# Logarithm Problem 51-60

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# Problem 51

**51.** Solve $\log_a[1+\log_b\{1+\log_c(1+\log_p x)\}]=0$

# Solution of Problem 51

**Solution:**

$$\text{Given, } \log_a[1+\log_b\{1+\log_c(1+\log_p x)\}]=0$$

$$1+\log_b\{1+\log_c(1+\log_p x)\}=1$$

$$\log_b\{1+\log_c(1+\log_p x)\}=0$$

$$1+\log_c(1+\log_p x)=1$$

$$\log_c(1+\log_p x)=0$$

$$1+\log_p x=1$$

$$\log_p x=0$$

$$x=1$$

# Problem 52

**52.** Solve $\log_7 \log_5(\sqrt{x+5} + \sqrt{x}) = 0$

# Solution of the Problem 52

**Solution:**

Given, $\log_7 \log_5(\sqrt{x+5}+\sqrt{x}) = 0$

$$\Rightarrow \log_5(\sqrt{x+5}+\sqrt{x}) = 1$$

$$\Rightarrow \sqrt{x+5}+\sqrt{x} = 5$$

$$\Rightarrow \sqrt{x+5} = 5-\sqrt{x}$$

Squaring both sides, we get

$$x+5 = 25+x-10\sqrt{x}$$

$$\sqrt{x=2} \Rightarrow x = 4$$

# Problem 53

**53.** Solve $\log_2 x + \log_4(x+2) = 2$

# Solution of Problem 53

**Solution:**

$$\log_2 x + \log_4 (x+2) = 2$$
$$\Rightarrow \log_2 x + \log_{2^2} (x+2) = 2$$
$$\Rightarrow \log_2 x + \frac{1}{2} \log_2 (x+2) = 2$$
$$\Rightarrow 2\log_2 x + \log_2 (x+2) = 4$$
$$\Rightarrow x^2 (x+2) = 16$$
$$\Rightarrow x = 2$$

# Problem 54

**54.** Solve $\frac{\log(x+1)}{\log x} = 2$

# Solution of Problem 54

**Solution:**

$$\frac{\log(x+1)}{\log x} = 2$$

$$\Rightarrow \log(x+1) = 2\log x = \log x^2$$

$$\Rightarrow x+1 = x^2$$

$$\Rightarrow x = \frac{1 \pm 5}{2}$$

$$\because x > 0, x = \frac{1+\sqrt{5}}{2}$$

# Problem 55

**55.** Solve $2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0[a > 0]$

# Solution of Problem 54

**Solution:**

$$2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0[a > 0]$$

$$\Rightarrow \frac{2}{\log_a x} + \frac{1}{\log_a ax} + \frac{3}{\log_a a^2x} = 0$$

$$\Rightarrow \frac{2}{\log_a x} + \frac{1}{\log_a a + \log_a x} + \frac{3}{\log_a a^2 + \log_a x} = 0$$

$$\Rightarrow \frac{2}{\log_a x} + \frac{1}{1 + \log_a x} + \frac{3}{2 + \log_a x} = 0$$

Let $\log_a x = z,$ so the above equation becomes

$$\frac{2}{z} + \frac{1}{z+1} + \frac{3}{z+2} = 0$$

$$\Rightarrow 6z^2 + 11z + 4 = 0$$

$$\Rightarrow z = -\frac{1}{2}, -\frac{4}{3}$$

$$\therefore x = \frac{1}{\sqrt{a}}, \frac{1}{\sqrt[3]{a^4}}$$

# Problem 56

**56.** Solve $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$

# Solution of Problem 56

**Solution:**

$$x + \log_{10}(1 + 2^x) = \log_{10} 5^x + \log_{10} 6$$

$$\Rightarrow \log_{10} 10^x + \log_{10}(1 + 2^x) = \log_{10} 5^x + \log_{10} 6$$

$$\Rightarrow \log_{10} 10^x(1 + 2^x) = \log_{10} 5^x * 6$$

$$\Rightarrow 2^x(1 + 2^x) = 2 * 3$$

$$\Rightarrow 2^x = 2, 1 + 2^x = 3 \Rightarrow x = 1$$

# Problem 57

**57.** Solve $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \sqrt{2}$

# Solution of Problem 57

**Solution:**

$$x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \sqrt{2}$$

Taking $\log_2$ of both sides

$$\left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right)\log_2 x = \frac{1}{2}\log_2 2$$

$$\left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right)\log_2 x = \frac{1}{2}$$

Let $z = \log_2 x$

$$\Rightarrow \left(\frac{3}{4}z^2 + z - \frac{5}{4}\right)z = \frac{1}{2}$$

Solving this cubic equation yields $x = 2, \frac{1}{4}, \frac{1}{\sqrt[3]{2}}$

## Problem 58

**58.** Solve $(x^2+6)^{\log_3 x} = (5x)^{\log_3 x}$

# Solution of Problem 58

**Solution:** $\log_3 x$ has a possible value of $0$, in which case $x = 1$. If $\log_3 x \neq 1$

$$x^2 + 6 = 5x \Rightarrow x = 2, 3$$

## Problem 59

**59.** Solve $(3+2\sqrt{2})^{x^2-6x+9}+(3-2\sqrt{2})^{x^2-6x+9}=6$

# Solution of Problem 59

**Solution:**

$$3+2\sqrt{2}=\frac{1}{3-2\sqrt{2}}$$

So given equation can be written as

$$(3+2\sqrt{2})^{x^2-6x+9}+(3+2\sqrt{2})^{-(x^2-6x+9)}=6$$

Let $z=(3+2\sqrt{2})^{x^2-6x+9}$

$$\Rightarrow z+\frac{1}{z}=6$$

$$\Rightarrow z=3\pm 2\sqrt{2}$$

$x=2,4$ because other roots are irrational.

# Problem 60

**60.**Solve $\log_8\left(\frac{8}{x^2}\right) \div (\log_8 x)^2 = 3$

# Solution of Problem 60

**Solution:**

$$\text{Given, } \log_8\left(\frac{8}{x^2}\right) \div (\log_8 x)^2 = 3$$

$$\Rightarrow \log_8 8 - \log_8 x^2 = 3(\log_8 x)^2$$

$$1 - 2\log_8 x = 3(\log_8 x)^2$$

$$\text{Let } z = \log_8 x$$

$$1 - 2z = 3z^2$$

$$z = -1, \frac{1}{3}$$

$$x = 2, \frac{1}{8}$$