Logarithm Problem 1-10

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Problem 1

1. Find the value of x where $\log_{\sqrt{8}} x = \frac{10}{3}$

Solution of Problem 1

Solution:

$$\log_{\sqrt{8}} x = \frac{10}{3} \Rightarrow \log_{2^{\frac{3}{2}}} x = \frac{10}{4}$$

$$\Rightarrow \frac{2}{3} \log_2 x = \frac{10}{3} \Rightarrow \log_2 x = 5$$

$$\Rightarrow x = 5^2 = 25$$

Problem 2

2. Prove that $\log_b a . \log_c b \log_a c = 1$

Solution of Problem 2

Solution:

$$L.H.S. = \log_b a . \log_c b \log_a c = \frac{\log a}{\log b} \frac{\log b}{\log c} \frac{\log c}{\log a} = 1 = R.H.S.$$

Problem 3

3. Prove that $\log_3 \log_2 \log_{\sqrt{5}}(625) = 1$

Solution of Problem 3

Solution:

$$\log_3 \log_2 \log_{\sqrt{5}}(625) = \log_3 \log_2 \log_{\sqrt{5}} 5^4$$
$$= \log_3 \log_2 8 = \log_3 3 = 1$$

Problem 4

4. If $a^2 + b^2 = 23ab$, then prove that $\log \frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$

Solution of Problem 4

Solution:

$$a^2 + b^2 = 23ab \Rightarrow (a+b)^2 = 25ab \Rightarrow \left(\frac{a+b}{5}\right)^2 = ab$$

Taking $\log$ of both sides

$$2\log\frac{a+b}{5} = \log(ab) \Rightarrow \log\frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$$

Problem 5

5. Prove that $7 \log \frac{16}{15} + 5 \log \frac{25}{24} + 3 \log \frac{81}{80} = \log 2$

Solution of Problem 5

Solution:

$$L.H.S. = 7[\log 2^4 - \log(3.5)] + 5[\log 5^2 - \log(8.3)] + 3[\log 3^4 - \log 16.5]$$
$$= 7[4\log 2 - \log 3 - \log 5] + 5[2\log 5 - 3\log 2 - \log 3] + 3[4\log 3 - 4\log 2 - \log 5]$$
$$= \log 2 = R.H.S.$$

Problem 6

6. Find the value of $\log \tan 1^\circ + \log \tan 2^\circ + ... + \log \tan 89^\circ$

Solution of Problem 6

Solution:

$$L.H.S. = \log\tan 1^\circ + \log\tan 2^\circ + ... + \log\tan 89^\circ$$

$$= \log(\tan 1^\circ . \tan 2^\circ . ... \tan 89^\circ)$$

$$= \log(\tan 1^\circ . \tan 89^\circ)(\tan 2^\circ . \tan 88^\circ) ... \tan 45^\circ$$

$$= \log(\tan 1^\circ . \cot 1^\circ)(\tan 2^\circ \cot 2^\circ) ... \tan 45^\circ$$

$$= \log(1.1.1. ... 1) = \log 1 = 0$$

Problem 7

7. Evaluate $\log_9 \tan \frac{\pi}{6}$

Solution of Problem 7

Solution:

$$\log_9 \tan \frac{\pi}{6} = \log_{3^2} \frac{1}{\sqrt{3}}$$

$$= \frac{1}{2} \log_3 3^{-\frac{1}{2}} = -\frac{1}{4}$$

Problem 8

8. Evaluate $\frac{\log_{a^2} b}{\log_{\sqrt{a}} b^2}$

Solution of Problem 8

Solution:

$$\frac{\log_{a^2} b}{\log_{\sqrt{a}} b^2} = \frac{2 \log_a b}{2.2 \log_a b}$$

$$= \frac{1}{8}$$

Problem 9

9. Evaluate $\log_{\sqrt{5}} .008$

Solution of Problem 9

Solution:

$$\log_{\sqrt{5}} .008 = \log_{\sqrt{5}} \frac{8}{1000}$$

$$= \log_{\sqrt{5}} 8 - \log_{\sqrt{5}} 1.125 = \log_{\sqrt{5}} 8 - \log_{\sqrt{5}} 8 - \log_{\sqrt{5}} 125 = -\log_{\sqrt{5}} 5^3 = -6$$

Problem 10

10. Evaluate $\log_{2\sqrt{3}} 144$

Solution of Problem 10

Solution:

$$\log_{2\sqrt{3}} 144 = \log_{12^{\frac{1}{2}}} 12^2 = 4$$