Arithmetic, Geometric and Harmonic Means Problems 21-30

Shiv Shankar Dayal

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Problem 21

21. Insert 17 A.M. between $\frac{7}{2}$ and $-\frac{83}{2}$.

Solution of Problem 21

Solution: Let $a_1, a_2, \ldots, a_{17}$ are required 17 A.M. Let d to be the common difference. We know that there will be a total of 19 terms in the A.P. Thus,

$$-\frac{83}{2} = \frac{7}{2} + 18d \Rightarrow d = -\frac{5}{2}$$

Now the means can be found easily.

Problem 22

22. Between 1 and 31, n A.M. are inserted such that ratio of 7th and $(n-1)$th means is $5:9$, find n.

Solution of Problem 22

Solution: Let the means are $a_1, a_2, \ldots, a_n$ betweeen 1 and 31 then $d = \frac{30}{n+1}$, where d is the common difference.

$$\frac{x_7}{x_{n-1}} = \frac{5}{9} \Rightarrow \frac{1+7d}{1+(n-1)d} = \frac{5}{9} \Rightarrow n = 14$$

Problem 23

23. Find the relation between x and y in order that rth mean between x and $2y$ may be the same as rth mean between $2x$ and y; if n arithmetic means are inserted in each case.

Solution of Problem 23

Solution: In first case $x_r = x + \frac{2y-x}{n+1}r$ and in second case $y_r = 2x + \frac{y-2x}{n+1}r$

Equating them we get $y = \frac{n+1-r}{r}x$

Problem 24

24. Insert 7 geometric means between 2 and 162.

Solution of Problem 24

Solution: If we insert 7 G.M. then total no. of terms would be 9, so if r is common ratio then $162 = 2.r^8$

$$\Rightarrow r = \sqrt{3}$$

Thus, G.M. will be $2\sqrt{3}, 6, 6\sqrt{3}, 18, 18\sqrt{3}, 54, 54\sqrt{3}$

Problem 25

25. Insert 6 geometric means between $\frac{8}{27}$ and $-\frac{81}{16}$

Solution of Problem 25

Solution: If we insert 6 G.M. then total no. of terms would be 8, so if r is the common ratio then $-\frac{81}{16} = \frac{8}{27}r^7 \Rightarrow r = -\frac{3}{2}$

Thus, G.M. will be $-\frac{4}{9}, \frac{2}{3}, -1, \frac{3}{2}, -\frac{9}{4}, \frac{27}{8}$

Problem 26

26. If odd number of geometric means are inserted between two given numbers a and b, show that the middle geometric mean is $\sqrt{ab}$.

Solution of Problem 26

Solution: Let $2n+1$ geometric means are inserted between a and b and that r is the common ratio. Then,

$$b = ar^{2n+2} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{2n+2}}$$

$$\text{Middle geometric mean } = g_{n+1} = a.r^{n+1} = \sqrt{ab}$$

Problem 27

27. Insert four harmonic means between 1 and $\frac{1}{11}$.

Solution of Problem 27

Solution: Let h_1, h_2, h_3, h_4 be four harmonic means between 1 and $\frac{1}{11}$. Thus corresponding A.P. will be $1, \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}, \frac{1}{h_4}, 11$

Since there are six terms in A.P. $11 = 1 + 5d \Rightarrow d = 2$. So A.P. will be $1, 3, 5, 7, 9, 11$ and corresponsing H.P. will be composed of reciprocals of these values.

Problem 28

28. n harmonic means are inserted between 1 and 4 such that first mean: last mean $= 1 : 3$, then find n.

Solution of Problem 28

Solution: After inserting n harmonic means there will be a total of $n+2$ terms. So in corresponding A.P. 1 will remain 1 but 4 will become $\frac{1}{4}$ and the ratio of first mean to last mean will also become its reciprocal i.e. $3:1$

$$\frac{1}{h_1} = 1 + d, \frac{1}{h_n} = 1 + nd, d = \frac{\frac{1}{4} - 1}{n+1} = -\frac{3}{4(n+1)}$$

Now n can be found to be 11.

Problem 29

29. Find n such that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be a single harmonic mean between a and b.

Solution of Problem 29

Solution: H.M. between a and $b = \frac{2ab}{a+b}$

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{2ab}{a+b}$$

$$a^{n+1}+ab^{n+1}+ba^{n+1}+b^{n+2} = 2a^{n+1}b+2ab^{n+1}$$

$$a^{n+2}-ab^{n+1}-ba^{n+1}+b^{n+2} = 0$$

$$\Rightarrow a^{n+1}-b^{n+1} = 0 \Rightarrow n = -1$$

Problem 30

30. If $H_1, H_2, \dots, H_n$ be n harmonic means between a and b, prove that $\frac{H_1+a}{H_1-a} + \frac{H_n+b}{H_n-b} = 2n$

Solution of Problem 30

Solution: We have evaluated previously that $H_1 = \frac{ab(n+1)}{a+nb}$ and $H_n = \frac{ab(n+1)}{na+b}$ Substituting in the given equality

$$\text{L.H.S.} = \frac{ab(n+1)+a^2+nab}{ab(n+1)-a^2-nab} + \frac{ab(n+1)+nab+b^2}{ab(n+1)-nab-b^2}$$

$$= \frac{a(a+b)+2nab}{a(b-a)} + \frac{b(a+b)+2nab}{b(a-b)}$$

$$= \frac{(a+b)+2nb}{b-a} + \frac{(a+b)+2na}{a-b}$$

$$= \frac{2n(b-a)}{b-a} = 2n$$