Problems 31 to 40

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Choosing numbers

- When you have to choose three numbers in arithmetic progression it is better to choose them as $a - d, a, a + d$
- When you have to choose four numbers in arithmetic progression it is better to choose them as $a - 3d, a - d, a + d, a + 3d$
- When you have to choose five numbers in arithmetic progression it is better to choose them as $a - 2d, a - d, a, a + d, a + 2d$

Problem 31

31. The sum of three numbers in A.P. is 27 and the sum of their squares is 293. Find the numbers.

Solution of problem 31

Solution: Let the three numbers in A.P. be $a-d, a, a+d$
Given, $a-d+a+a+d=27 \Rightarrow 3a=27 \therefore a=9$
and $(a-d)^2+a^2+(a+d)^2=293$
$\Rightarrow a^2+d^2-2ad+a^2+a^2+d^2+2ad=293$
$3a^2+2d^2=293 \Rightarrow 3.9^2+2d^2=293$
$243+2d^2=293 \Rightarrow 2d^2=50 \Rightarrow d^2=25$
$d=\pm 5$
If $d=-5$, the three numbers are $14, 9, 4$
If $d=5$, the three numbers are $4, 9, 14$

Problem 32

32. The sum of four integers in A.P. is 24 and their product is 945. Find the numbers.

Solution of problem 32

Solution: Let the four numbers be $a-3d, a-d, a+d, a+3d$
Given, $a-3d+a-d+a+d+a+3d=24$
$\Rightarrow 4a=24 \Rightarrow a=6$
and $(a-3d)(a-d)(a+d)(a+3d)=945$
$(a^2-9d^2)(a^2-d^2)=945$
$(36-9d^2)(36-d^2)=945$
$d^4-40d^2+144=105$
$d^4-40d^2+39=0$
$(d^2-1)(d^2-39)=0$
Since numbers are integers $\therefore d^2 \neq 39, \therefore d^2=1, d=\pm 1$
Hence integers are, $3,5,7,9$ or $9,7,4,3$

Problem 33

33. If the p^{th} term of an A.P. is q and the q^{th} term is p, find the first term and common difference. Also, show that $(p+q)^{th}$ term is zero.

Solution of problem 33

Solution: Let x be the first term and d be the common difference.
$t_p = x + (p-1)d = q$
$t_q = x + (q-1)d = p$
Subtracting these two we get, $q - p = (p-q)d \Rightarrow d = -1$
Substituting this value for t_p, we get $q = x + (p-1).-1$
$x = p + q - 1$
$t_{p+q} = x + (p+q-1)d = p + q - 1 + (p+q-1).-1 = 0$

Problem 34

34. For an A.P. show that $t_m + t_{2n+m} = 2t_{m+n}$.

Solution of problem 34

Solution: Let a be the first term and d be the common difference.
$t_m = a + (m-1)d$
$t_{2n+m} = a + (2n+m-1)d$
$2t_{m+n} = 2[a + (m+n-1)d]$
$t_m + t_{2n+m} = 2a + (m-1+2n+m-1)d = 2[a + (m+n-1)d] = 2t_{m+n}$

Problem 35

35. Divide 15 into three parts which are in A.P. and the sum of their squares is 83.

Solution of problem 35

Solution: Let $a-d, a, a+d$ be three such numbers.
Given, $a-d+a+a+d=15 \Rightarrow 3a=15 \therefore a=5$
also, $(a-d)^2+a^2+(a+d)^2=83$
$a^2+d^2-2ad+a^2+a^2+d^2+2ad=83$
$3a^2+2d^2=83 \Rightarrow 3.5^2+2d^2=83$
$2d^2=8 \Rightarrow d=\pm 2$
Therefore, numbers are $3,5,7$ or $7,5,3$

Problem 36

36. Three numbers are in A.P. Their sum is 27 and the sum of their squares is 275. Find the numbers.

Solution of problem 36

Solution: Let $a-d, a, a+d$ be three such numbers.
Given, $a-d+a+a+d=27 \Rightarrow 3a=27 \therefore a=9$
also, $(a-d)^2+a^2+(a+d)^2=275$
$a^2+d^2-2ad+a^2+a^2+d^2+2ad=275$
$3a^2+2d^2=275 \Rightarrow 3.9^2+2d^2=275$
$2d^2=32 \Rightarrow d=\pm 4$
Therefore, numbers are $5, 9, 13$ or $13, 9, 5$

Problem 37

37. The sum of three numbers in A.P. is 12 and the sum of their cubes is 408, find them.

Solution of problem 37

Solution: Let $a-d, a, a+d$ be three such numbers.
Given, $a-d+a+a+d=12 \Rightarrow 3a=12 \therefore a=4$
also, $(a-d)^3+a^3+(a+d)^3=408$
$a^3-3a^2d+3ad^2-d^3+a^3+a^3+3a^2d+3ad^2+d^3=408$
$3a^3+6ad^2=408$
$3.4^3+6.4.d^2=408$
$24d^2=216 \Rightarrow d^2=9 \therefore d=\pm 3$
Therefore, numbers are $1,4,7$ or $7,4,1$.

Problem 38

38. Divide 20 into four parts which are in A.P. such that the product of first and fourth is to product of second and third is $2 : 3$.

Solution of problem 38

Solution: Let $a-3d, a-d, a+d, a+3d$ be the numbers.
Given, $a-3d+a-d+a+d+a+3d=20 \Rightarrow a=5$
also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{2}{3}$
$\frac{a^2-9d^2}{a^2-d^2}=\frac{2}{3}$
$3a^2-27d^2=2a^2-2d^2$
$a^2=25d^2 \Rightarrow d=\pm 1$
Therfore, numbers are $2,4,6,8$ or $8,6,4,2$

Problem 39

39. The sum of three numbers in A.P. is -3 and their product is 8. Find the numbers.

Solution: Let $a-d, a, a+d$ be the numbers.
Given, $a-d+a+a+d=-3 \Rightarrow 3a=-3 \therefore a=-1$
also, $(a-d)a(a+d)=8$
$a(a^2-d^2)=8$
$-1(1-d^2)=8$
$d^2=9 \Rightarrow d=\pm 3$
Therefore, numbers are $-4, -1, 2$ or $2, -1, -4$

Problem 40

40. Divide 32 into four parts which are in A.P. such that the ratio of product of extremes to the product of means is 7 : 15.

Solution of problem 40

Solution: Let $a - 3d, a - d, a + d, a + 3d$ be the numbers.
Given, $a - 3d + a - d + a + d + a + 3d = 32 \Rightarrow a = 8$
also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$
$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$
$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$
$960 - 135d^2 = 448 - 7d^2$
$128d^2 = 512 \Rightarrow d^2 = 4 \therefore d = \pm 2$
Therfore, numbers are $2, 6, 10, 14$ or $14, 10, 6, 2$