Complex Numbers Problems 61-70

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Problem 61

61. If $z_n = \cos\frac{\pi}{(2n+1)(2n+3)} + i\sin\frac{\pi}{(2n+1)(2n+3)}$ then find $z_1 z_2 z_3 \ldots \infty$.

Solution of Problem 61

Solution: $z_n = \cos\left(\frac{1}{2n+1} - \frac{1}{2n+3}\right).\frac{\pi}{2} + i\sin\left(\frac{1}{2n+1} - \frac{1}{2n+3}\right).\frac{\pi}{2}$

$\therefore z_1 z_2 z_3 \ldots \infty = \cos\left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} \ldots \infty\right).\frac{\pi}{2} + i\sin\left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} \ldots \infty\right).\frac{\pi}{2}$

$= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

Problem 62

62. If z_1, z_2 be two complex numbers and a, b are two real numbers, then prove that $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$.

Solution of Problem 62

Solution: Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (ax_1 - bx_2)^2 + (ay_1 - by_2)^2 + (bx_1 + ax_2)^2 + (by_1 + ay_2)^2$

$= a^2x_1^2 + b^2x_2^2 - 2abx_1x_2 + a^2y_1^2 + b^2y_2^2 - 2aby_1y_2 + b^2x_1^2 + a^2x_2^2 + 2abx_1x_2 + b^2y_1^2 + a^2y_2^2 + 2aby_1y_2$

$= (a^2 + b^2)(x_1^2 + y_1^2 + x_2^2 + y_2^2)$

$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$