Logarithm Problem 81-90

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Problem 81

81. Solve $(5+2\sqrt{6})x^2 - 3 + (5-2\sqrt{6})^{x^2-3} = 10$

Solution of Problem 81

Solution:

$$\text{Given, } (5 + 2\sqrt{6})x^2 - 3 + (5 - 2\sqrt{6})^{x^2-3} = 10$$

$$\Rightarrow (5 + 2\sqrt{6})^{x^2-3} + (5 + 2\sqrt{6})^{-(x^2-3)} = 10$$

Let $z = (5 + 2\sqrt{6})^{x^2-3}$, then we can rewrite above as

$$z + \frac{1}{z} = 10$$

$$z = 5 \pm 2\sqrt{6}$$

$$\therefore x = \pm 2, \pm\sqrt{2}$$

Problem 82

82. For $x > 1$, show that $2\log_{10} x - \log_x .01 \geq 4$

Solution of Problem 82

Solution:

$$2\log_{10} x - \log_x .01 = 2\log_{10} x - \log_x 10^{-2}$$

$$= 2\log_{10} x + 2\log_x 10 = 2\log_{10} x + 2\frac{1}{\log_{10} x}$$

$$= 2\left(\log_{10} x + \frac{1}{\log_{10} x}\right)$$

$$= 2\left[\left(\sqrt{\log_{10} x} - \frac{1}{\sqrt{\log_{10} x}}\right)^2 + 2\right] \geq 4$$

Problem 83

83. Show that $|\log_b a + \log_a b| > 2$

Solution of Problem 83

Solution: Let $E = |\log_b a + \log_a b|$

Also, let $z = \log_b a$, then we can rewrite above as $E = \left|z + \frac{1}{z}\right|$

Clearly, $z \neq 0$, or the problem will be undefined. When $z > 0$, $E = \left(\sqrt{z} - \frac{1}{\sqrt{z}}\right)^2 + 2 > 2$

When $z < 0$, let $z = -y$, then $E = \left|z + \frac{1}{z}\right| = \left|-y - \frac{1}{y}\right| = y + \frac{1}{y} > 2$

Problem 84

84. Solve $\log_{0.3}(x^2 + 8) > \log_{0.3} 9x$

Solution of Problem 84

Solution:

$$\text{Given, } \log_{0.3}(x^2+8) > \log_{0.3} 9x$$
$$\Rightarrow x^2 + 8 < 9x$$
$$\Rightarrow (x-1)(x-8) < 0$$
$$\Rightarrow 1 < x < 8$$

Problem 85

85. Solve $\log_{x-2}(2x-3) > \log_{x-2}(24-6x)$

Solution of Problem 85

Solution:

$$\text{Given, } \log_{x-2}(2x-3) > \log_{x-2}(24-6x)$$

$$\text{Case I: When } 0 < x-2 < 1 \Rightarrow 2 < x < 3$$

$$\text{Given inequality becomes } 2x-3 < 24-6x \Rightarrow x < \frac{27}{8}$$

$$\text{But } x < 3 \text{ so } 3 \text{ is still limiting value of } x$$

$$\text{Case II: When } x-2 > 1 \Rightarrow x > 3$$

$$2x-3 > 24-6x \Rightarrow x > \frac{27}{8}$$

However, for logarithm to be defined $2x-3>0$ and $24-6x>0$ and also $x-2>0$. Combining all these we get $2 < x < 3$

Problem 86

86. Find the interval in which x will lie if $\log_{0.3}(x-1) < \log_{0.09}(x-1)$

Solution of Problem 86

Solution:

$$\text{Given, } \log_{0.3}(x-1) < \log_{0.09}(x-1)$$
$$\Rightarrow \log_{0.3}(x-1) < \log_{0.3^2}(x-1)$$
$$(x-1)^2 > (x-1)$$
$$\Rightarrow x^2 - 3x + 2 > 0$$
$$\Rightarrow x < 1, x > 2$$

