Geometric Progression
Problems 1-10

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Problem 1

1. How many terms are in the G.P. $5, 20, 80, \ldots, 5120$?

Solution of problem 1

Solution: Given $a = 5$ and $r = \frac{20}{5} = 4$
Let there are n terms in G.P.
The formula for t_n is $t_n = ar^{n-1}$

$$t_n = ar^{n-1} = 5120 \Rightarrow 5.4^{n-1} = 5120$$

$$\Rightarrow 4^{n-1} = 1024 = 4^5 \Rightarrow n - 1 = 5$$

$$\therefore n = 6$$

Problem 2

2. How many terms are in the G.P. $0.03, 0.06, 0.12, \ldots, 3.84$?

Solution of problem 2

Solution: Given $a = 0.03$ and $r = \frac{0.06}{0.03} = 2$
Let there are n terms in G.P.
The formula for t_n is $t_n = ar^{n-1}$

$$t_n = ar^{n-1} = 3.84$$
$$\Rightarrow 0.03.2^{n-1} = 3.84$$
$$\Rightarrow 2^{n-1} = 128 = 2^7$$
$$\Rightarrow n - 1 = 7 \Rightarrow n = 8$$

Problem 3

3. A boy agrees to work at the rate of one rupee the first day, two rupee the second day, four rupees the third day, eight rupees the fourth day and so on. How much would he get on $20th$ day?

Solution of problem 3

Solution: Clearly, the money gained by the boy is in G.P. with $a = 1$ and $r = 2$
Thus, the money made by boy on 20th day

$$t_{20} = 1.2^{20-1} = 2^{19}$$

$$\therefore t_{20} = 524288$$

Problem 4

4. The population of a city in January 1987 was $20,000$. It increased at the rate of 2% per annum. Find the population of the city in January 1997.

Solution of problem 4

Solution: Population in 1988 $= 20000 + \frac{2}{100} \times 20000 = 20000 * 1.02$
Population in 1989 $= 20000 * 1.02 + \frac{2}{100} \times 20000 * 1.02 = 20000 * (1.02)^2$
Thus, we see that it is a geometric progression with $a = 20000$ and $r = 1.02$
Thus, after 10 years population in 1997 $= 20000 * (1.02)^{10} = 24379$

Problem 5

5. The sum of n terms of a sequence is $2^n - 1$, find its nth term. Is the sequence in G.P.?

Solution of problem 5

Solution: Given, $S_n = 2^n - 1 \therefore S_{n-1} = 2^{n-1} - 1$

$$t_n = S_n - S_{n-1} = 2^n - 1 - 2^{n-1} + 1 = 2^n - 2^{n-1} = 2^{n-1}(2-1) = 2^{n-1}$$

$$\therefore \frac{t_n}{t_{n-1}} = \frac{2^{n-1}}{2^{n-2}} = 2$$

Since the ratio of consecutive terms is a constant and independent of n the sequence is in G.P.

Problem 6

6. If the fifth term of a G.P. is 81 and second term is 24. Find the G.P.

Solution of problem 6

Solution: Let a be the first term and r be the common ratio of the G.P.

$$t_2 = ar = 24 \text{ and } t_5 = ar^4 = 81$$

Dividing we get,

$$r^3 = \frac{81}{24} = \frac{27}{8}$$

$$r = \frac{3}{2}$$

Substituring the value of r for t_2

$$t_2 = ar = 24 \Rightarrow a = \frac{24}{r} = \frac{24.2}{3} = 16$$

Therefore, the G.P. is $16, 24, 36, 54, 81, \ldots$

Problem 7

7. The seventh term of a G.P. is 8 times the fourth term. Find the G.P. when its 5th term is 48.

Solution of problem 7

Solution: Let a be the first term and r be the common ratio of the G.P.

$$t_7 = ar^6 \text{ and } t_4 = ar^3$$

Given, $t_7 = 8.t_4$

$$ar^6 = 8.ar^3 \Rightarrow r^3 = 8 \Rightarrow r = 2$$

Also, given $t_5 = 48 \Rightarrow ar^4 = 48 \Rightarrow a = \frac{48}{2^4} = 3$
Thus, G.P. is 3, 6, 12, 24, ...

Problem 8

8. If the 5th and 8th terms of a G.P. be 48 and 384 respectively, find the G.P.

Solution of problem 8

Solution: Let a be the first term and r be the common ratio of G.P.
Given, $t_5 = ar^4 = 48$ and $t_8 = ar^7 = 384$
Dividing we get,

$$\frac{t_8}{t_5} = \frac{ar^7}{ar^4} = r^3 = \frac{384}{48} = 8$$

$$\Rightarrow r = 2$$

Substituting the value of r in $t_5, a.2^4 = 48 \Rightarrow a = 3$
Thus, G.P. is 3, 6, 12, 24, ...

Problem 9

9. If the 6th and 10th terms of a G.P. are $\frac{1}{16}$ and $\frac{1}{256}$ respectively, find the G.P.

Solution of problem 9

Solution: Let a be the first term and r be the common ratio of G.P.
Given, $t_6 = ar^5 = \frac{1}{16}$ and $t_{10} = ar^9 = \frac{1}{256}$
Dviding we get, $\frac{t_{10}}{t_6} = r^4 = \frac{1}{256}16 = \frac{1}{16}$

$$\Rightarrow r = \pm\frac{1}{2}$$

Substituting the value of r in t_6, we get

$$t_6 = a \pm \frac{1}{32} = \frac{1}{16} \Rightarrow a = \pm 2$$

Thus, the G.P. is either $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ or $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$.

Problem 10

10. If the pth, qth and rth terms of a G.P. be $a, b, c (a, b, c > 0)$, then prove that $(q - r)\log a + (r - p)\log b + (p - q)\log c = 0$

Solution of problem 10

Solution: Let x be the first term and y be the common ratio. Then,

$$a = xy^{p-1} \Rightarrow \log a = \log x + (p-1)\log y$$

$$b = xy^{q-1} \Rightarrow \log b = \log x + (q-1)\log y$$

$$c = xy^{r-1} \Rightarrow \log c = \log x + (r-1)\log y$$

Now,

$$(q-r)\log a + (r-p)\log b + (p-q)\log c = (q-r)[\log x + (p-1)\log y + (r-p)[\log x + (q-1)\log y] + (p-q)[\log x + (r-1)\log y]$$

$$= \log x(p-q+r-p+p-q) + \log y[(q-r)(p-1) + (r-p)(q-1) + (p-q)(r-1)]$$

$$= \log y(pq - q - pr + r + qr - r - pq + p + pr - p - qr + q)$$

$$= 0$$