Problems and Solutions on A.P.

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August 16, 2019

Problem 1

1. If nth term of a sequence is $2n^2 + 1$, find the sequence. Is this seuquence in A.P.?

Solution of Problem 1

Solution: Given $t_n = 2n^2 + 1$
Putting $n = 1$, we get $t_1 = 2.1^2 + 1 = 3$
Putting $n = 2$, we get $t_2 = 2.2^2 + 1 = 9$
Putting $n = 3$, we get $t_3 = 2.3^2 + 1 = 19$
Putting $n = 4$, we get $t_4 = 2.4^2 + 1 = 33$
...
Hence, given sequence is $3, 9, 19, 33, \ldots$
$t_{n-1} = 2.(n-1)^2 + 1$
$t_n - t_{n-1} = 2n^2 + 1 - 2.(n-1)^2 + 1$
$= 2n^2 + 1 - [2(n^2 - 2n + 1) + 1)]$
$= 4n - 2$
The difference is not independent of n i.e. it is not a constant. Thus given seuqence is not in A.P.

Problem 2

2. Find the first five terms of the sequence for which $t_1 = 1, t_2 = 2$ and $t_{n+2} = t_n + t_{n+1}$

Solution of Problem 2

Solution: Putting $n = 1$, we get $t_3 = t_1 + t_2 = 1 + 2 = 3$
Putting $n = 2$, we get $t_4 = t_2 + t_3 = 2 + 3 = 5$
Putting $n = 3$, we get $t_5 = t_3 + t_4 = 3 + 5 = 8$

Problem 3

3. Write the sequence whose nth term is $3n + 5$

Solution of Problem 3

Solution: Putting $n = 1$, we get $t_1 = 3.1 + 5 = 8$
Putting $n = 2$, we get $t_2 = 3.2 + 5 = 11$
Putting $n = 3$, we get $t_3 = 3.3 + 5 = 14$

Problem 4

4. Write the sequence whose nth term is $2n^2 + 3$

Solution of Problem 4

Solution: Putting $n = 1$, we get $t_1 = 2.1^2 + 3 = 5$
Putting $n = 2$, we get $t_2 = 2.2^2 + 3 = 11$
Putting $n = 3$, we get $t_3 = 2.3^2 + 3 = 21$

Problem 5

5. Write the sequence whose nth term is $\frac{3n}{2n+4}$

Solution of Problem 5

Solution: Putting $n = 1$, we get $t_1 = \frac{3.1}{2.1+4} = \frac{3}{6} = \frac{1}{2}$
Putting $n = 2$, we get $t_2 = \frac{3.2}{2.2+4} = \frac{6}{8} = \frac{3}{4}$
Putting $n = 3$, we get $t_3 = \frac{3.3}{2.3+4} = \frac{9}{10}$

Problem 6

6. Write the first three terms of sequence defined by $t_1 = 2$, $t_{n+1} = \frac{2t_n+1}{t_n+3}$

Solution of Problem 6

Solution: Putting $n = 1$, we get $t_{1+1} = \frac{2t_1+1}{t_1+3} = \frac{2.2+1}{2+3} = 1$
Putting $n = 2$, we get $t_{2+1} = \frac{2.1+1}{1+3} = \frac{3}{4}$

Problem 7

7. If nth term of a sequence is $4n^2 + 1$, find the sequence. Is this sequence an A.P.?

Solution of Problem 7

Solution: Putting $n = 1$, we get $t_1 = 4.1^2 + 1 = 5$
Putting $n = 2$, we get $t_2 = 4.2^2 + 1 = 17$
Putting $n = 3$, we get $t_3 = 4.3^2 + 1 = 37$
$t_n - t_{n-1} = 4n^2 + 1 - 4(n-1)^2 - 1 = 8n + 4$, which is not independent of n i.e. it is not a constant. Therefore, the sequence is not an A.P.

Problem 8

8. If nth term of a sequence is $2an + b$, where a, b are constants, is this sequence an A.P.?

Solution of Problem 8

Solution: $t_n - t_{n-1} = 2an + b - 2an - 1 - b = 2a$ which is independent of n i.e. a constant. Therefore, the sequence is an A.P.

Problem 9

9. Find the 5th term of the sequence whose first three terms are $3, 3, 6$ and each term after the second is the sum of two preceding terms.

Solution of Problem 9

Solution: Since each term after the second is the sum of two preceding terms $t_n = t_{n-1} + t_{n-2}$
Putting $n = 4$, we get $t_4 = t_3 + t_2 = 6 + 3 = 9$
Putting $n = 5$, we get $t_5 = t_4 + t_3 = 9 + 6 = 15$

Problem 10

10. Consider the sequence defined by $t_n = an^2 + bn + c$. If $t_1 = 1$, $t_2 = 5$ and $t_3 = 11$ then find the value of t_{10}

Solution of Problem 10

Solution: $t_1 = 1 \Rightarrow a + b + c = 1$
$t_2 = 5 \Rightarrow 4a + 2b + c = 5$
$t_3 = 11 \Rightarrow 9a + 3b + c = 11$
Solving the three equations we get $a = 1, b = 1, c = -1$
$t_{10} = 100 + 10 - 1 = 109$