Problems 21 to 30

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Problem 21

21. Find the first negative term of the sequence $2000, 1995, 1990, \ldots$

Solution of problem 21

Solution: Clearly, $a = 2000, d = 2000 - 1995 = 1995 - 1900 = 5$
Let t_n be the first negative term, then we have
$a + (n-1)d < 0 \Rightarrow 2000 + (n-1)5 < 0 \Rightarrow -5n + 2005 < 0$
$\Rightarrow n > 401 \therefore n = 402$ least value for which $n > 401$
Thus, 402^{nd} term will be first negative term.
$t_{402} = 2000 + (402 - 1)5 = -5$

Problem 22

22. How many terms are identical in two arithmetic progressions $2, 4, 6, 8, \ldots$ up to 100 terms and $3, 6, 9, \ldots$ up to 80 terms.

Solution of problem 22

Solution: Let r terms be identical. Now the sequence of identical terms is $6, 12, 18, \ldots$
$t_r = a + (r-1)d = 6 + (r-1)6 = 6r$
100^{th} term of the sequence $2, 4, 6, \ldots = 2 + (100-1)2 = 200$
80^{th} term of the sequence $3, 6, 9, \ldots = 3 + (80-1)3 = 240$
Thus, r^{th} term of the sequence of identical terms cannot be greater than 200
$6r \leq 200 \Rightarrow r \leq 33\frac{1}{3} \Rightarrow r = 33$
Hence, 33 terms are identical.

Problem 23

23. Find the number of all positive integers of 3 digits which are divisible by 5.

Solution of problem 23

Solution. Smallest 3 digit number divisible by 5is 100 and largest is 995.
Clearly, we have $a = 100, d = 5, t_n = 995$
$t_n = 100 + (n - 1)5 \Rightarrow 995 = 95 + 5n \Rightarrow 900 = 5n \Rightarrow n = 180$

Problem 24

24. Is 105 a term of the arithmetic progression $4, 9, 14, \ldots$?

Solution of problem 24

Solution: $a = 4, d = 9 - 4 = 14 - 9 = 5$
Let 105 be n^{th} term of the arithmetic progression.
$t_n = a + (n-1)d \Rightarrow 105 = 4 + (n-1)5 \Rightarrow 106 = 5n$
Since n is not an integer 105 is not a member of the given arithmetic progression.

Problem 25

25. Find the first negative term of the sequence $999, 995, 991, \ldots$

Solution of problem 25

Solution. $a = 999, d = 995 - 999 = 991 - 995 = -4$
Let n^{th} term is first negative number.
$t_n = a + (n-1)d = 999 + (n-1)(-4) = 1003 - 4n < 0 \Rightarrow n > \frac{1003}{4}$
Least integral value of n is 251.
$\therefore t_n = 999 + (251-1)(-4) = -1$

Problem 26

26. Each of the series $3 + 5 + 7 + \ldots$ and $4 + 7 + 10 + \ldots$ is continued to 100 term. Find how many terms are identical?

Solution of problem 26

Solution: $a = 7$ and $d = 6$
Last term of first series $t_{100} = 3 + (100 - 1)2 = 201$
Last term of second series $t_{100} = 4 + (100 - 1)3 = 301$
Clearly, last term of series of common terms $t_n < 201$
$7 + (n - 1)6 < 201 \Rightarrow 6n < 200 \Rightarrow n < 33\frac{1}{3}$
Least integral value of $n = 33$

Problem 27

27. If m tims the m^{th} term of an A.P. is equal to n times the n^{th} term, find its $(m+n)^{th}$ term.

Solution of problem 27

Solution: Let a be the first term and d be the common difference of the A.P.
Given $nt_n = mt_m$
$\therefore n[a + (n-1)d] = m[a + (m-1)]d$
$(m-n)a = d[n(n-1) - m(m-1)]$
$(m-n)a = [n^2 - n - m^2 + m]d$
$(m-n)a = -(m-n)(m+n-1)d$
$a = -(m+n-1)d$
Now, $t_{m+n} = a + (m+n-1)0 = a - a = 0$

Problem 28

28. If a, b, c be the p^{th}, q^{th} and r^{th} terms respectively of an A.P., prove that $a(q-r)+b(r-p)+c(p-q)=0$

Solution of problem 28

Solution: Let x be the first term and d be the common difference of A.P.
$t_p = a = x + (p-1)d$
$t_q = b = x + (q-1)d$
$t_r = c = x + (r-1)d$
$t_p - t_r = a - c = (p-r)d$
$t_q - t_p = b - a = (q-p)d$
$t_r - t_q = c - b = (r-q)d$
Now, $a(q-r) + b(r-p) + c(p-q) = q(a-c) + r(b-a) + c(p-q)$
$= q(p-r)d + r(q-p)d + p(r-q)d$
$= 0$

Problem 29

29. Find the number of integers between 100 and 1000 that are divisible by 7 and not divisible by 7.

Solution of problem 29

Solution: First number between 100 and 1000 divisible by $7 = 105$ and last number $= 994$
Hencer $a = 105, d = 7, t_n = 994$
$t_n = 994 = 105 + (n-1)5$
$896 = 7n \therefore n = 128$
Numbers not divisible by $7 =$ Total number of numbers between 100 and 1000 - number of numbers between 100 and 1000 divisible by 7
$= 899 - 128 = 771$

Problem 30

30. If a, b, c be the p^{th}, q^{th} and r^{th} terms respectively of an A.P., prove that $(a-b)r + (b-c)p + (c-a)q = 0$

Splution of problem 30

Solution: Let x be the first term and d be the common difference of the A.P.

$t_p = a = x + (p-1)d$
$t_q = b = x + (q-1)d$
$t_r = c = x + (r-1)d$
$t_p - t_q = a - b = (p-q)d$
$t_q - t_r = b - c = (q-r)d$
$t_r - t_p = c - a = (r-p)d$
Now, $(a-b)r + (b-c)p + (c-a)q$
$= d[(p-q)r + (q-r)p + (r-p)q]$
$= 0$