REGRESSION WITH TIME SERIES VARIABLES

INTRODUCTION

- Regression modelling goal is complicated when the researcher uses time series data since an explanatory variable may influence a dependent variable with a time lag. This often necessitates the inclusion of lags of the explanatory variable in the regression.
- If "time" is the unit of analysis we can still regress some dependent variable, Y, on one or more independent variables

INTRODUCTION

- The form of a regression model with one explanatory variable is:

$$y_t = x_t'\beta + \varepsilon_t, t=1,2,\ldots,n$$

β is a k × 1 vector of parameters to be estimated and ε_t is an error term.

Assumptions about ε_t, the “error term”:

i. $E(\varepsilon_t)$ = 0, zero mean

ii. $E(\varepsilon_t) = \sigma^2$, constant variance

iii. $E(\varepsilon_t, X_t)$ = 0, no correlation with X

iv. $E(\varepsilon_t, \varepsilon_{t-i})$ =0, no autocorrelation.

v. ε_t ~ Normally distributed.

- Assumption four is especially important and most likely not to be met when using time series data.

INTRODUCTION

- The most common situation occurs when i = 1, which is called a first-order autocorrelation;

$$E(\varepsilon_t \varepsilon_{t-1}) \neq 0.$$

Then, we will have

A simple linear model has the usual form

$$\boxed{Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t}$$

But now the errors are related by the (linear) simple regression function:

$$\boxed{\varepsilon_t = \rho\varepsilon_{t-1} + v_t}$$

SOME REGRESSION MODELS WHEN VARIABLES ARE TIME SERIES

Static Time Series Regression Model (*also called the **Levels Model**):

$$Y_t = \beta_0 + \alpha_0 X_t + \varepsilon_t$$

Distributed Lag Time Series Regression Model:

$$Y_t = \beta_0 + \alpha_0 X_t + \alpha_1 X_{t-1} + \cdots + \alpha_q X_{t-q} + \varepsilon_t$$

Autoregressive Distributed Lag Time Series Regression Model (**ADL Model**):

(Also referred to as the ARDL or ARX model)

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} + \alpha_0 X_t + \alpha_1 X_{t-1} + \cdots + \alpha_q X_{t-q} + \varepsilon_t$$

STATIC MODEL (Levels Model)

- A contemporaneous relation between y and x can be captured by a static model:

$$Y_t = \beta_0 + \alpha_0 X_t + \varepsilon_t,\ t = 1, 2, \ldots, n.$$

When to use?

How to estimate?

AUTOREGRESSIVE DISTRIBUTED LAG (ADL) MODEL

- Augment AR(p) with lags of explanatory variables produces ADL model (*where we hve also included a deterministic linear trend $\boldsymbol{\delta t}$):

$$Y_t = \beta_0 + \boldsymbol{\delta t} + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} + \alpha_0 X_t + \alpha_1 X_{t-1} + \cdots + \alpha_q X_{t-q} + \varepsilon_t$$

- **p lags of Y, q lags of X $\Rightarrow$ ADL(p,q).**

AUTOREGRESSIVE DISTRIBUTED LAG (ADL) MODEL

- Estimation and interpretation of the ADL(p,q) model depends on whether Y and X are stationary or have unit roots.

- Before you estimate an ADL model you should test both Y and X for unit roots using the Augmented Dickey-Fuller (ADF) test.

TIME SERIES REGRESSION WHEN X AND Y ARE STATIONARY

- Minimal changes (e.g. OLS fine, testing done in standard way, etc.), except for the interpretation of results.
- Lag lengths, p and q can be selected using sequential tests.
- It is convenient to rewrite ADL model as:

$$\nabla Y_t = \beta_0 + \boldsymbol{\delta t} + \varphi Y_{t-1} + r_1 \nabla Y_{t-1} + \cdots + r_{p-1} \nabla Y_{t-p+1} + \theta X_t$$
$$+ \omega_0 \nabla X_t + \omega_1 \nabla X_{t-1} + \cdots + \omega_{q-1} \nabla X_{t-q+1} + \varepsilon_t$$

TIME SERIES REGRESSION WHEN X AND Y ARE STATIONARY

- Effect of a slight change in X on Y in the long run.
- To understand the long run multiplier:

❖Suppose X and Y are in an equilibrium or steady state.

