Rob J Hyndman

Forecasting using R

11. Dynamic regression

OTexts.com/fpp/9/1/

Outline

Regression with ARIMA errors

Regression models

$$y_t = b_0 + b_1 x_{1,t} + \cdots + b_k x_{k,t} + n_t$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \dots, x_{k,t}$.
- Usually, we assume that n_t is WN.
- Now we want to allow n_t to be autocorrelated.

Example: n_t = ARIMA(1,1,1)

$$y_t = b_0 + b_1 x_{1,t} + \cdots + b_k x_{k,t} + n_t$$

$$\text{where} \quad (1 - \phi_1 B)(1 - B)n_t = (1 - \theta_1 B)e_t$$

and e_t is white noise .

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- Be careful in distinguishing n_t from e_t.
- n_t are the "*errors*" and e_t are the "*residuals*".
- In ordinary regression, n_t is assumed to be white noise and so $n_t = e_t$.

After differencing all variables

$$y'_t = b_1 x'_{1,t} + \cdots + b_k x'_{k,t} + n'_t.$$

Now a regression with ARMA(1,1) error

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Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Original data

$$y_t = b_0 + b_1 x_{1,t} + \cdots + b_k x_{k,t} + n_t$$

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Modeling procedure

Problems with OLS and autocorrelated errors

1. OLS no longer the best way to compute coefficients as it does not take account of time-relationships in data.
2. Standard errors of coefficients are incorrect — most likely too small. This invalidates tests and prediction intervals.

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- Estimation only works when all predictor variables are deterministic or stationary and the errors are stationary.
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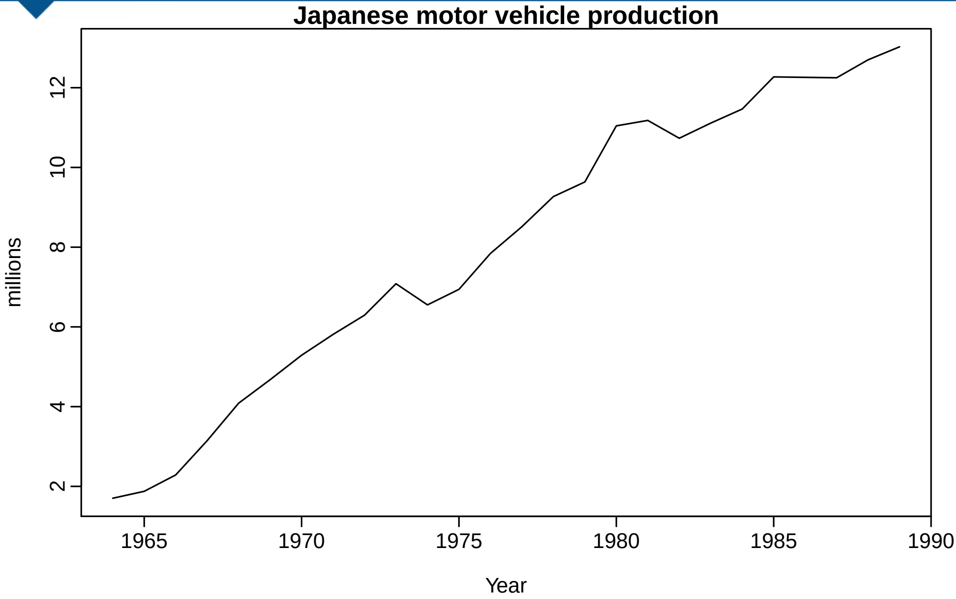
1. Regression with ARIMA errors
2. **Example: Japanese cars**
3. Using Fourier terms for seasonality
4. Example: Sales of petroleum & coal products

