Logarithm Problem 41-50

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Problem 41

41. If $\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q}$, prove that $x^{q+r} y^{p+r} z^{p+q} = x^p y^q z^r$

Solution of Problem 41

Solution:

$$\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q} = k(\text{let})$$

$$\Rightarrow \log x = k(q-r), \log y = k(r-p), \log z = k(p-q)$$

We have to prove that $x^{q+r}y^{p+r}z^{p+q} = x^p y^q z^r$. Taking $\log$ of both sides

$$(q+r)\log x + (p+r)\log y + (p+q)\log z = p\log x + q\log y + r\log z$$

$$k(q^2-r^2) + k(r^2-p^2) + k(p^2-q^2) = k(pq-pr+qr-pq+pr-qr)$$

$$0 = 0$$

Problem 42

42. If $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$, prove that $x = a^{\frac{1}{1-\log_a z}}$

Solution of Problem 42

Solution: Given $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$

$$\Rightarrow z = a^{\frac{1}{1-\log_a a^{\frac{1}{1-\log_a x}}}}$$

$$z = a^{\frac{1}{1-\frac{1}{1-\log_a x}}}$$

Taking $\log$ of both sides with base a, we get

$$\log_a z = \frac{1}{1 - \frac{1}{1-\log_a x}}$$

$$= \frac{1 - \log_a x}{-\log_a x} = 1 - \frac{1}{\log_a x}$$

$$x = a^{\frac{1}{1-\log_a z}}$$

Problem 43

43. Let $f(x) = \frac{1}{1-\log_e x}$. If $f(y) = e^{f(z)}$ and $z = e^{f(x)}$, prove that $x = e^{f(y)}$

Solution of Problem 43

Solution:

$$f(y) = e^{\frac{1}{1-\log_a z}}, z = e^{\frac{1}{1-\log_e x}}$$

$$\Rightarrow f(y) = e^{\frac{1}{1-\log_e e^{\frac{1}{1-\log_e x}}}}$$

$$f(y) = e^{\frac{1}{1-\frac{1}{1-\log_e x}}}$$

Following like previous problem

$$x = e^{f(y)}$$

Problem 44

44. Show that $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + ... + \frac{1}{\log_{43} n} = \frac{1}{\log_{43!} n}$

Solution of Problem 44

Solution:

$$L.H.S. = \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + ... + \frac{1}{\log_{43} n}$$

$$= \log_n 2 + \log_n 3 + \log_n 4 + ... + \log_n 43$$

$$= \log_n (2.3.4. \dots 43) = \log_n 43! = \frac{1}{\log_{43!} n}$$

Problem 45

45. Show that $2(\log a + \log a^2 + \log a^3 + ... + \log a^n) = n(n+1)\log a$

Solution of Problem 45

Solution:

$$L.H.S. = 2(\log a + \log a^2 + \log a^3 + ... + \log a^n)$$

$$= 2\log a(1 + 2 + 3 + ... + n) = 2\log a\frac{n(n+1)}{2}$$

$$= n(n+1)\log a$$

Problem 46

46. Find the number of digits in 12^{12}, without actual computation. Given $\log 2 = 0.301$ and $\log 3 = 0.477$

Solution of Problem 46

Solution: We will make use of the fact that positive characteristics of n of a logarithm there are $n+1$ digits in the number.

Let $y = 12^{12} \Rightarrow \log y = 12 \log 12 = 12 \log(2.2.3) = 12[2 * 0.301 + 0.477]$

$= 12.96$

Thus, number of digits is 13.

Problem 47

47. How many positive integers have characteristics 2 when base is 3?

Solution of Problem 47

Solution: Number of positive integers having base b and characteristics n is $b^{n+1} - b^n$

Thus, number of integers with base 3 and characteristics 2 is $3^3 - 3^2 = 18$.

Problem 48

48. How many zeros are there between the decimal point and the first significant digit in 0.0504^{10}. Given, $\log 2 = 0.301, \log 3 = 0.477, \log 7 = 0.845$

Solution of Problem 48

Solution: Let $y = 0.0504^{10}$

$\log_{10} y = 10\log_{10} 0.0504 = 10\log_{10}(504 * 10^{-4})$

$= 10\log_{10}[-4 + \log(2^3.3^2.7)]$

$= -12.98$

Thus, characteristics is -13, therefore number of zeros after decimal and first significant digit $= 12$

Problem 49

49. Find the number of digits in 72^{15} without actual computation. Given $\log 2 = 0.301, \log 3 = 0.477$.

Solution of Problem 49

Solution: Let $x = 72^{15} \therefore \log_{10} x = 15 \log_{10} 72$

$= 15 \log_{10}(2^3 * 3^2) = 15 \log_{10}[3 \log_{10} 2 + 2 \log_{10} 3]$

$= 15[3 * 0.301 + 2 * 0.477] = 27.855$

So characteristics is 27, therefore, the number of digits will be 28.

Problem 50

50. How many positive integers have characteritics 2 when base is 5?

Solution of Problem 50

Solution: Number of integers with base b and characteristics n is $b^{n+1} - b^n$.

$\therefore$ number of integers is $5^3 - 5^2 = 100$