Geometric Progression
Problems 51-60

Shiv Shankar Dayal

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Problem 51

51. Find $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$ to n terms.

Solution of Problem 51

Solution: This is a G.P. with $a = 1, r = -\frac{1}{2}, n = n$

$$S = \frac{1\left(1 - \frac{1}{(-2)^n}\right)}{1 + \frac{1}{2}}$$

$$= \frac{2}{3}[1 - (-1)^n/2^n]$$

Problem 52

52. If you had a choice of a salary of a salary of \$1000 a day for a month of 31days or \$1 for the first day, doubling every day which choice would you make?

Solution of Problem 52

Solution: In the first case total salary $= 1000 + 1000 + \ldots$ to 31 terms $= \$31000$
In the second case total salary $= 1 + 2 + 4 + \ldots$ to n terms $= \frac{(2^{31}-1)}{2-1} = \$2^{31} - 1$
$\because 2^5 = 32 \Rightarrow 2^{10} = 1024 \Rightarrow 2^{20} = 1048576$
CLearly, $2^{31} - 1 > 31000$, therefore, second choice should be made.

Problem 53

53. How many terms of the series $1 + 3 + 3^2 + 3^3 + \ldots$ must be taken to make 3280?

Solution of Problem 53

Solution: Let the sum of n terms of the given series be 3280

$$S_n = \frac{a(r^n - 1)}{r - 1} \therefore 3280 = \frac{1(3^n - 1)}{3 - 1}$$

$$\Rightarrow 3^n - 1 = 6560 \Rightarrow 3^n = 6561 \Rightarrow n = 8$$

Problem 54

54. Find the least value of n for which $1 + 3 + 3^2 + \ldots + 3^{n-1} > 1000$

Solution of Problem 54

Solution: Given, $1 + 3 + 3^2 + \ldots + 3^{n-1} > 1000$

$$1.\left(\frac{3^n - 1}{3 - 1}\right) > 1000 \Rightarrow 3^n > 2001$$

Now we know that $3^6 = 729, 3^7 = 2187$ so the least value of n would be 7.

Problem 55

55. Find $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ to ∞

Solution of Problem 55

Solution: This is a G.P. with $a = 1, r = \frac{1}{2}, |r| < 1$

$$\therefore S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$$

Problem 56

56. A person starts collecting \$ 1 first day, \$ 3 second day, \$ 9 third day and so on. What will be his collection in 20 days.

Solution of Problem 56

Solution: This is a G.P. with $a = 1, r = 3$

$S_{20} = \frac{1(3^{20}-1)}{3-1} = \frac{3^{20}-1}{2} = 1743392200$

Problem 57

57. Find the sum of $\left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 + \frac{1}{x^4} + 5\right) + \left(x^6 + \frac{1}{x^6} + 8\right) + \ldots$ to n terms

Solution of Problem 57

Solution: Given series can be rewritten as

$$(x^2 + x^4 + x^6 + \ldots) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \ldots\right) + (2 + 5 + 8 + \ldots)$$

$$= \frac{x^2(x^{2n} - 1)}{x^2 - 1} + \frac{1}{x^2}\frac{1 - \frac{1}{x^{2n}}}{1 - \frac{1}{x^2}} + \frac{n}{2}[4 + (n-1)3]$$

$$= \frac{x^{2n-1} - 1}{x^2 - 1}\left(x^2 + \frac{1}{x^{2n}}\right) + \frac{n(3n+1)}{2}$$

Problem 58

58. How many terms of the series $1 + 2 + 2^2 + \ldots$ must be taken to make 511?

Solution of Problem 58

Solution: Let we need n terms to make sum 511

$$S_n = \frac{1.(2^n - 1)}{2 - 1} = 511 \Rightarrow 2^n = 512 \Rightarrow n = 9$$

Problem 59

59. Find the least value of n such that $1 + 2 + 2^2 + \ldots + 2^{n-1} \geq 300$

Solution of Problem 59

Solution. Given, $1 + 2 + 2^2 + \ldots + 2^{n-1} \geq 300$

$$\frac{1.(2^n - 1)}{2 - 1} \geq 300$$

$$2^n \geq 301$$

We know that $2^8 = 256, 2^9 = 512$ this least value of n will be 9.

Problem 60

60. Determine the no. of terms of a G.P. if $a_1 = 3$, $a_n = 96$ and $S_n = 189$

Solution of Problem 60

Solution: Let r be the common ratio of G.P., then

$$a_n = a_1 r^{n-1} = 3r^{n-1} = 96 \Rightarrow r^{n-1} = 32$$

$$S_n = a_1 \frac{r^n - 1}{r - 1} = 189$$

$$3 \frac{r^{n-1}.r - 1}{r - 1} = 189$$

$$\frac{32r - 1}{r - 1} = 63 \Rightarrow r = 2 \Rightarrow n = 6$$