## Differentiation DATE DELLE

Let f be real-valued function defined on an open interval containing a point a we say fix differentiable at a, is finite. Then, f'(a) = lim f(v)-f(a) exists and is finite. Then, f'(a) = lim f(v)-f(a) x-a dom(f') = dom(f') = dom(f')

& f(x)=x" +xER

 $\Rightarrow f(x) - f(a) = x^{n} - a^{n} = (x - a) (x^{n-1} + ax^{n-2} + a^{2}x^{n-3} + \dots + a^{n-1})$ :.  $f(x) - f(a) = x^{n-1} + ax^{n-2} + \cdots + a^{n-2}x + a^{n-1}$  for  $x \neq a$ 

Then f'(a) = lim f(x)-f(a) = an-1+an-1 + - an in times)

Proof: f'(a) = Lim f(x)-f(a) (Griven)

To prove: lim f(x)= f(a)

We have: f(x)= (x-a).f(x)-f(a) +f(a) x-a yxedom(1), x+a.

Now, lim f(x) = lim (x-a). (im f(x)-f(a) + lim f(a)

= 0 + f(a) = f(a)

Theorem: Let I and g be function differentiable at a Then cf, ftg, fg, f/g is also differentiable at a, except f/g if g(a) = 0. The formulas are: (1) (cf)(a) = c.f.(a)

 $(2) (f \pm g)'(a) = f'(a) \pm g'(a)$ 

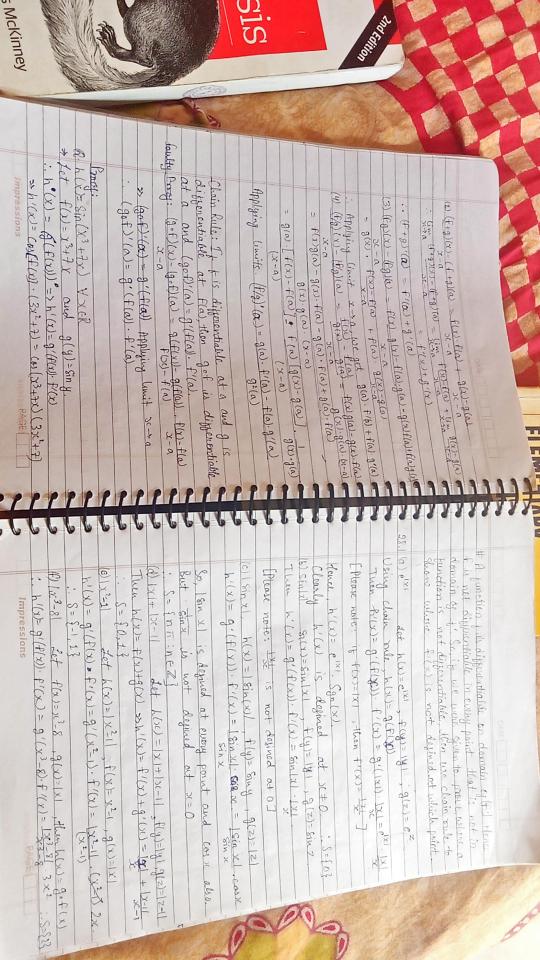
(3) [ product rule] (fg)'(a)= f(a)g'(a)+f'(a)g(a)

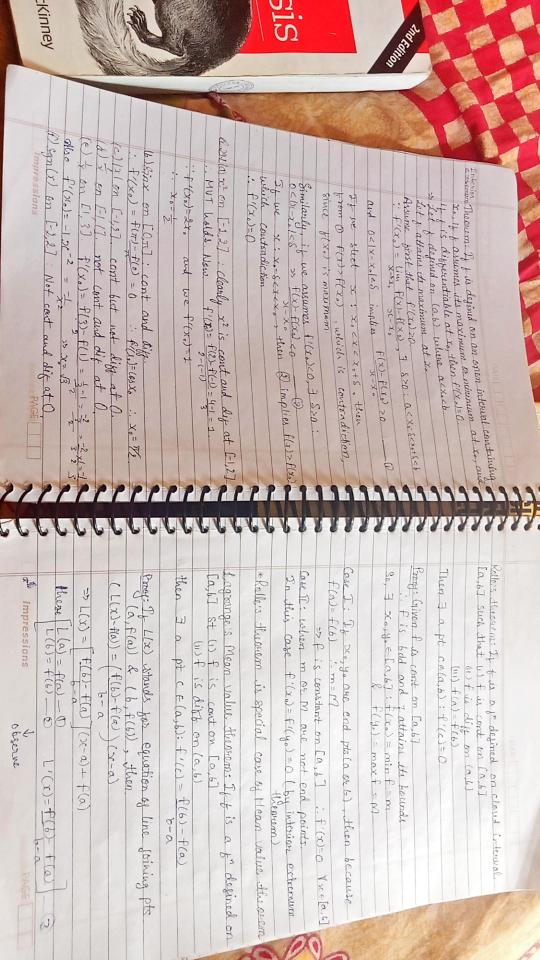
(4) [ quotient onle] (+/g)'(a) = [g(a) +/a) - +(a) g'(a)]

