Logarithm Problem 71-80

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Problem 71

71. Solve $\log_5\left(5^{\frac{1}{x}} + 125\right) = \log_5 6 + 1 + \frac{1}{2x}$

Solution of Problem 71

Solution:

$$\text{Given, } \log_5\left(5^{\frac{1}{x}}+125\right)=\log_5 6+1+\frac{1}{2x}$$

$$\Rightarrow \log_5\left(5^{\frac{1}{x}}+126\right)-\log_5 6=1+\frac{1}{2x}$$

$$\Rightarrow \log_5\left(\frac{5^{\frac{1}{x}}+125}{6}\right)=1+\frac{1}{2x}$$

$$\Rightarrow \frac{5^{\frac{1}{x}}+125}{6}=5^{1+\frac{1}{2x}}$$

$$\Rightarrow 5^{\frac{1}{x}}+126=30.5^{\frac{1}{2x}}$$

$$\text{Let } z=5^{\frac{1}{2x}}$$

$$\Rightarrow z^2-30z+125=0$$

$$z=5,25\Rightarrow x=\frac{1}{2},\frac{1}{4}$$

Problem 72

72. Solve for x and y: $\log_{100}|x+y| = 2$ and $\log_{10} y - \log_{10}|x| = \log_{100} 4$

Solution of Problem 72

Solution:

$$\text{For } \log_{100}|x+y| = \frac{1}{2}$$

$$\Rightarrow (x+y)^2 = 100$$

$$\text{For } \log_{10} y - \log_{10}|x| = \log_{100} 4$$

$$\Rightarrow \log_{10}\frac{y}{|x|} = \log_{10} 2$$

$$\Rightarrow y = 2|x| \Rightarrow y^2 = 4x^2$$

$$\text{When } x > 0, x = \frac{10}{3}, \text{ when } x < 0, x = -10 \Rightarrow y = \frac{20}{3}, 20$$

Problem 73

73. Solve $2\log_2\log_2 x+\log_{\frac{1}{2}}\log_2(2\sqrt{2}x)=1$

Solution of Problem 73

Solution:

$$\text{Given, } 2\log_2 \log_2 x + \log_{\frac{1}{2}} \log_2(2\sqrt{2}x) = 1$$

$$\Rightarrow \log_2(\log_2 x)^2 - \log_2 \log_2(2\sqrt{2}x) = 1$$

$$\Rightarrow \log_2\left(\frac{(\log_2 x)^2}{\log_2(2\sqrt{2}x)}\right) = 1$$

$$\Rightarrow (\log_2 x)^2 = 2\log_2(2\sqrt{2}x)$$

$$\Rightarrow (\log_2 x)^2 - 3 - 2\log_2 x = 0$$

$$\Rightarrow z^2 - 2z - 3 = 0, \text{ where } z = \log_2 x$$

$$\Rightarrow z = -1, 3$$

$$\Rightarrow x = \frac{1}{2}, 8$$

However, for log to be defined $x > 0, \log_2 x > 0, \log_2 2\sqrt{2}x > 0$ and thus $x = 8$ is only acceptable solution.

Problem 74

74. Solve $\log_{\frac{3}{4}} \log_8(x^2+7) + \log_{\frac{1}{2}} \log_{\frac{1}{4}}(x^2+7)^{-1} = -2$

Solution of Problem 74

Solution:

$$\text{Given, } \log_{\frac{3}{4}} \log_8(x^2+7) + \log_{\frac{1}{2}} \log_{\frac{1}{4}}(x^2+7)^{-1} = -2$$

$$\Rightarrow \log_{\frac{3}{4}} \log_{2^3}(x^2+7) + \log_{\frac{1}{2}} \log_{2^{-2}}(x^2+7)^{-1} = -2$$

$$\Rightarrow \log_{\frac{3}{4}}[\frac{1}{3}\log_2(x^2+7)] + \log_{\frac{1}{2}}[\frac{1}{2}\log_2(x^2+7)] = -2$$

$$\text{Let } y = \log_2(x^2+7)$$

$$\Rightarrow \log_{\frac{3}{4}} \frac{y}{3} + \log_{\frac{1}{2}} \frac{1}{2} + \log_{\frac{1}{2}} y = -2$$

$$\Rightarrow \log_{\frac{3}{4}} y - \log_{\frac{3}{4}} 3 + 1 - \log_2 y = -2$$

$$\Rightarrow \log_2 y(\log_{\frac{3}{4}} 2 - 1) = -3 + \log_{\frac{3}{4}} 3$$

$$\Rightarrow \log_2 y \left(\log_{\frac{3}{4}} 2 - \log_{\frac{3}{4}} \frac{3}{4}\right) = \log_{\frac{3}{4}} \left(\frac{3}{4}\right)^{-3} + \log_{\frac{3}{4}} 3$$

$$\Rightarrow \log_2 y \log_{\frac{3}{4}} \frac{8}{3} = \log_{\frac{3}{4}} \frac{64}{9} = 2\log_{\frac{3}{4}} \frac{8}{3}$$

$$\log_2 y = 2 \Rightarrow y = 4 \Rightarrow \log_2(x^2+7) = 4 \Rightarrow x = \pm 3$$

Problem 75

75. Solve the following equations for x and y :

$$\log_{10} x + \log_{10} x^{\frac{1}{2}} + \log_{10} x^{\frac{1}{4}} + \ldots \infty = y$$

$$\frac{1 + 3 + 5 + \ldots + (2y - 1)}{4 + 7 + 10 + \ldots + (3y + 1)} = \frac{20}{7 \log_{10} x}$$