For logarithm to be defined $x - 1 > 0$ i.e. $x > 1$, thus the interval for x would be $(2, \infty]$

Problem 87

87. Solve $\log_{\frac{1}{2}} x \geq \log_{\frac{1}{3}} x$

Solution of Problem 87

Solution:

$$\text{Given, } \log_{\frac{1}{2}} x \geq \log_{\frac{1}{3}} x$$

$$\Rightarrow \log_{\frac{1}{2}} x \geq \log_{\frac{1}{2}} x \log_{\frac{1}{3}} \frac{1}{2}$$

$$\Rightarrow \log_{\frac{1}{2}} x \left[1 - \log_{\frac{1}{3}} \frac{1}{2}\right] \geq 0$$

$$\Rightarrow \log_{\frac{1}{2}} x \left[1 - \log_{3^{-1}} 2^{-1}\right] \geq 0$$

$$\Rightarrow \log_{\frac{1}{2}} x \left[1 - \log_3 2\right] \geq 0$$

$$\Rightarrow \log_{\frac{1}{2}} x \geq 0$$

$$\Rightarrow x \leq 1$$

For logarithm to be defined $x > 0$, thus range of x would be $(0, 1]$

Problem 88

88. Solve $\log_{\frac{1}{2}} \log_4(x^2 - 5) > 0$

Solution of Problem 88

Solution:

$$\text{Given, } \log_{\frac{1}{2}} \log_4(x^2 - 5) > 0$$

$$\Rightarrow \log_4(x^2 - 5) < 1$$

$$\Rightarrow x^2 - 5 < 4$$

$$\Rightarrow x^2 < 9 \Rightarrow -3 < x < 3$$

For logarithm to be defined $x^2 - 5 > 0$ and $\log_4(x^2 - 5) > 0 \Rightarrow x^2 - 5 > 1 \Rightarrow x < -\sqrt{6}, x > \sqrt{6}$.
Thus, the two ranges for x are $(-3, -\sqrt{6})$ and $(\sqrt{6}, 3)$

Problem 89

89. Solve $\log(x^2 - 2x - 2) \leq 0$

Solution of Problem 89

Solution:

$$\text{Given, } \log(x^2 - 2x - 2) \leq 0$$

$$\Rightarrow x^2 - 2x - 2 \leq 1$$

$$\Rightarrow (x-3)(x+1) \leq 0$$

$$-1 \leq x \leq 3$$

For logarithm to be defined $x^2 - 2x + 2 > 0 \Rightarrow x < 1 - \sqrt{3}, x > 1 + \sqrt{3}$
Thus, the ranges are $[-1, 1-\sqrt{3}), (1+\sqrt{3}, 3]$

Problem 90

90.

Solve $\log_{2^2}(x-1)^2 - \log_{0.5}(x-1) > 5$

Solution of Problem 90

Solution:

$$\text{Given, } \log_{2^2}(x-1)^2 - \log_{0.5}(x-1) > 5$$

$$\Rightarrow (2\log_2|x-1|)^2 - \log_{0.5}(x-1) > 5$$

$$\Rightarrow 4[\log_2(x-1)]^2 + \log_2(x-1) > 5$$

$$[\because \text{ for } \log_{0.5}(x-1) \text{ to be defined } x-1>0 \therefore |x-1| = x-1]$$

$$\log_2(x-1) < \frac{-5}{4}, \log_2(x-1) > 1$$

$$\text{When } \log_2(x-1) < \frac{-5}{4} \Rightarrow x < 1 + \frac{1}{2\sqrt[4]{2}}$$

For logarithm to be defined $x-1>0 \Rightarrow 1 < x < 1 + \frac{1}{2\sqrt[4]{2}}$

$$\text{When } \log_2(x-1) > 2 \Rightarrow x > 3$$

Thus, ranges are $\left(1, 1+\frac{1}{2\sqrt[4]{2}}\right), (3, \infty]$