Prog: (1) (cf)(a)= \(\int\_{\text{sin}}\) (cf)(\(\text{cf})(a) = \(\int\_{\text{cf}}\)(a) = \(\int\_{\text{con}}\) (cf)(a) = \(\int\_{\text{con}}\) \(\text{con}\) \(\text{con}

Impressions

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Renneth A. Ross Similarly, 7 g= (x,x2): g(y2) < g(x2), 90 x0 ±x2 xo is in (x, x2), so g(x)=0, by Sutorioz extremum. Incurasing and decreasing function in interval I Dof (Inc): A & f defined on R is said in be an +1(x0)= 81(x0)+C-C. increasing for it for x122 22 f(x1) = f(x1) Phenoun: Let F. me-to one contif on I and let To T- f(D) Strictly inc: f(1) < f(x2) Similarly in dec To bis diff of xoEI and if f(xo) to then b' is differentiable at yo= f(x) and (f-1)'(yo) = f'(x) (1) + is shortly in or die it f(x)>0 or f(x) ko Vx eta. (). (1) f is inc as due if f'(1) o or f'(1) to 4 relad Add let f(x)=x-sinx. We have to show f(x)>0 4 x>0 fi(x)=1-cosx. Hore cosxe[-1,1]  $\Rightarrow$  (1) (onsider  $\times$ ,  $\times_2 \in (a,b)$  st  $a \leq \times_1 \in \times_2 \leq b$ . By MVI, for some x e(x, x2), we have f(x) = f(x) + f(x) 1. f1(x)20 2312 F(x) = tanx-x then P'(x) = Sco2x-1 = (052x-1) we know that cos2x (0,1) : f'(x)>0,: f(0)=0 Also since it is ine for & feloso It implies  $f(x_2) - f(x_2) > f(x_2) > f(x_3) > f(x_3)$ (b) fot f(x)= >C Lystoictly inc .. f((x) = Sinx - xcox . On (0, 7/2) Sin x (0,1) X COSX E ( DIME) [:x,-x, >0] 342 x C(0,1) YSIS Intermediate Value theorem for Derivatives Clearly, Sin2x 70 Let file a differentiable from (a,6) ushenews Nous, we have toprove Sins & cost >> toun > I which is proved In port (a) acx, <x2 < b and c lies bet f'(x1) and f'(x2), PEPPER I Lat least one x in (x, x2) st f'(x)=c. : f(x)>0 on (0, 1/2) (c) f(x) = \$28mx-x f'(x)= \$72 coix -1 > Proof: Assume f'(x) < c < f'(x) we thave to prove 1/2 core 20 which is the Let g(x)=f(x)-cx + sce(a,b) Now, 18 cosx = 2 20 cosx = 2/11 20 20 500000 0.88=0.004x [Note: we took f(x)-cx becaus g'(x)=f'(x)-() Then, we have g'(DC+)<0<g'(X2) from 1 1: x 20.004 1 ((x)20 and vice ver2a. V: \$(0.001) is winimum and \$(0.004) = 0 (aprox) According to Maximum value theorem, g assumes Hs minimum on [x,,x2] at x. 6[x,x2] :. f(x) >0 >> TERMEZY Clearly cox ((0,1): f(x) >0 g'(x,)= (1 g(y)-g(x1) <0, Now f(0)=0: f(x) > 0 >> F(8hx > x Since y-x, >0, : 9(y)-g(x)) must be negative for 13. Let h(x) = g(x) - f(x) . Also - f'(x) +gr(x) =0 y closer to and larger than x, :. h'(x) ≥0 also h(0)=0 : h(x) is inc in [0,00] >> g(x)>f(x) in [0,00) In particular, I y,  $E(x_1, x_2)$ :  $g(y_1) < g(x_1)$ .

I g doesn't take minimum at x, so we must have  $x_0 \neq x_1$ . les McKinney