Example: Japanese cars



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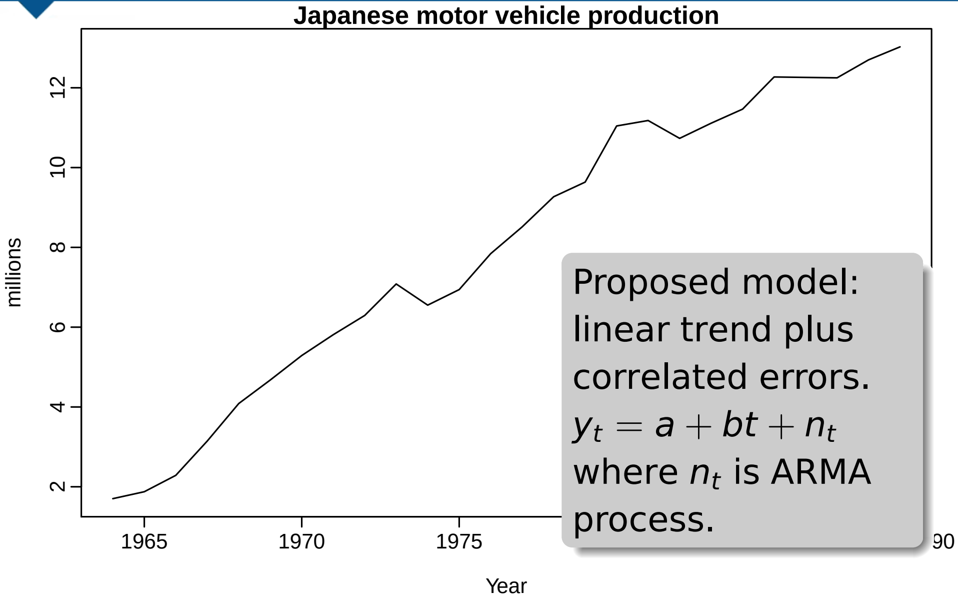

Proposed model: linear trend plus correlated errors.

$y_t = a + bt + n_t$

where n_t is ARMA process.

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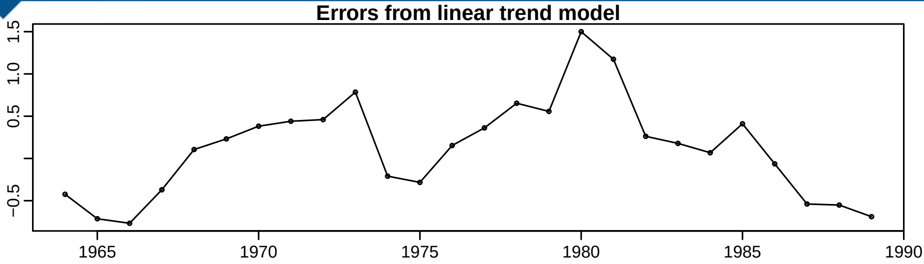

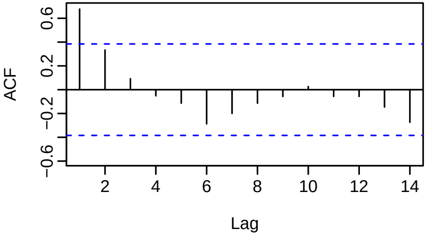

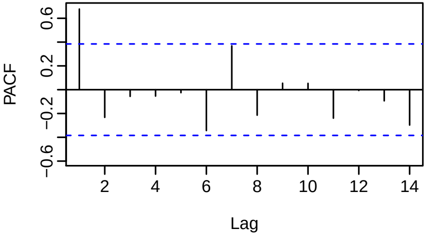

Example: Japanese cars

1. We will fit a linear trend model:

$$y_t = a + bx_t + n_t$$

where $x_t = t - 1963$.

2. `auto.arima` chooses an AR(1) model for n_t.
3. Full model is

$$y_t = a + bx_t + n_t \qquad \text{where} \quad n_t = \phi_1 n_{t-1} + e_t$$

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AR(1)	ϕ_1	0.736	0.152
Intercept	a	1662.	504.1
Slope	b	463.6	32.31

