Homework 1

2.4 Prove that if node u is distance i f rom node v in a hypercube, then there are i! different paths of length i from u to v (though some hypercube edges may appear in more than one path).

We can use mathematical induction to prove that. It's easy to see that proposition is right when i=1,2,3. Suppose when i=k-1 the proposition is right. When i=k, suppose the d_1,d_2,…,d_k digits in u are different from v. Then there are k nodes adjacent to node v each of which is only one digit different from node v and is the same as u in the digit. Hence these k nodes are distance k-1 from node u. So there are (k-1)! different paths from u to those k nodes. Sum them up we can get k*(k-1)!=k! paths. Q.E.D.

2.5 Prove that if node u is distance i f mm node v in a hypercube, then there are i paths of length i from u to v that share no edges.

Again, We can use mathematical induction to prove that. It's easy to see that proposition is right when i=1,2,3. Suppose when i=k-1 the proposition is right. When i=k, suppose the d_1,d_2,…,d_k digits in u are different from v. Then there are k nodes adjacent to node v each of which has only one digit that is different from node v and is the same as u in the digit. Hence these k nodes are distance k-1 from node u. So there are (k-1) different paths from u to each of those k nodes. However, the distance between these k

nodes and node v is only 1. There is only one path between each pair. So there are i paths of length i from u to v that share no edges. Q.E.D.

2.11

a. How many switching elements are in the network?

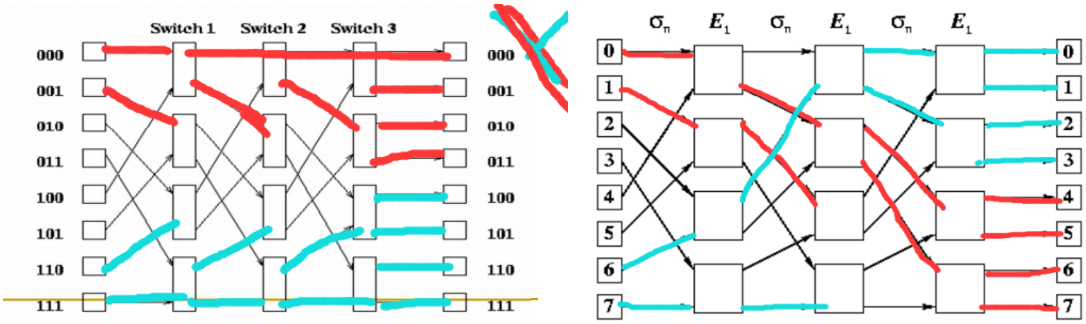
$$k * 2^{k-1} = \frac{n}{2}\log_2 n$$

b. What is the diameter of the network?

$$1 + \log_2 n$$

c. What is the bisection width of the network?

$$\frac{n}{2}\log_2 n$$



d. What is the maximum number of edges per switching node?

4.

e. Does the network have constant edge length as the number of nodes increases?

No. But it has constant edges per node.

2.14 Given a processor array containing eight processing elements, each capable of performing 10 million integer operations per second, determine the performance in millions of operations per second of this processor array adding two integer vectors, for all vector sizes from 1 to 50.

For any two vectors with size s, we need $0.1\mu s * \lceil s/8 \rceil$ to add them.

$$\text{Performance} = \frac{1\text{operations}}{0.1\mu s * \lceil s/8 \rceil} = \frac{10}{\lceil s/8 \rceil}\text{operations/second}$$

2.18 A directory-based protocol is a popular way to implement cache coherence on a distributed multiprocessor.

a. Why should the directory be distributed among the multiprocessor's local memories?

To prevent accesses to the cache directory from becoming a performance bottleneck, the directory itself should be distributed among the computer's local memories. Besides, when a directory of a processor is broken, directories of other processors can continue to be used.

b. Why are the contents of the directory not replicated?

The directory information about a particular memory block is in exactly one location. Otherwise, we still have to implement the directory coherence, which is absurd as it causes more network overhead.