- All of a sudden, X changes slightly.
- This affects Y, which will change and, in the long run, move to a new equilibrium value.
- The difference between the old and new equilibrium values for Y = the long run multiplier effect of X on Y.
- Can show that the long-run multiplier is $-\theta/\phi$.
- For system to be stable (a concept we will not formally define), we need $\phi < 0$.

EXAMPLE: THE EFFECT OF FINANCIAL LIBERALIZATION ON ECONOMIC GROWTH

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- Time series data for 98 quarters for a country
- Y = the percentage change in GDP
- X = the percentage change in total stock market capitalization
- Assume Y and X are stationary (*this is not a necessary condition to utilize the ADL model as we know the ADL model is suitable for all I(0), all I(1) and, mixed I(0) and I(1). It is a very flexible model indeed.)

EXAMPLE: THE EFFECT OF FINANCIAL LIBERALIZATION ON ECONOMIC GROWTH

- ADL(2,2) with Deterministic Trend Model

$$\nabla Y_t = \beta_0 + \boldsymbol{\delta t} + \varphi Y_{t-1} + r_1 \nabla Y_{t-1} + \theta X_t + \omega_0 \nabla X_t + \omega_1 \nabla X_{t-1} + \varepsilon_t$$

	Coeff.	Stand. Error	t-Stat	P-val	Lower 95%	Upper 95%
Inter.	-.028	.041	-.685	.495	-.110	.054
Y_{t-1}	-.120	.013	-9.46	4.E-15	-.145	-.095
ΔY_{t-1}	.794	.031	25.628	7.E-43	.733	.856
X_t	.125	.048	2.605	.011	.030	.221
ΔX_t	.838	.044	19.111	3.E-33	.750	.925
ΔX_{t-1}	.002	.022	.103	.918	-.041	.046
time	.001	.001	.984	.328	-.001	.002

EXAMPLE: THE EFFECT OF FINANCIAL LIBERALIZATION ON ECONOMIC GROWTH

- Estimate of long run multiplier:

$$-(0.125/-0.120)=1.042$$

- Remember that the dependent and explanatory variables are % changes:

- The long run multiplier effect of financial liberalization on GDP growth is 1.042 percent.

- If X permanently increases by one percent, the equilibrium value of Y will increase by 1.042 percent.

TIME SERIES REGRESSION WHEN Y AND X HAVE UNIT ROOTS: SPURIOUS REGRESSION

- Now assume that Y and X have unit roots.
- Consider the standard regression of Y on X:

$$Y_t = \alpha + \beta X_t + \varepsilon_t$$

- OLS estimation of this regression can yield results <u>which are completely wrong</u>.

- These problems carry over to the ADL model.

TIME SERIES REGRESSION WHEN Y AND X HAVE UNIT ROOTS: SPURIOUS REGRESSION

- Even if the true value of β is 0, OLS can yield an estimate, $\hat{\beta}$, which is very different from zero.
- Statistical tests (using the t-stat or P-value) may indicate that β is not zero.
- If β=0, then the R^2 should be zero. In the present case, the R^2 will often be quite large.
- This is called the ***spurious regression*** *problem*.
- **Practical Implication:**
- *With the one exception that we note below, you should never run a regression of Y on X if the variables have unit roots.*
- *The exception occurs if Y and X are* ***cointegrated****.*

SPURIOUS REGRESSION

Set-up:

$$y_t = y_{t-1} + u_t \; ; \; u_t \propto iid(0, \sigma_u^2)$$

$$x_t = x_{t-1} + v_t \; ; \; v_t \propto iid(0, \sigma_v^2) \qquad ; \; E(u_t, v_s) = 0 \quad \forall t, s$$

$$E(u_t u_{t-k}) = E(v_t v_{t-k}) = 0 \quad \forall k$$

Regress

$$\boxed{y_t = \alpha + \beta x_t + \varepsilon_t}$$

What do you expect to get?