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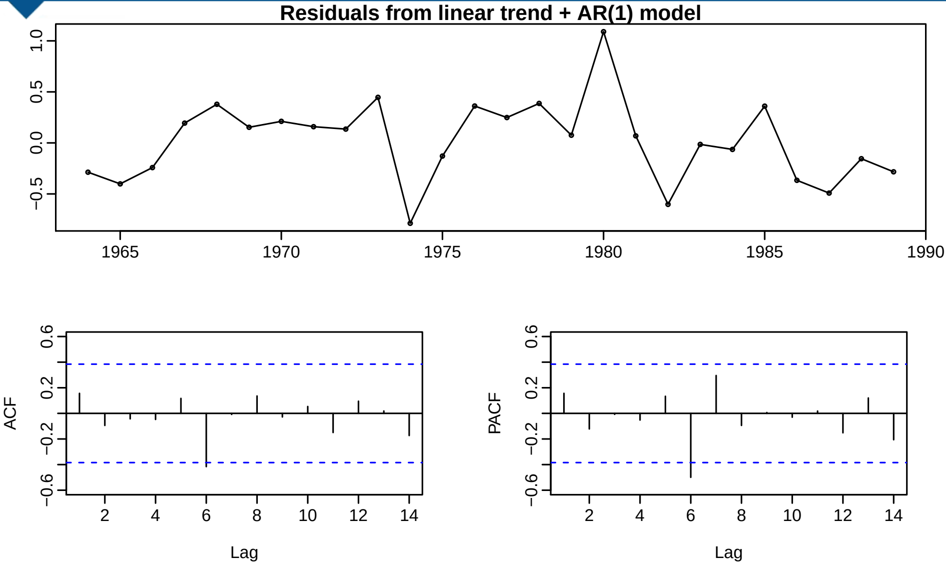

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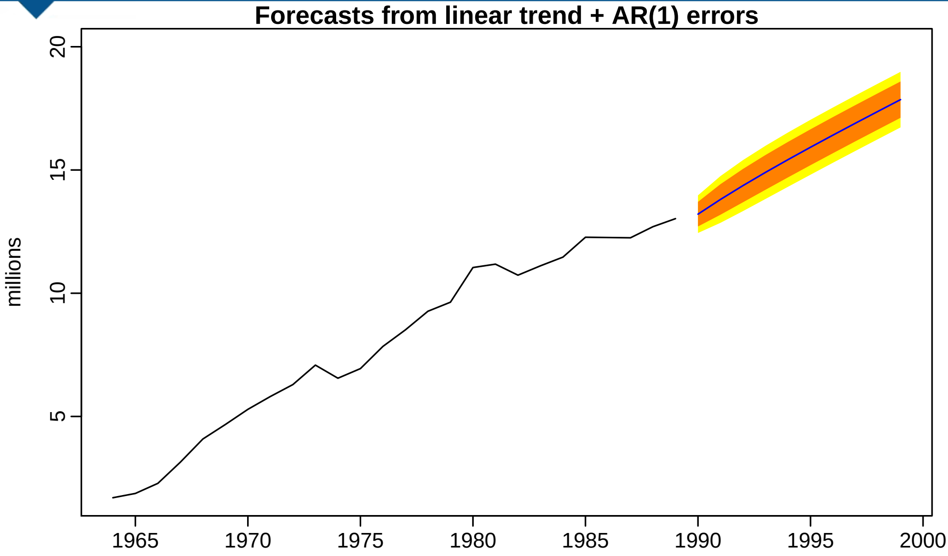

Example: Japanese cars

```
> fit2 <- auto.arima(x)
ARIMA(0,1,0) with drift

Coefficients:
       drift
      0.4530
s.e.  0.0836

sigma^2 estimated as 0.1749:
log likelihood = -13.68
AIC = 31.36   AICc = 31.9   BIC = 33.8
```

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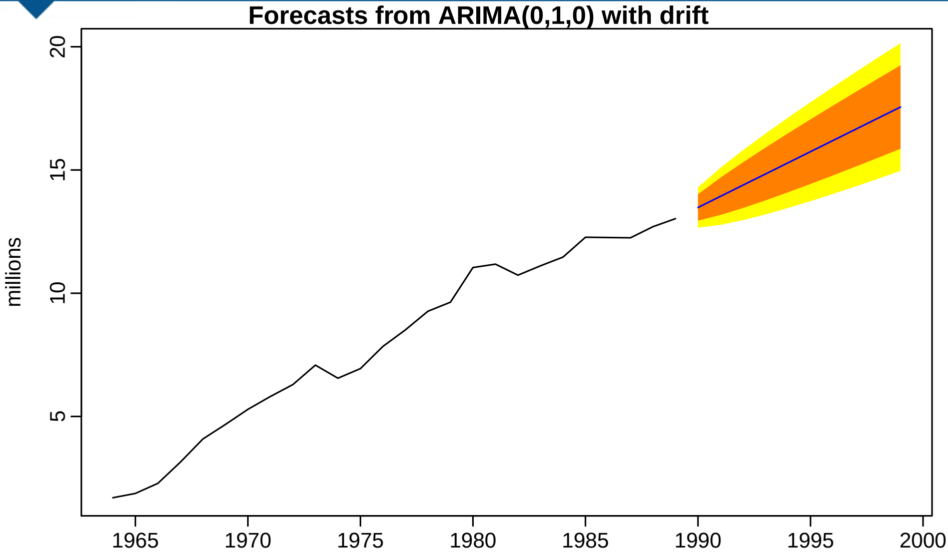

