Summation of Series
Problems 41-48

Shiv Shankar Dayal

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Problem 41

41. Find the sum of the series $1 + 9 + 24 + 46 + 75 + ...$ to n terms.

Solution of Problem 41

41.

$$S = 1 + 9 + 24 + 46 + 75 + ... + t_n$$

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Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 1 + 8 + 15 + 22 + 29 + ... + \text{ to } n \text{ terms } - t_n$$

$$t_n = \frac{n}{2}[2 + (n-1)7] = \frac{1}{2}(7n^2 - 5n)$$

$$\therefore S_n = \frac{1}{6}n(n+1)(7n-4)$$

Problem 42

42. Find the nth term of the series

$$2 + 4 + 7 + 11 + 16 + \ldots$$

Solution of Problem 42

42.

$$S = 2 + 4 + 7 + 11 + 16 + ... + t_n$$

$$S = 2 + 4 + 7 + 11 + 16 + ... + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 2 + [2 + 3 + 4 + 5 + ... + \text{ to } (n-1) \text{ terms }] - t_n$$

$$t_n = 2 + \frac{n-1}{2}[2.2 + (n-2).1] = \frac{1}{2}(n^2 + n + 2)$$

Problem 43

43. Find the sum to 10 terms of the series $1 + 3 + 6 + 10 + ...$

Solution of Problem 43

Solution:

$$S = 1 + 3 + 6 + 10 + ... + t_n$$

$$S = 1 + 3 + 6 + 10 + ... + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 1 + 2 + 3 + 4 + ... + \text{ to } n \text{ terms } - t_n$$

$$t_n = \frac{n(n+1)}{2}$$

$$S_n = \frac{1}{2}\left(\sum n^2 + \sum n\right)$$

$$\Rightarrow S_{10} = 220$$

Problem 44

44. The odd natural numbers have been divided in groups as $(1, 3), (5, 7, 9, 11), (13, 15, 17, 19, 21, 23), ...$ Show that the sum of numbers in the nth group is $4n^3$.

Solution of Problem 44

Solution: First group has 2 numbers, second has 4 numbers and so on. So nth term will have $2n$ terms. Also, total no. of numbers till $n-1$th group is $S = 2+4+6+...+2n-2$

$$S = \frac{n-1}{2}[2.2+(n-2)2] = n(n-1)$$

So first term of the nth group will be

$$t_{n(n-1)} + 2 = 1 + (n^2-n-1).2 + 2 = 2n^2 - 2n + 1$$

Thus, required sum

$$S_{2n} = \frac{2n}{2}[2(2n^2-2n+1)+(2n-1).2]$$
$$= n[4n^2-4n+2+4n-2] = 4n^3$$

Problem 45

45. Show that the sum of numbers in each of the following groups is an square of an odd positive integer $(1), (2, 3, 4), (3, 4, 5, 6, 7), ...$

Solution of Problem 45

Solution: First term has one number, second has three numbers and third has five numbers therefore nth group will have $2n-1$ numbers.

Also, the first number is an A.P. with first term being 1 and common difference 1 so the first term of nth group is n

Thus, $S = \frac{2n-1}{2}[2.n + (2n-2).2] = (2n-1)^2$

which is square of an odd positive integer.

Problem 46

46. Find the sum to n terms of the series $2 + 5 + 14 + 41 + ...$

Solution to Problem 46

Solution:

$$S = 2 + 5 + 14 + 41 + ... + t_n$$

$$S = 2 + 5 + 14 + 41 + ... + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 2 + [3 + 9 + 27 + ... + \text{ to } (n-1) \text{ terms }] - t_n$$

$$t_n = 2 + \frac{3(3^{n-1} - 1)}{2}$$

$$S_n = \frac{1}{2} \sum 3^n + \frac{1}{2} \sum 1$$

$$= \frac{3}{4}(3^n - 1) + \frac{n}{2}$$

Problem 47

47. Find the sum to n terms of the series $1.1 + 2.3 + 4.5 + 8.7 + \ldots$

Solution of Problem 48

Solution: Clearly the first number in t_n would be nth term of a G.P. with first term 1 and common ratio 2 i.e. 2^{n-1}. The second number in t_n would be nth term of an A.P. whose first term is 1 and common difference 2 i.e. $2n-1$. Thus, $t_n = 2^{n-1}(2n-1) = n2^n - 2^{n-1}$

This is an arithmetico gerometric series. So we apply the formula

$$S_n = \frac{dr^n}{(r-1)^2} + \frac{[a+(n-1)d]r^n}{r-1} - \frac{a}{r-1} - \frac{dr}{(r-1)^2}$$

$$= \frac{1.2^n}{(2-1)^2} + \frac{[1+(n-1).1]2^n}{(2-1)} - \frac{1}{2-1} + \frac{1.2}{(2-1)^2}$$