \ldots + (a - 2d) + (a - d) + a$$

Adding these two we get

$$2S_n = (a+t_n) + (a+t_n) + (a+t_n) + \ldots + (a+t_n) + (a+t_n) + (a+t_n)$$

$$= (a+t_n) + (a+t_n) + \ldots \text{to } n \text{ terms}$$
$$= n(a+t_n)$$
$$\therefore S_n = \frac{n}{2}(a+t_n)$$
$$= \frac{n}{2}[a + a + (n-1)d]$$
$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

Properties of an A.P.

If the same quantity is added to or subtracted from all the terms of an A.P., the resulting progression is also an arithmetic progression.

Proof:

Given A. P.	Sequence after adding k to each term of given A.P.	Sequence after subtracting k from each term of given A.P.
$t_1 = a$	$t_1 = a + k$	$t_1 = a - k$
$t_2 = a + d$	$t_1 = a + d + k$	$t_1 = a + d - k$
$t_3 = a + 3d$	$t_1 = a + 2d + k$	$t_1 = a + 2d - k$
...	...	...
$t_n = a + (n - 1)d$	$t_n = a + (n - d) - k$	$t_n = a + (n - 1)d - k$

Clearly, each term changes by k but the common difference remains same making resulting series arithmetic progression as well.

Properties of an A.P.

If the corresponsing terms of two arithmetic progressions be added or subtracted, the resulting progression is also an arithmetic progression.

Proof:

Terms of first A. P.	Terms of second A.P.	Terms of A.P. after addition
a_1	a_2	$a_1 + a_2$
$a_1 + d_1$	$a_2 + d_2$	$a_1 + a_2 + d_1 + d_2$
$a_1 + 2d_1$	$a_2 + 2d_2$	$a_1 + a_2 + 2d_1 + 2d_2$
...	...	...

Clearly, addition results in a new A.P. with first terms as sum of first terms and common difference as sum of common differences.

Properties of an A.P.

If all the terms of an A.P. are multiplies or divided by some constant then the resulting progression is also an A. P.

Proof:

Original A.P.	A.P. after multuplication	A.P. after division
a	ak	$\frac{a}{k}$
$a+d$	$(a+d)k$	$\frac{a+d}{k}$
$a+2d$	$(a+2d)k$	$\frac{a+2d}{k}$
...	...	...

Clearly, in case of multiplication new first term is ak and common difference is dk while in case of division new first term is $\frac{a}{k}$ and common difference is $\frac{d}{k}$