$$\hat{\beta} \xrightarrow{p} 0$$

$$R^2 \xrightarrow{p} 0$$

$$t_{\hat{\beta}} \Rightarrow t_{distribution}$$

Spurious Regression (continued)

What happened really

$$\hat{\beta} \Rightarrow \text{some distribution}$$

$$R^2 \Rightarrow \text{some distribution}$$

$$DW \xrightarrow{p} 0$$

$$T^{-1/2} t_{\hat{\beta}} \Rightarrow \text{some distribution}$$

Spurious Regression (continued)

Spurious Regression Problem:

Regression of an integrated series on another unrelated integrated series produces t-ratios on the slope parameter which indicates a relationship much more often than they should at the nominal test level. This problem does not dissapear as the sample size is increased.

In a Spurious Regression the errors would be correlated and the standard t-statistic will be wrongly calculated because the variance of the errors is not consistently estimated. In the I(0) case the solution is:

Spurious Regression (continued)

How do we detect a Spurious Regression (between I(1) series)?

Looking at the correlogram of the residuals and also by testing for a unit root on them.

How do we convert a Spurious Regression into a valid regression?

Quick solution (may not be optimal): By taking differences. That is, use ***the difference model****:*

$$\nabla Y_t = \alpha + \beta \nabla X_t + \varepsilon_t$$

Does this solve the SPR problem?

It solves the statistical problems but not the economic interpretation of the regression. Think that by taking differences we are loosing information and also that it is not the same information contained in a regression involving growth rates than in a regression involved the levels of the variables.

Spurious Regression (continued)

Does it make sense to preform a regression between two I(1) variables?

Yes if the regression error is I(0).

Can this be possible?

The same question was asked by David Hendry to Clive Granger some time ago.

Clive answered NO WAY!!!!! but he also said that he would think about. In the plane trip back home to San Diego, Clive thought about it and concluded that YES IT IS POSSIBLE. It is possible when both variables share the same source of the I(1)'ness (co-I(1)), that is, when both variables move together in the long-run (co-move), ... That is, when both variables are

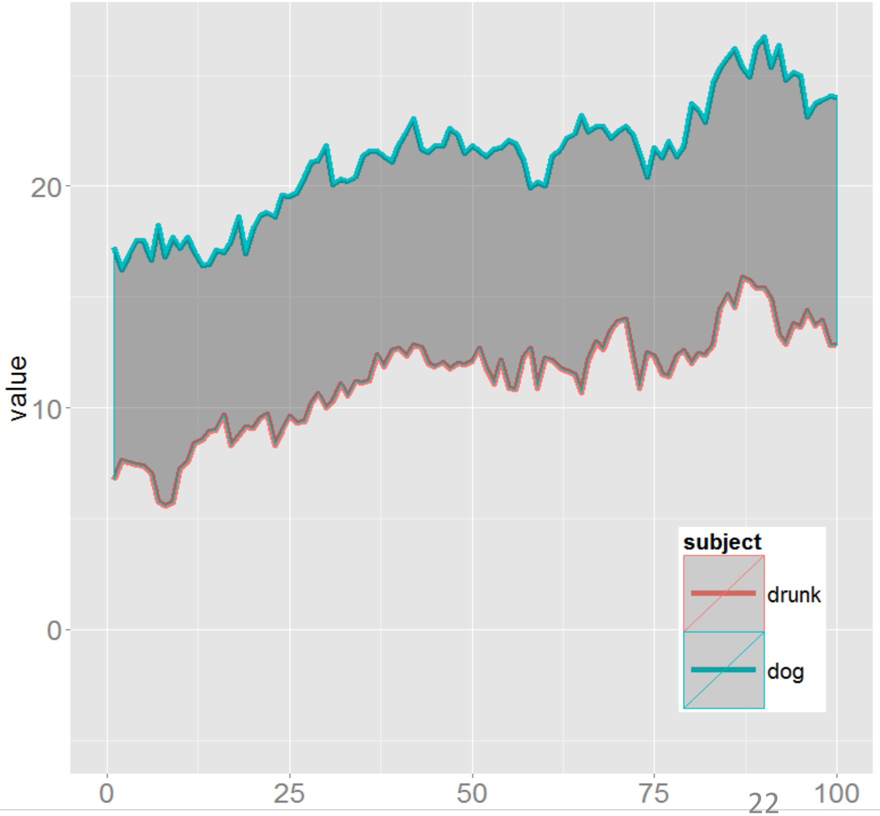
COINTEGRATED!