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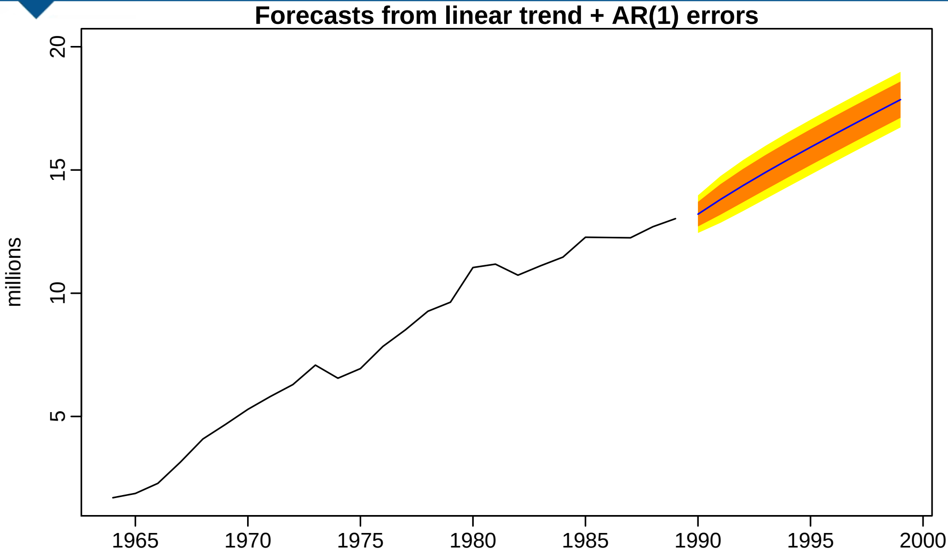

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Fourier series

Any periodic function of period m can be approximated using

$$\sum_{k=1}^{K} \left\{ \sin\left(\frac{2\pi kt}{m}\right) + \cos\left(\frac{2\pi kt}{m}\right) \right\}$$

- The approximation can be made exact with large K.
- Each k represents a harmonic.
- `fourier` generates a matrix of Fourier terms.

Example

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fit <- auto.arima(y, seasonal=FALSE,
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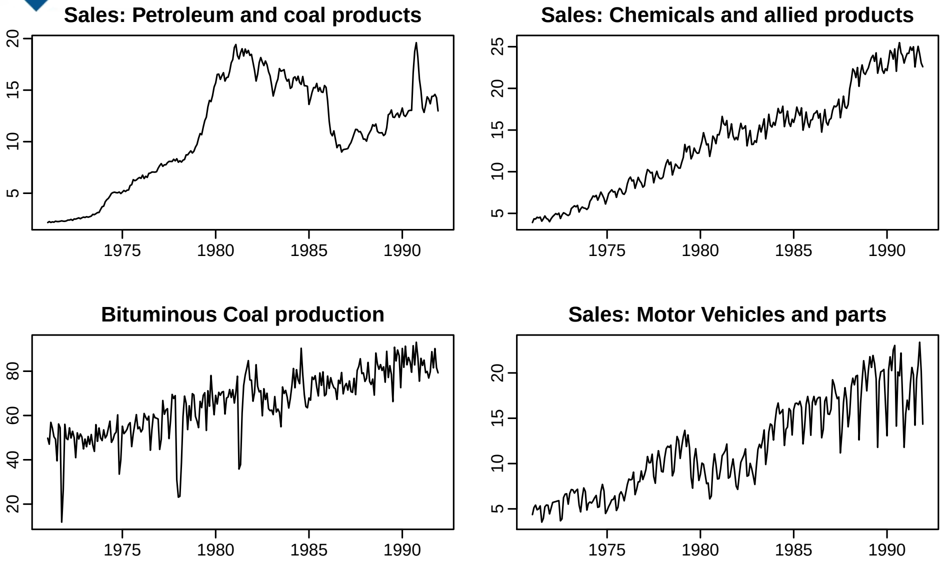

U.S. monthly sales: Jan 1971 – Dec 1991.

