Geometric Progression
Problems 11-20

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May 26, 2021

Problem 11

11. If the $(p+q)$th term of a G.P. is a and the $(p-q)$th term is b, show that its pth term is $\sqrt{ab}$.

Solution of problem 11

Solution: Let x be the first term and y be the common ratio. Then we have,

$$t_{p+q} = xy^{p+q-1} = a \text{ and } t_{p-q} = xy^{p-q-1} = b$$

Multiplying both we get

$$x^2y^{2p-2} = ab$$

$$(xy^{p-1})^2 = ab$$

$$xy^{p-1} = \sqrt{ab}$$

$$\Rightarrow t_p = \sqrt{ab}$$

Problem 12

12. If the pth, qth and rth terms of a G.P. be x, y and z respectively, prove that $x^{q-r}.y^{r-p}.z^{p-q} = 1$

Solution of problem 12

Solution: Let a be the first term and b be the common ratio of the G.P. Then we have,

$$t_p = x = ab^{p-1}$$

$$t_q = y = ab^{q-1}$$

$$t_r = z = ab^{r-1}$$

$$\therefore x^{q-r}.y^{r-p}.z^{p-q} = a^{(q-r+r-p+p-q)}b^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$

$$= a^0 b^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q} = b^0 = 1$$

Problem 13

13. The first term of a G.P. is 1. The sum of third and fifth terms is 90. Find the common ratio of G.P.

Solution of problem 13

Solution: Let r be the common ratio. Since first term is 1 we have,

$$t_3 = r^2 \text{ and } t_5 = r^4$$

Given that

$$t_3 + t_5 = 90$$
$$\Rightarrow r^4 + r^2 = 90$$
$$r^4 + r^2 - 90 = 0$$
$$r^4 + 10r^2 - 9r^2 - 90 = 0$$
$$(r^2 + 10)(r^2 - 9) = 0$$
$$\Rightarrow r = \pm 3$$

Problem 14

14. Fifth term of a G.P. is 2. Find the product of its first nine terms.

Solution of problem 14

Solution: Let a be the first term and r be the common ratio of the G.P. Given, $t_5 = ar^4 = 2$. Also,

$$t_1.t_2.t_3 \dots .t_9 = a^9 r^{1+2+3+\dots+8}$$

$$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$$

Problem 15

15. The fourth, seventh and last term of a G.P. are 10, 80 and 2560 respectively. Find the first term and number of terms in the G.P.

Solution of problem 15

Solution: Let a be the first term and r be the common ratio. Let there be n terms in G.P. We have,

$$t_4 = ar^3 = 10, t_7 = ar^6 = 80, ar^{n-1} = 2560$$

So we can have following:

$$\frac{t_7}{t_4} = \frac{ar^6}{ar^3} = \frac{80}{10}$$

$$r^3 = 8 \Rightarrow r = 2$$

Substituting the value of r in t_4, we get

$$a.2^3 = 10 \Rightarrow a = \frac{10}{8} = \frac{5}{4}$$

Substituting the values of a and r for last term, we get

$$\frac{5}{4}2^{n-1} = 2560$$

$$2^{n-1} = 2048 = 2^{n-1} = 2^{11}$$

$$\Rightarrow n = 12$$

Problem 16

16. Three numbers are in G.P. If we double the middle term they form an A.P. Find the common ratio of the G.P.

Solution of problem 16

Solution: Let a be the first term and r be the common ratio. Let a, ar, ar^2 be the terms of the G.P. If we double the middle term then $a, 2ar, ar^2$ are in A.P.
Thus, we can write

$$4ar = a + ar^2$$

$$r^2 - 4r + 1 = 0$$

$$r = \frac{4 \pm \sqrt{4^2 - 4.1.1}}{2} = 2 \pm \sqrt{3}$$

Problem 17

17. If p, q and r are in A.P. show that pth, qth and rth term of a G.P. are in G.P.

Solution of problem 17

Solution: Let p, q, r be in A.P. i.e. $q - p = r - q$
Let a be the first term and b be the common ratio of G.P. Thus, we have

$$t_p = ab^{p-1}, t_q = ab^{q-1}, t_r = ab^{r-1}$$

$$\frac{t_q}{t_p} = b^{q-p}$$

$$\frac{t_r}{t_q} = b^{r-q}$$

$$\because q - p = r - q, \therefore \frac{t_q}{t_p} = \frac{t_r}{t_q}$$

Thus, pth, qth and rth terms of a G.P. are in G.P.

Problem 18

18. If a, b, c and d are in G.P., show that $(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

Solution of problem 18

Solution: Let r be the common ratio then $b = ar, c = ar^2, d = ar^3$

$$L.H.S. = (a^2r + a^2r^3 + a^2r^5)^2 = a^4r^2(1 + r^2 + r^4)^2$$

$$\begin{aligned} R.H.S. &= (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6) \\ &= a^2(1 + r^2 + r^4)a^2r^2(1 + r^2 + r^4) \\ &= a^4r^2(1 + r^2 + r^4)^2 \end{aligned}$$

Thus, L.H.S. = R.H.S.

Problem 19

19. Three non-zero numbers a, b and c are in A.P. Increasing a by 1 or increading c by 2,the numbers are in G.P. Then find b

Solution of problem 19

Solution: Because a, b, c are in A.P. $\therefore 2b = a + c$
Also, by increasing a by 1 or by increasing c by 2 the numbers are in G.P. so we can write

$$b^2 = (a+1)c, b^2 = a(c+2)$$

Thus,

$$(a+1)c = a(c+2)$$
$$\Rightarrow c = 2a$$
$$\therefore b^2 = (a+1)2a$$

Also, from the A.P. relationship

$$2b = a + 2a = 3a \Rightarrow b = \frac{3a}{2}$$

Substituting this back

$$\frac{9a^2}{4} = 2a^2 + 2a$$
$$\Rightarrow a = 8$$
$$\Rightarrow c = 16$$
$$b = \frac{a+c}{2} = 12$$

Problem 20

20. Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the mean by 5, they will be in A.P. Find the numbers.

Solution of problem 20

Solution: Let the numbers be a, ar and ar^2. Then,

$$a(1 + r + r^2) = 70$$

Given that $4a, 5ar, 4ar^2$ are in A.P. Therefore,

$$10ar = 4a + 4ar^2$$

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$$\Rightarrow r = 2, \frac{1}{2}$$

Putting values of r in the first equation we obtain a to be 10 or 40 with r as 2 and $\frac{1}{2}$ respectively. Thus, the numbers are either 10, 20, 40 or 40, 20, 10.