Logarithm Problem 111-120

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Problem 111

111. If n is a natural number such that $n = p_1^{a_1} p_2^{a_2} p_3^{a_3} \dots p_k^{a_k}$ and $p_1, p_2, p_3, \dots, p_k$ are distinct primes, then show that $\log n \geq k \log 2$

Solution of Problem 111

Solution: Since n is a natural number and $p_1, p_2, p_3, \ldots, p_k$ are distinct primes, therefore $a_1, a_2, \ldots, a_k$ are also natural numbers.

Now, $n = p_1^{a_1} p_2^{a_2} p_3^{a_3} \ldots p_k^{a_k}$

$\log n = a_1 \log p_1 + a_2 \log p_2 + \ldots + a_k \log p_k$ $\log n \geq \log 2 + \log 2 + \ldots + \log 2$ [since bases are p_is are primes so minimum value is 2 and powers are natural numbers so they are greater than 1]

$\log n \geq k \log 2$

Problem 112

112. Prove that $\log_4 18$ is an irrational number.

Solution of Problem 112

Solution: $\log_4 18 = \log_{2^2}(2.3^2) = \frac{1}{2} + \log_2 3$

Thus, it will be enough to prove that $\log_2 3$ is a irrational number.

Let $\log_2 3 = \frac{p}{q}$ where $p, q \in I$

$2^{\frac{p}{q}} = 3 \Rightarrow 2^p = 3^q$

However, 2^p is an even number and 3^q is an odde number, and hence the equality will never be achieved. Therefore, $\log_2 3$ is an irrational number.

Problem 113

113. Find the value of $\log_{30} 8$, if $\log_{30} 3 = a$ and $\log_{30} 5 = b$.

Solution of Problem 113

Solution:

$$\log_{30} 8 = 3\log_{30} 2 = 3\log_{30} \frac{30}{15}$$
$$= 3\log_{30} 30 - 3\log_{30} 15 = 3 - 3(\log_{30} 3 + \log_{30} 5)$$
$$= 3(1 - a - b)$$

Problem 114

114. Find the value of $\log_{54} 168$, if $\log_7 12 = a$ and $\log_{12} 24 = b$.

Solution of Problem 114

Solution: Given, $\log_7 12 = a$ and $\log_{12} 24 = b$

Multiplying, $ab = \log_7 24$

Adding 1 on both sides

$ab + 1 = \log_7 24 + \log_7 7 = \log_7 168$

Similalry, $8a = \log_7 12^8$ and $5ab = \log_7 24^5$

$\frac{ab+1}{8a-5ab} = \frac{\log_7 168}{\log_7 12^8 - \log_7 24^5}$

$= \frac{\log_7 168}{\log_7 \frac{12^8}{24^5}} = \frac{\log_7 168}{\log_7 54} = \log_{54} 168$

Problem 115

115. If $a \neq 0$ and $\log_x(a^2+1) < 0$ then find the interval in which x lies.

Solution of Problem 115

Solution: In all the cases $x > 0$ for logarithm to exist.

Case I: When $x > 1, x > a^2 + 1$. Also, $a^2 + 1 > 1 \therefore x > 1$

Case II: When $x < 1, x < a^2 + 1$. Also, $a^2 > 0 \therefore x < 1$

Problem 116

116. If $\log_{12} 18 = a$ and $\log_{24} 54 = b$, prove that $ab + 5(a - b) = 1$

Solution of Problem 116

Solution:

$$ab + 5(a-b) = \frac{\log 18 \log 54}{\log 12 \log 24} + 5\left(\frac{\log 18}{\log 12} - \frac{\log 54}{\log 24}\right)$$

$$= \frac{\log 18 \log 54 + 5(\log 18 \log 24 - \log 54 \log 12)}{\log 12 \log 24}$$

$$\log 18 = \log 2 + 2\log 3, \log 12 = 2\log 2 + \log 3, \log 24 = 3\log 2 + \log 3, \log 54 = \log 2 + 3\log 3$$

Substituting and simplifying we will obtain the desired result.

Problem 117

117. If $a, a_1, a_2, \dots, a_n$ are in G.P. and $b, b_1, b_2, \dots, b_n$ in A.P. with positive terms and also the common difference of A.P. and common ratios of G.P. are positive, show that there exists a system of logarithm for which
$\log a_n - b_n = \log a - b$ for any n. Find base of the system.

Solution of Problem 117

Solution: Let r be the common ratio of G.P. and d be the common difference.

$$\log a_n - b_n = \log a + n \log r - (b + nd) = \log a - d$$

$$n \log r - nd = 0 \Rightarrow \log r = d$$

Thus, base $= r^{\frac{1}{d}}$

Problem 118

118. If $\log_3 2, \log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P., find the value of x.

Solution of Problem 118

Solution:

Given, $\log_3 2, \log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P.

$$\Rightarrow 2\log_3(2^x - 5) = \log_3\left(2^x - \frac{7}{2}\right) + \log_3 2$$

$$(2^x - 5)^2 = 2(2^x - \frac{7}{2})$$

Let $z = 2^x$, then we have

$$z^2 - 10z + 25 = 2x - 7 \Rightarrow z^2 - 12z + 32 = 0$$

$$\Rightarrow z = 8, 4 \Rightarrow x = 2, 3$$

But if $x = 2, 2^x - 5 < 0$ which cannot be so only acceptable value of x is 3.

Problem 119

119. Prove that $\log_2 7$ is an irrational number.

Solution of Problem 119

Solution: Let $\log_2 7 = \frac{p}{q}$ where $p, q \in I$

$\Rightarrow 2^p = 7^2$, however, 2^p is an even number and 7^q is an odd number. Thus, our assumption is wrong and $\log_2 7$ is an irrational number.

Problem 120

120. If $\log_{0.5}(x-2) < \log_{0.25}(x-2)$, then find the interval in which x lies.

Solution of Problem 120

Solution:

$$\text{Given, } \log_{0.5}(x-2) < \log_{0.25}(x-2)$$
$$\Rightarrow (x-2)^2 > (x-2)$$
$$(x-2)(x-3) > 0$$
$$\Rightarrow x < 2, x > 3$$

Thus, $x > 3$ for logarithm function to be defined.