Practice Assignment 4by Shiv Shankar Saran

I.

A. Depict lake pollution model in the constant flowrate setting; taking cin = 4, V = 30, F = 52 for ten different values of the initial concentration. What happens to the concentration as t increases?
B. Modify the syntax to account for a seasonal flowrate given by F(t) =8 + 4 cos(2πt), retaining the other values as they are in Part A. What changes do you observe in the two plots? Write as a comment under the plot.

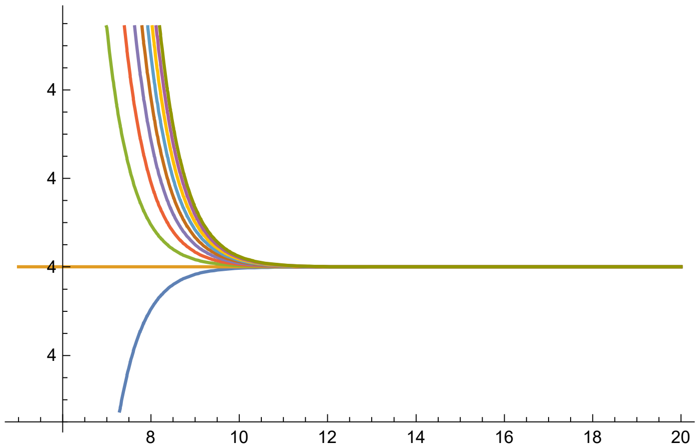
A. Constant flowrate

In[338]:=
```
cin = 4;
V = 30;
F = 52;
de1 = D[C[t], t] == (F / V) * (cin - C[t])
sol1 = DSolve[{de1, C[0] == C0}, C[t], t]
Plot[Evaluate[C[t] /. sol1 /. C0 → Range[3, 12]], {t, 5, 20}]
```

Out[341]= $C'[t] == \frac{26}{15}(4 - C[t])$

Out[342]= $\{\{C[t] \to e^{-26\,t/15}\,(-4 + C0 + 4\,e^{26\,t/15})\}\}$

Out[343]=




```
(*As t increases,concentration level become stable near 6
 If the inflowing pollutants exceed the lake's natural capacity for
 absorption or dilution,pollution levels may gradually increase over
 time,leading to an upward trend in the graph.On the other hand,if
 the flow rate brings in pollutants below the lake's capacity for
 handling them,pollution levels may decrease over time,resulting in
 a downward trend in the graph.*)
```

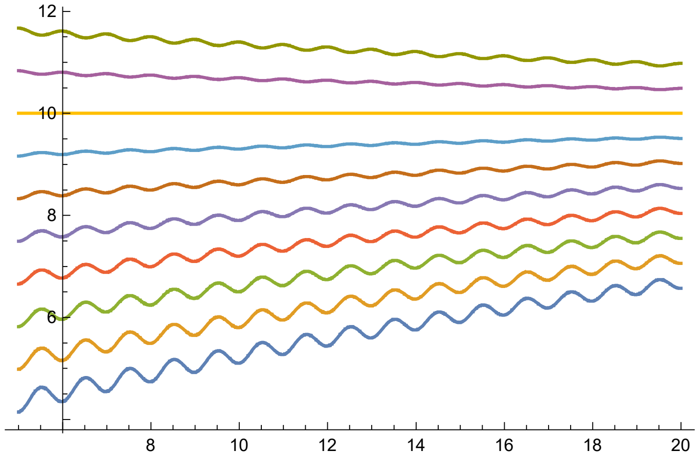
B. Seasonal flowrate

In[169]:=
```
V = 28;
de2 = D[C[t], t] == ((1 + 6 Sin[2 * Pi * t]) * (10 - C[t])) / V
sol2 = DSolve[{de2, C[0] == C0}, C[t], t]
Plot[Evaluate[C[t] /. sol2 /. C0 → Range[3, 12]], {t, 5, 20}]
```

Out[170]= $C'[t] == \frac{1}{28} (10 - C[t]) (1 + 6 \text{Sin}[2 \pi t])$

Out[171]= $\left\{\left\{C[t] \to e^{-\frac{3}{28\pi} - \frac{t}{28}} \left(10\, e^{\frac{3}{28\pi} + \frac{t}{28}} - 10\, e^{\frac{3 \text{Cos}[2\pi t]}{28\pi}} + C0\, e^{\frac{3 \text{Cos}[2\pi t]}{28\pi}}\right)\right\}\right\}$

Out[172]=




```
(* In graph 1,with increase in t,concentration level become stable near 6
 In graph 2 with seasonal flow rate,the pollution level graph exhibit
 periodic fluctuations.During periods of high flow rates (e.g.,during
   rainy seasons),the influx of pollutants may increase,causing pollution
 levels to rise.This would be reflected as peaks or upward spikes in the
 graph.During periods of low flow rates (e.g.,during dry seasons),inflow
 of pollutants may decrease,resulting in reduced pollution levels.This
 would be represented as valleys or downward dips in the graph.The graph
 shows a repeating pattern of peaks and valleys corresponding to the
 seasonal changes in the flow rate*)
```

2.

A. Plot standard Lotka Volterra system of One Predator-One Prey interaction, taking appropriate values of the constants of your choice for the plot. Comment on the behaviour of the two populations as t increases.

B. Modify the syntax to include one more prey in the model, assuming that the two prey species do not interact with each other. What happens as time progresses? Comment on the basis of the plot obtained.