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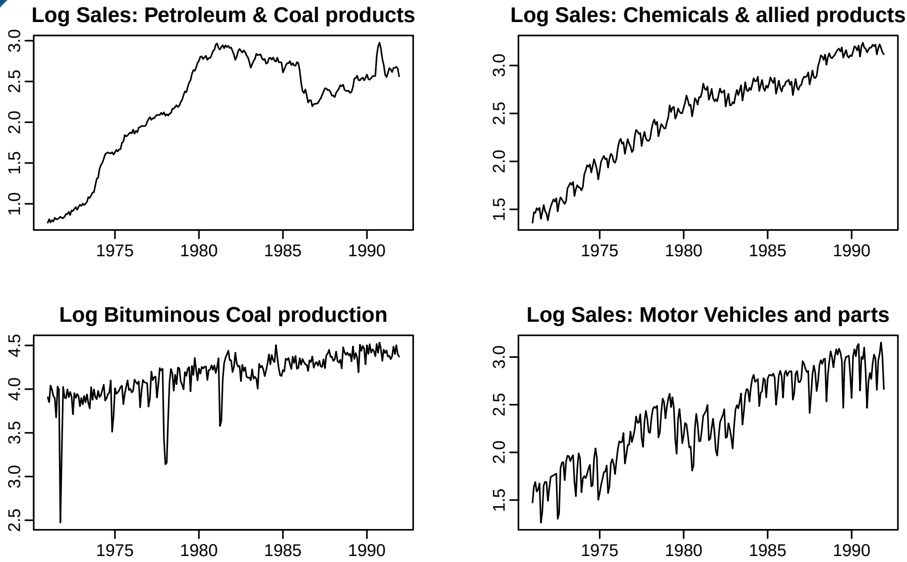

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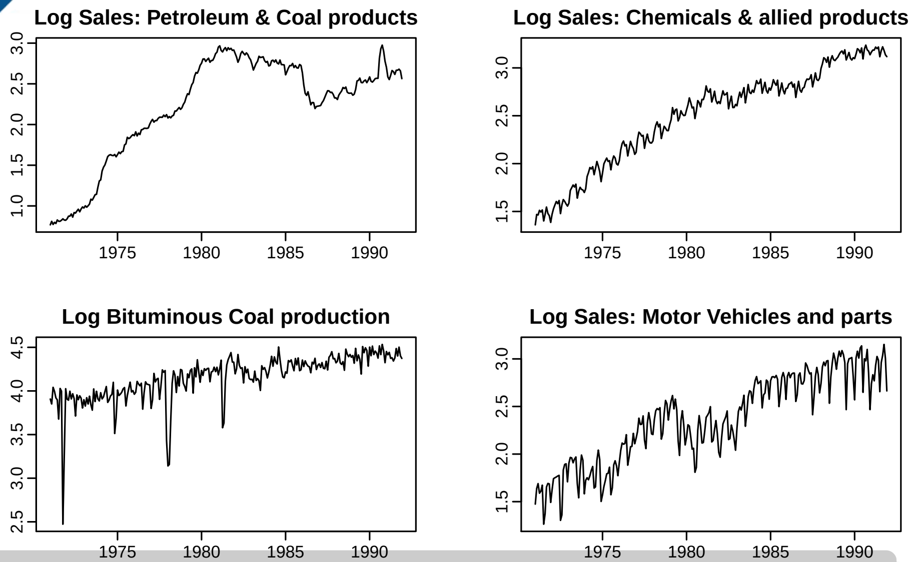

Series clearly non-stationary, so difference.

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- $y_t =$ log petroleum sales
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- `auto.arima` selects an ARIMA(0,0,1)(1,0,2)$_{12}$ model for n_t.
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Log Chemicals	b_1	0.296	0.065
Log Coal	b_2	-0.029	0.013
Log Vehicles	b_3	-0.014	0.023

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