

Miscellaneous Problems on A.P., G.P. and H.P. Problems 121-130

Shiv Shankar Dayal

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Problem 121

121. The sum of first ten terms of an A.P. is equal to 155, and the sum of first two terms of a G.P. is 9. Find these progressions if the first term of the A.P. equals the common ratio of the G.P. and the first term of G.P. equals the common difference of A.P.

Solution of Problem 121

Solution: Let a be the first term and d be the common difference of A.P. and thus d will be the first term and a be the common ratio of the G.P. Given,

$$155 = \frac{10}{2}[2a + (10 - 1)d] \Rightarrow 2a + 9d = 31$$

$$d + ad = 9$$

$$\Rightarrow a = \frac{25}{2}, 2 \Rightarrow d = \frac{2}{3}, 3$$

Thus, A.P. is $2, 5, 8, \dots$ or $\frac{25}{2}, \frac{79}{6}, \frac{83}{6}, \dots$ and the G.P. is $3, 6, 12, \dots$ or $\frac{2}{3}, \frac{25}{3}, \frac{625}{6}, \dots$

Problem 122

122. If a, b, c be in H.P., prove that $\left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c}\right) \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right) = \frac{4}{ac} - \frac{3}{b^2}$

Solution of Problem 122

Solution: Since a, b, c are in H.P. therefore $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow b = \frac{2ac}{a+c} \Rightarrow \frac{3}{b} - \frac{2}{c} = \frac{1}{a} + \frac{1}{b} - \frac{1}{c} \text{ and } \frac{3}{b} - \frac{2}{a} = \frac{1}{b} + \frac{1}{c} - \frac{1}{a}$$

$$\begin{aligned} \left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c}\right) \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right) &= \left(\frac{3}{b} - \frac{2}{c}\right) \left(\frac{3}{b} - \frac{2}{a}\right) \\ &= \frac{9ac - 6ab - 6bc + 4b^2}{acb^2} = \frac{4}{ac} + \frac{9}{b^2} - \frac{6b(a+c)}{acb^2} \\ &= \frac{4}{ac} + \frac{9}{b^2} - \frac{6b}{acb^2} \cdot \frac{2}{b} \\ &= \frac{4}{ac} - \frac{3}{b^2} \end{aligned}$$