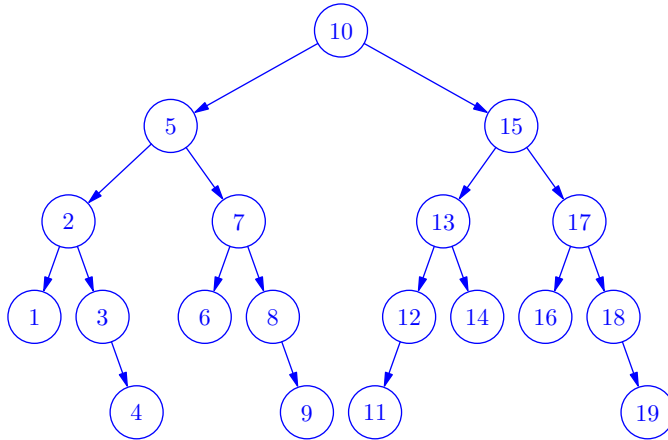# Binary Trees

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Binary Tree

Definitions

A *binary tree* is a finite set of elements that is either empty or partitioned into three disjoint subset. The first subset contains a single element called the *root* of the tree. Two other subsets themselves are binary trees, called the *left* or *right* subtrees of the original tree. Either or both of the subtrees can be empty. Each element of a binary tree is called a *node* of the binary tree.

If A is the root of a binary tree and B is the root of its left or right suhtree, then A is said to be the *father* of B and B is said to be the left or right *son* of A. A node that has no sons is called a *leaf* node.

A node A is an *ancestor* of some node B(this is *descendant* of ancestor node) if A is either the father of B or the father of some ancestor of B. Two nodes are *brothers* if they are left and right sons of the same father.

If every nonleaf node in a binary tree has nonempty left and right subtrees, the tree is called a *strictly binary tree*. A strictly binary tree with n leaves always contains $2n - 1$ nodes.

The root of the tree has level 0, and the level of any other node in the tree is one more than the level of its father. The *depth* of a binary tree is the maximum level of any leaf in the tree. This is equal to the lenght of the longest path from the root to any leaf.

A *complete binary tree* of depth d is the strictly binary trree all of whose leaves are ay level d.

A complete binary tree contains 1 node at level $0, 2$ at level $1, 4$ at level 2 and so on. On dth level it will contain 2^d leaves. Thus, total no. of nodes $= 2^0 + 2^1 + 2^2 + ... + 2^d = 2^{d+1} - 1$. Since all the leaves at at level d total no. of leaves 2^d and total no. of nonleaf nodes is $2^d - 1$.

A binary tree of depth d is an allmost complete binary tree if:

1. Any node at level less than $d - 1$ has two sons.
2. For any node in the tree with a right descendant at level dm it must have a left son and every left desendant of it is either a leaf at level d or has two sons.