Solution of Problem 75

Solution:

$$\log_{10} x + \log_{10} x^{\frac{1}{2}} + \log_{10} x^{\frac{1}{4}} + \ldots \infty = y$$

$$\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots \infty\right] \log_{10} x = y$$

$$\frac{1}{1 - \frac{1}{2}} \log_{10} x = y$$

$$\Rightarrow \log_{10} x = \frac{y}{2}$$

$$\frac{1 + 3 + 5 + \ldots + (2y - 1)}{4 + 7 + 10 + \ldots + (3y + 1)} = \frac{20}{7 \log_{10} x}$$

$$\Rightarrow \frac{\frac{y}{2}[2 + (y - 1)2]}{\frac{y}{2}[8 + (y - 1)3]} = \frac{20}{7.\frac{y}{2}}$$

$$\Rightarrow \frac{2y}{3y + 5} = \frac{40}{7y}$$

$$\Rightarrow 7y^2 - 60y - 100 = 0$$

$$y = 10, \frac{-10}{7}$$

Since no. of terms cannot be a fraction, therefore $y = 10$. Hence $x = 10^5$

Problem 76

76. Solve $18^{4x-3} = (54\sqrt{2})^{3x-4}$

Solution of Problem 76

Solution:

Given, $18^{4x-3} = (54\sqrt{2})^{3x-4}$

Taking $\log$ of both sides, we get

$$(4x-3)\log 18 = (3x-4)\log(18.3\sqrt{2}) = (3x-4)\log 18^{\frac{3}{2}}$$

$$\Rightarrow 4x - 3 = (3x-4)\frac{3}{2}$$

$$\Rightarrow x = 6$$

Problem 77

77. Solve $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$

Solution of Problem 77

Solution:

$$\text{Given, } 4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$$

$$\Rightarrow 4^{\log_{3^2} 3} + 9^{\log_2 2^2} = 10^{\log_x 83}$$

$$\Rightarrow 4^{\frac{1}{2}\log_3 3} + 9^{2\log_2 2} = 10^{\log_x 83}$$

$$\Rightarrow 4^{\frac{1}{2}} + 9^2 = 10^{\log_x 83}$$

$$\Rightarrow 83 = 10^{\log_x 83}$$

Taking $\log_{10}$ of both sides, we gte

$$\log_{10} 83 = \log_x 83 \Rightarrow x = 10$$

Problem 78

78. Solve $3^{4\log_9(x+1)} = 2^{2\log_2 x} + 3$

Solution of Problem 78

Solution:

$$\text{Given, } 3^{4\log_9(x+1)} = 2^{2\log_2 x} + 3$$
$$\Rightarrow 3^{4\log_{3^2}(x+1)} = 2^{\log_2 x^2} + 3$$
$$\Rightarrow 3^{\log_3 (x+1)^2} = x^2 + 3$$
$$\Rightarrow (x+1)^2 = x^2 + 3 \Rightarrow x = 1$$

Problem 79

79. Solve $\frac{6}{5}a^{\log_a x \log_{10} a \log_a 5} - 3^{\log_{10}\left(\frac{x}{10}\right)} = 9^{\log_{100} x + \log_4 2}$

Solution of Problem 79

Solution:

$$\text{Given, } \frac{6}{5}a^{\log_a x \log_{10} a \log_a 5} - 3^{\log_{10}\left(\frac{x}{10}\right)} = 9^{\log_{2^2} x + \log_{2^2} 2}$$

$$\Rightarrow \frac{6}{5}a^{\log_a x \log_{10} a \log_a 5} - 3^{\log_{10} x - \log_{10} 10} = 9^{\log_{10^2} x + \log_{2^2} 2}$$

$$\Rightarrow \frac{6}{5}a^{\log_a x \log_{10} a \log_a 5} - 3^{\log_{10} x - 1} = 9^{\frac{1}{2}\log_{10} x + \frac{1}{2}\log_2 2}$$

$$\Rightarrow \frac{6}{5}(a^{\log_a 5})^{\log_{10} x} = 3^{\log_{10} x - 1} = 3^{\log_{10} x + 1}$$

$$\Rightarrow \frac{6}{5}5^{\log_{10} x} = 3^{\log_{10} x - 1}[1 + 3^2]$$

$$\Rightarrow \left(\frac{5}{3}\right)^{\log_{10} x - 1} = \frac{10}{6} = \frac{5}{3}$$

$$\Rightarrow \log_{10} x - 1 = 1 \Rightarrow x = 100$$

Problem 80

80. Solve $2^{3x+\frac{1}{2}} + 2^{x+\frac{1}{2}} = 2^{\log_2 6}$

Solution of Problem 80

Solution:

$$\text{Given, } 2^{3x+\frac{1}{2}} + 2^{x+\frac{1}{2}} = 2^{\log_2 6}$$

$$\Rightarrow 2^{3a}\sqrt{2} + 2^{x}\sqrt{2} = 6$$

$$\Rightarrow (2x^2)^3 + 2^x = 3\sqrt{2}$$

$$\Rightarrow 2^x = \sqrt{2}, \frac{-\sqrt{2} \pm \sqrt{5}i}{2}$$

Ignoring complex roots we have $2^x = \sqrt{2} \Rightarrow x = \frac{1}{2}$