

# Summation of Series Problems 41-48

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## Problem 41

**41.** Find the sum of the series  $1 + 9 + 24 + 46 + 75 + \dots$  to  $n$  terms.

## Solution of Problem 41

41.

$$S = 1 + 9 + 24 + 46 + 75 + \dots + t_n$$

$$S = 1 + 9 + 24 + 46 + 75 + \dots + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 1 + 8 + 15 + 22 + 29 + \dots + \text{to } n \text{ terms} - t_n$$

$$t_n = \frac{n}{2}[2 + (n-1)7] = \frac{1}{2}(7n^2 - 5n)$$

$$\therefore S_n = \frac{1}{6}n(n+1)(7n-4)$$

## Problem 42

**42.** Find the  $n$ th term of the series

$$2 + 4 + 7 + 11 + 16 + \dots$$

## Solution of Problem 42

42.

$$S = 2 + 4 + 7 + 11 + 16 + \dots + t_n$$

$$S = 2 + 4 + 7 + 11 + 16 + \dots + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 2 + [2 + 3 + 4 + 5 + \dots + \text{to } (n - 1) \text{ terms}] - t_n$$

$$t_n = 2 + \frac{n-1}{2} [2 \cdot 2 + (n-2) \cdot 1] = \frac{1}{2} (n^2 + n + 2)$$

## Problem 43

**43.** Find the sum to 10 terms of the series  $1 + 3 + 6 + 10 + \dots$

## Solution of Problem 43

**Solution:**

$$S = 1 + 3 + 6 + 10 + \dots + t_n$$

$$S = 1 + 3 + 6 + 10 + \dots + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 1 + 2 + 3 + 4 + \dots + \text{to } n \text{ terms} - t_n$$

$$t_n = \frac{n(n+1)}{2}$$

$$S_n = \frac{1}{2} \left( \sum n^2 + \sum n \right)$$

$$\Rightarrow S_{10} = 220$$

## Problem 44

**44.** The odd natural numbers have been divided in groups as  $(1, 3), (5, 7, 9, 11), (13, 15, 17, 19, 21, 23), \dots$  Show that the sum of numbers in the  $n$ th group is  $4n^3$ .



## Solution of Problem 44

**Solution:** First group has 2 numbers, second has 4 numbers and so on. So  $n$ th term will have  $2n$  terms. Also, total no. of numbers till  $n - 1$ th group is  $S = 2 + 4 + 6 + \dots + 2n - 2$

$$S = \frac{n-1}{2} [2 \cdot 2 + (n-2)2] = n(n-1)$$

So first term of the  $n$ th group will be

$$t_{n(n-1)} + 2 = 1 + (n^2 - n - 1) \cdot 2 + 2 = 2n^2 - 2n + 1$$

Thus, required sum

$$\begin{aligned} S_{2n} &= \frac{2n}{2} [2(2n^2 - 2n + 1) + (2n - 1) \cdot 2] \\ &= n[4n^2 - 4n + 2 + 4n - 2] = 4n^3 \end{aligned}$$

## Problem 45

**45.** Show that the sum of numbers in each of the following groups is a square of an odd positive integer  
 $(1), (2, 3, 4), (3, 4, 5, 6, 7), \dots$

## Solution of Problem 45

**Solution:** First term has one number, second has three numbers and third has five numbers therefore  $n$ th group will have  $2n - 1$  numbers.

Also, the first number is an A.P. with first term being 1 and common difference 1 so the first term of  $n$ th group is  $n$

$$\text{Thus, } S = \frac{2n-1}{2} [2.n + (2n-2).2] = (2n-1)^2$$

which is square of an odd positive integer.

## Problem 46

**46.** Find the sum to  $n$  terms of the series  $2 + 5 + 14 + 41 + \dots$

## Solution to Problem 46

**Solution:**

$$S = 2 + 5 + 14 + 41 + \dots + t_n$$

$$S = 2 + 5 + 14 + 41 + \dots + t_n$$

Subtracting first term of lower series with second term of upper series, second term of lower series from third term of upper series and so on, we get

$$0 = 2 + [3 + 9 + 27 + \dots + \text{to } (n-1) \text{ terms}] - t_n$$

$$t_n = 2 + \frac{3(3^{n-1} - 1)}{2}$$

$$\begin{aligned} S_n &= \frac{1}{2} \sum 3^n + \frac{1}{2} \sum 1 \\ &= \frac{3}{4} (3^n - 1) + \frac{n}{2} \end{aligned}$$

## Problem 47

**47.** Find the sum to  $n$  terms of the series  $1.1 + 2.3 + 4.5 + 8.7 + \dots$

## Solution of Problem 48

**Solution:** Clearly the first number in  $t_n$  would be  $n$ th term of a G.P. with first term 1 and common ratio 2 i.e.  $2^{n-1}$ . The second number in  $t_n$  would be  $n$ th term of an A.P. whose first term is 1 and common difference 2 i.e.  $2n - 1$ . Thus,  $t_n = 2^{n-1}(2n - 1) = n2^n - 2^{n-1}$

This is an arithmetico geometric series. So we apply the formula

$$\begin{aligned} S_n &= \frac{dr^n}{(r-1)^2} + \frac{[a + (n-1)d]r^n}{r-1} - \frac{a}{r-1} - \frac{dr}{(r-1)^2} \\ &= \frac{1 \cdot 2^n}{(2-1)^2} + \frac{[1 + (n-1) \cdot 1]2^n}{(2-1)} - \frac{1}{2-1} + \frac{1 \cdot 2}{(2-1)^2} \end{aligned}$$