Arithmetic Progression
Problems 71 to 75

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Problem 71

71. If the sum of m terms of an A.P.is n and the sum of its n terms is m, show that sum of $(m+n)$ terms is $-(m+n)$

Solution of problem 71

Solution: Let a be the first term and d be the common difference of the A.P.
Given, $S_m = n$ and $S_n = m$

$$\therefore \frac{m}{2}[2a + (m-1)d] = n \Rightarrow 2a + (m-1)d = \frac{2n}{m}$$

and,

$$\frac{n}{2}[2a + (n-1)d] = m \Rightarrow 2a + (n-1)d = \frac{2m}{n}$$

Subtracting, we get

$$(m-n)d = \frac{2(n^2 - m^2)}{mn} \Rightarrow d = -\frac{2(m+n)}{mn}$$

Substituting d in S_m, we get

$$\frac{m}{2}[2a + (m-1)d] = n$$

$$2a = \frac{2n}{m} + (m-1)\frac{2(m+n)}{mn}$$

$$2a = \frac{2n^2 + 2m^2 + 2mn - 2m - 2n}{mn}$$

Thus,

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

Substituting for $2a$ and d, we get

$$S_{m+n} = \frac{m+n}{2}\left[\frac{2n^2+2m^2+2mn-2m-2n}{mn} - (m+n-1)\frac{2(m+n)}{mn}\right]$$

$$S_{m+n} = \frac{m+n}{2}\left[\frac{2n^2+2m^2+2mn-2m-2n-2m^2-2mn-2mn-2n^2+2m+2n}{mn}\right]$$

$$S_{m+n} = -(m+n)$$

Problem 72

72. If S be the sum of $2n+1$ terms of an A.P., and S_1 that of alternate terms beginning with the first, then show that $\frac{S}{S_1} = \frac{2n+1}{n+1}$

Solution of problem 72

Solution. Let a be the first term and d be the common difference.

$$S = \frac{2n+1}{2}[2a + 2nd]$$

$$S_1 = \frac{n+1}{2}[2a + (n+1-1)2d]$$

$$\therefore \frac{S}{S_1} = \frac{2n+1}{n+1}$$

Problem 73

73. If a, b, c be the 1st, 3rd, nth terms respectively of an A.P., prove that the sum of n terms is $\frac{c+a}{2} + \frac{c^2-a^2}{b-a}$.

Solution: Let d be the common difference, then we have

$$t_1 = a, b = a + 2d, c = a + (n-1)d$$

Thus,

$$d = \frac{b-a}{2} \text{ and } (n-1)d = c - a \Rightarrow n = \frac{2(c-a)}{b-a} + 1$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \left(\frac{2(c-a)}{(b-a)} + 1\right)[2a + c - a]$$

$$= \frac{c+a}{2} + \frac{c^2 - a^2}{b-a}$$

Problem 73

73. If the pth term of an A.P. is x and qth term is y. Show that the sum of $(p+q)$ terms is $\frac{p+q}{2}\left(x+y+\frac{x-y}{p-q}\right)$

Solution of problem 73

Solution: Let a be the first term and d be the common difference.

$$t_p = a + (p-1)d = x$$

$$t_q = a + (q-1)d = y$$

Subtracting, we get

$$(p-q)d = x - y \Rightarrow d = \frac{x-y}{p-1}$$

Substituting value of d in t_p, we get

$$a + (p-1)\frac{x-y}{p-q} = x$$

$$a = x - (p-1)\frac{x-y}{p-q}$$

$$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$$= \frac{p+q}{2}\left[2x - 2(p-1)\frac{x-y}{p-q} + (p+q-1)\frac{x-y}{p-q}\right]$$

$$= \frac{p+q}{2}\left[2x + \frac{x-y}{p-q}(-2p+2+p+q-1)\right]$$

$$= \frac{p+q}{2}\left[2x + \frac{x-y}{p-q}(q-p+1)\right]$$

$$= \frac{p+q}{2}\left[2x - x + y + \frac{x-y}{p-q}\right]$$

$$= \frac{p+q}{2}\left[x + y + \frac{x-y}{p-q}\right]$$

Problem 74

74. The sum of n terms of two series in A.P. are in ratio $(3n+8):(7n+15)$. Find the ratio of their 12th terms.

Solution of problem 74

Solution: Let a_1, a_2 and d_1, d_2 be first terms and common differences of the two A.P. respectively. Then, we have

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{3n+8}{7n+15}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{3n+8}{7n+15}$$

Substituting $n = 23$ in the above equation, we get

$$\frac{2a_1 + 22d_1}{2a_2 + 22d_2} = \frac{77}{176}$$

$$\frac{a_1 + 11d_1}{a_2 + 11d_2} = \frac{7}{16}$$

Thus, the ratio of 12th terms is $\frac{7}{16}$

Problem 75

75. If the ratio of the sum of m terms and n terms of an A.P. is $m^2 : n^2$, prove that the ratio of its mth and nth term wil be $(2m - 1) : (2n - 1)$

Solution of problem 75

Solution: Let a be the first term and d be the common difference of the A.P. Given,

$$\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

Ratio of mth and nth terms is

$$\frac{a+(m-1)d}{a+(n-1)d} = \frac{2a+(2m-1-1)d}{2a+(2n-1-1)d}$$

Substituting $m = 2m-1$ and $n = 2n-1$ in the equation for ratio of sums, we get

$$\frac{2a+(2m-1-1)d}{2a+(2n-1-1)d} = \frac{2m-1}{2n-1}$$

Thus, ratio of mth and nth term is $(2m-1):(2n-1)$