Cointegration, Example 1

- **Drunk woman and a stray dog**

Drunk's position is a random walk along real line: $X_t = X_{t-1} + u_t$, where u_t is white noise
The dog also wanders aimlessly as a random walk: $Y_t = Y_{t-1} + w_t$, where w_t is white noise

- **What if the dog belongs to the drunk?**
  - They wouldn't be far away from each other
  - Drunk's current position X_t is not only affected by her previous position X_{t-1}, but also affected her distance from her dog previously, i.e. $Y_{t-1} - X_{t-1}$
  - Dog's current position Y_t is not only affected by her previous position Y_{t-1}, but also affected her distanc from her dog previously, i.e. $Y_{t-1} - X_{t-1}$



Cointegration, Example 2

Theory of Purchasing Power Parity (PPP)

"Apart from transportation costs, good should sell for the same effective price in two countries"

$$P_t = S_t \, P_t^*$$

An index of the price level in the USA

\$ per £

Price Index for UK

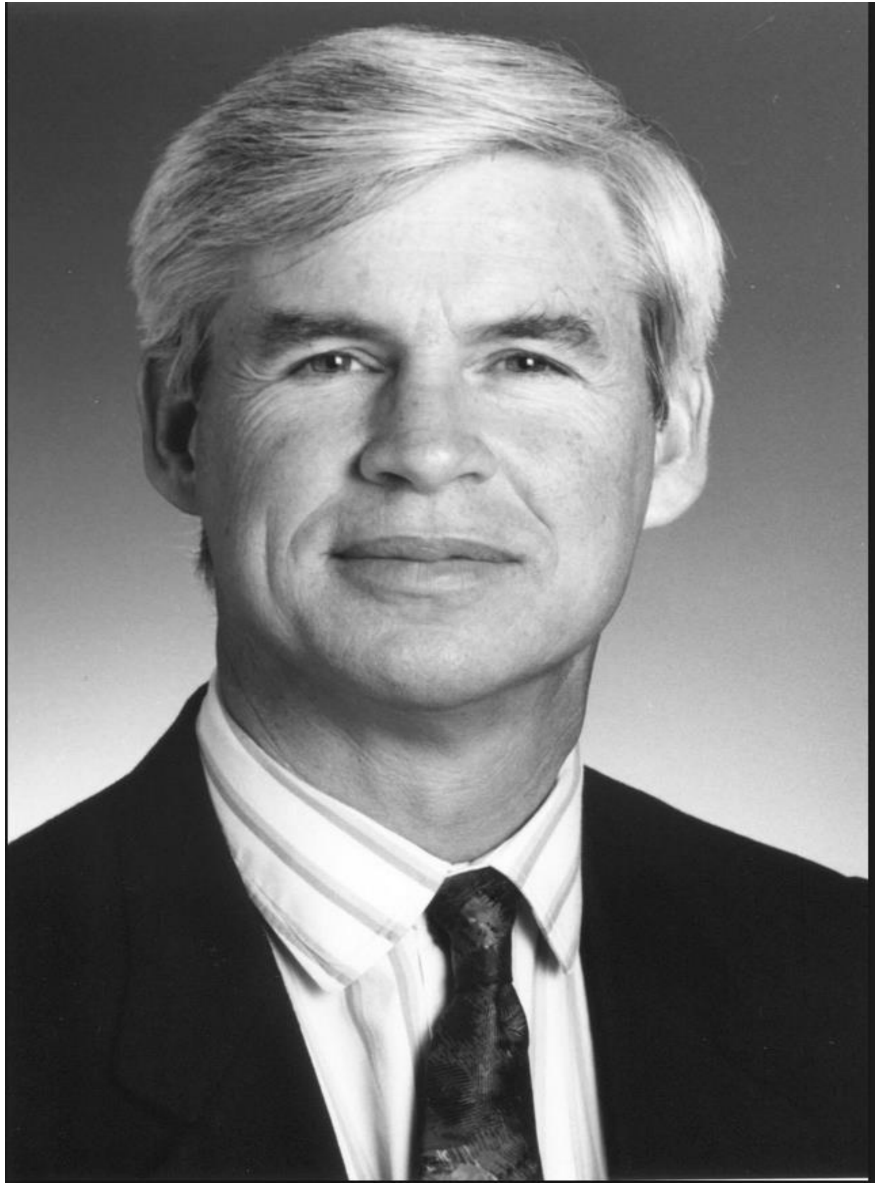
In logs :