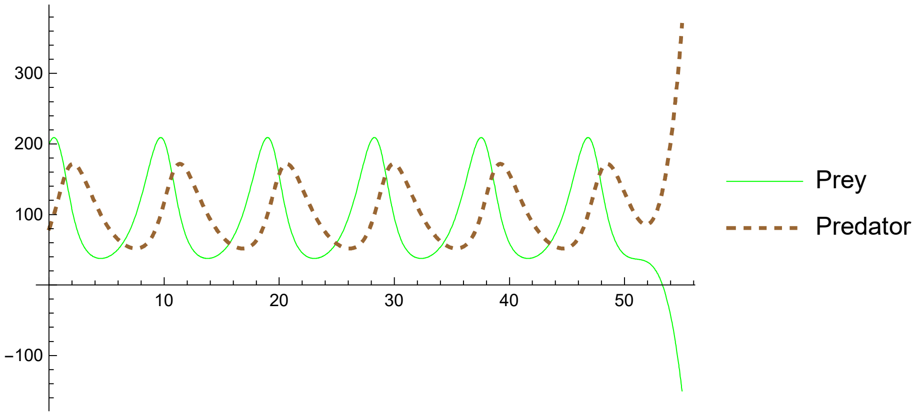
A. One Prey- One Predator

In[333]:=
```
b1 = 1; a2 = 0.5; c1 = 0.01; c2 = 0.005;
de3 = x'[t] == b1 * x[t] - c1 * x[t] * y[t]
de4 = y'[t] == c2 * x[t] * y[t] - a2 * y[t]
sol3 = NDSolve[{de3, de4, x[0] == 200, y[0] == 80}, {x[t], y[t]}, {t, 0, 50}];
Plot[Evaluate[{x[t], y[t]} /. sol3], {t, 0, 55},
 PlotRange → All, PlotLegends → {"Prey", "Predator"},
 PlotStyle → {{Green, Thin, Block}, {Brown, Dashed, Thick}}]
```

Out[334]= x′[t] == x[t] - 0.01 x[t] y[t]

Out[335]= y′[t] == -0.5 y[t] + 0.005 x[t] y[t]

Out[337]=



When the prey population increases, the predator population flourishes as it has food supply. Similarly, when When the prey population decreases, the predator po decreases as it has less food available.

B. Two Prey- One Predator

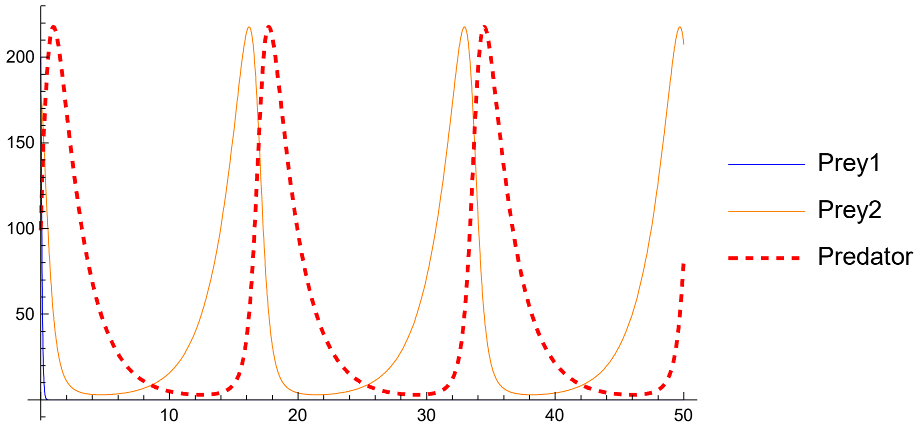
In[350]:=
```
b1 = 1; b2 = 0.5; b3 = 0.5; c1 = 0.1; c2 = 0.01; c3 = 0.01; c4 = 0.01;
de5 = x'[t] == b1 * x[t] - c1 * x[t] * z[t]
de6 = y'[t] == b2 * y[t] - c2 * y[t] * z[t]
de7 = z'[t] == -b3 * z[t] + c3 * x[t] * z[t] + c4 * y[t] * z[t]
sol4 = NDSolve[{de5, de6, de7, x[0] == 200, y[0] == 180, z[0] == 100},
   {x[t], y[t], z[t]}, {t, 0, 50}];
Plot[Evaluate[{x[t], y[t], z[t]} /. sol4], {t, 0, 50}, PlotRange → All,
 PlotLegends → {"Prey1", "Prey2", "Predator"},
 PlotStyle → {{Blue, Thin}, {Orange, Thin}, {Red, Thick, Dashed}}]
```

Out[351]= $x'[t] == x[t] - 0.1\, x[t]\, z[t]$

Out[352]= $y'[t] == 0.5\, y[t] - 0.01\, y[t]\, z[t]$

Out[353]= $z'[t] == -0.5\, z[t] + 0.01\, x[t]\, z[t] + 0.01\, y[t]\, z[t]$

Out[355]=



The graph of prey 1 vanishes whithin a short time at approx x = 0.6 . The graph of prey 2 also fall down initially in same time period, while predator population flourishes. Then again , due to shortage of food, predator population starts declining. With decline in population of predator, prey population starts flourishing. Again with prey population flourishment, predation population increases, and same pattern is repeated.

3.

Syntax related question:
A. What is the difference between DSolve and NDSolve commands?
B. What is the use of PlotLegends command?
C. How does PlotStyle command help in getting advanced plots?

Answer:
A. DSolve is used when we have to solve one differntial equation,while NDSolve is used when we have to solve 2 or more than 2 differential equation.
B. PlotLegends command is used to display a legend for a plot, providing a visual guide to the meaning of different plotted elements or functions.
C. Using PlotStyle, we can do different type of styling in graph, like we can change colour,thickness,style and many more of curves in graph.