$$p_t = s_t + p_t^*$$

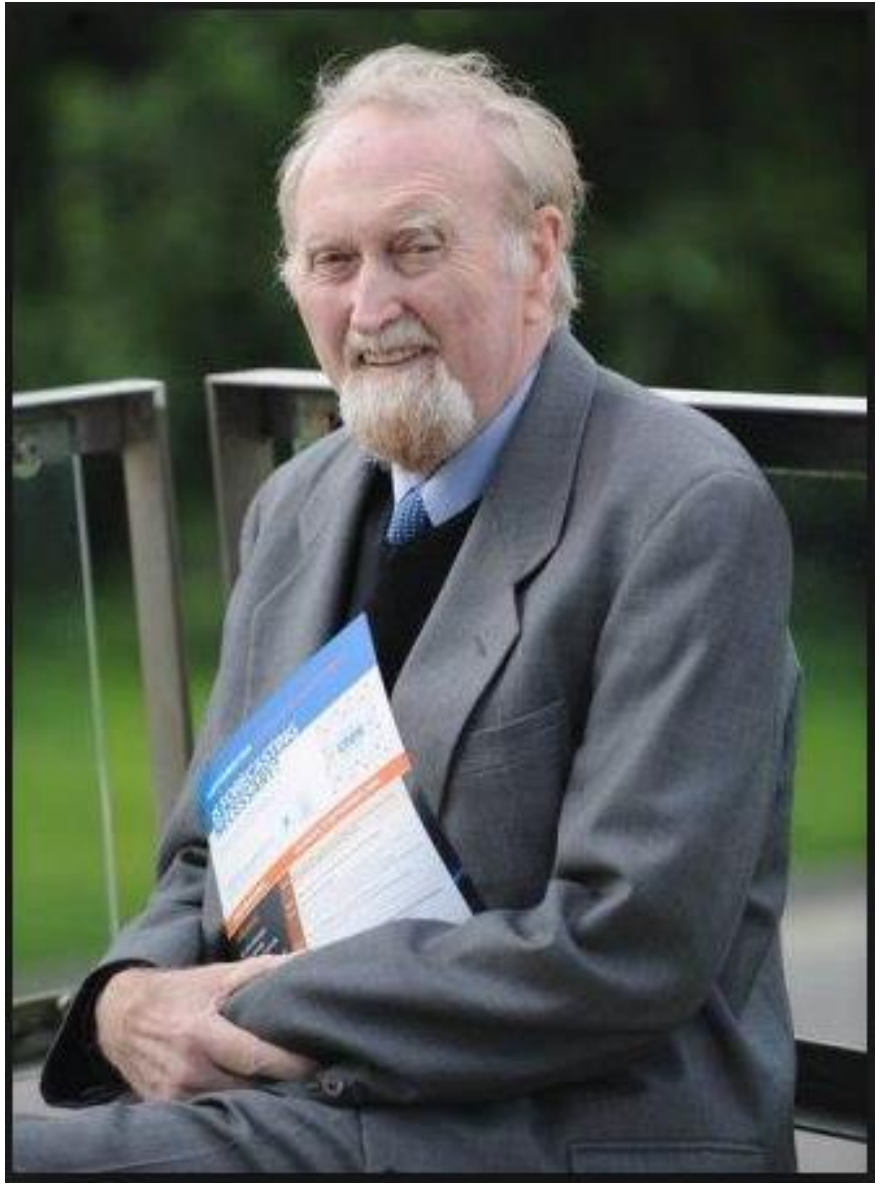
A weaker version of the PPP:

$$p_t = s_t + p_t^* + z_t$$

If the three variables are I(1) and z_t is I(0) then the PPP theory is implying cointegrating between p_t, s_t and p^*_t .



Robert F. Engle (born in 1942)
American economist, currently teaches at New York University, Stern School of Business



Clive Granger (1934 – 2009)
British economist, taught at University of Nottinghan in Britain & University of California, San Diego in US

They shared the Nobel Prize in Economic s in 2003 for developing the ARCH model (Engle) and Cointegration (Granger)

Test for Cointegration, the Engle-Granger Test

Cointegration:

There are two $I(1)$ series, X_t and Y_t, if there exists β s.t. $Y_t - \beta X_t \sim I(0)$, we say X_t and Y_t are cointegrated, or there is one cointegrating relation between X_t and Y_t, and the state of $Y_t - \beta X_t \sim I(0)$ is called the long-run equilibrium between X_t and Y_t

Test for cointegration:

- Engle Granger test
  - Fit OLS – $Y_t = \hat{\alpha} + \hat{\beta} X_t + \hat{Z}_t$
  - Test $\hat{Z}_t$ is I(0) or I(1)

How to test I(0) v.s. I(1):

Dickey-Fuller test & Augmented Dickey-Fuller test

EG test

- R: `fit5 = egcm(x,y)`
- SAS: `proc AUTOREG; model y = x/ nlag=1 stationarity =(adf=1);`

The Error Correction Model (ECM)

When two I(1) series are co-integrated, the best regression model between the two is the error correction model as it features both their long-run relationship, as well as their short-term relationship.

EG two-step fitting:

- Fit OLS: $Y_t = \hat{\alpha} + \hat{\beta} X_t + \hat{u}_t$
  (Since X_t & Y_t are cointegrated, $\hat{\alpha}$ & $\hat{\beta}$ are cointegrated)
- Fit OLS: $\nabla Y_t = \hat{\theta} \nabla X_t + \hat{\phi}\left(Y_{t-1} - \hat{\alpha} - \hat{\beta} X_{t-1}\right) + \hat{\varepsilon}_t$

Relationship between the Error Correction Model (ECM) and the Autoregressive Distributed Lag (ADL) Model

They are indeed equivalent to each other (with the ADL being more flexible):

- ECM:

$$\nabla Y_t = \theta \nabla X_t + \phi(Y_{t-1} - \alpha - \beta X_{t-1}) + \varepsilon_t$$

can be re-written as:

- ADL:

$$Y_t = -\alpha\phi + (1+\phi)Y_{t-1} + \theta X_t - (\theta + \beta\phi)X_{t-1} + \varepsilon_t$$

and vice versa.

Further Comparison of the Error Correction Model (ECM) and the Autoregressive Distributed Lag (ADL) Model

- The ECM model enjoys a clear interpretation by linking incorporating both the short term relationship and the long term relationship in the same regression model.

- While the ECM model is designed when all variables are I(1), the ADL Model is applicable when (1) all variables are I(1) , and (2) when we have a mixture of I(1) and I(0) variables.

To be continued … for more complicated scenarios such as more regressors and more cointegration relationships.

"Seven 'provided', thirteen 'but's' and twenty seven 'if's' - this is the best stockmarket forecast to clients I've ever read."